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Perturbed Random Differential Inclusions with Integral Boundary Conditions

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Abstract: In this article, we prove existence of random solution and extrtemal random solution for perturbed boundary value problem of random differtential inclusions with integral boundary condtions under mixed generalized Lipschitz and caratheodory condtions.
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I. STATEMENT OF THE PROBLEM

Consider the perturbed second order random nonlinear boundary value problem with integral boundary conditions

$$x''(t,\omega)\in F(t,\, x(t,\omega)\,\omega)+G(t,\, x(t,\omega)\,\omega)\ \text{ for a.e. } t\in[0,1], \qquad (1.1)$$

$$x(0,\omega)-k_1\,x'(0,\omega)=\int_0^1 h_1(x(s,\omega))\,ds, \qquad (1.2)$$

$$x(1,\omega)+k_2\,x'(1,\omega)=\int_0^1 h_2(x(s,\omega))\,ds, \qquad (1.3)$$

Where $\omega\in\Omega$, $F,G:[0,1]\times R\times\Omega\longrightarrow P(R)$ are compact-valued and multivalued maps, $P(R)$ is the family of all nonempty subsets of R, $h_i: R\times\Omega\longrightarrow R$ are continuous functions and k_i are nonnegative constants (i = 1, 2). Boundary value problems with integral boundary conditions form a very interesting and important class of problems. For boundary value problems with integral boundary conditions, we refer the papers of Brykalov[3] , Denche and Marhoune[5] , Gallardo[10], Lomtatidze and Malaguti[14].Recently, Belarbi and Benchora[2] considered the particular problem (1.1)-(1.3) with indeterministic functional differential inclusions G=0 and obtained existence results when the right hand side has convex as well as non-convex values. The present paper is motivated by Dhage, Gatsori and Ntouyas [6] deterministic functional differential inclusions which the existence of solution for first order perturbed functional differential inclusions. We prove the existence result for the problem (1.1)–(1.3) under the mixed generalized Lipschitz and caratheodory conditions for second order perturbed random differential inclusions.

II. AUXILIARY RESUTS

We introduce the notation, definitions, and preliminary facts from multivalued analysis .
Let $C([0,1],R)$ is the Banach space of all continuous functions from $[0,1]\longrightarrow R$ with the norm

$$\|x\|_\infty=\sup\{|x(t)|: 0\le t\le 1\}.$$

Let
$L^1([0,1],R)$ denotes the Banach space of measurable functions $x:(0,1)\longrightarrow R$ which are Lebesgue integrable normed by

$$\|x\|_{L^1}=\int_0^1|x(t)|\,dt \ \text{ for all } x\in L^1([0,1],R).$$

$AC^1([0,1],R)$ is the space of differentiable functions $x:(0,1)\longrightarrow R$ whosefirst derivative x' is absolutely continuous.

1) *Definition 2.1.* A multivalued map $F: [0,1]\times R\times\Omega\longrightarrow P(R)$ is said to be random carathéodory if
a) $(t,\omega)\mapsto F(t,y,\omega)$ is measurable for each $y\in R$, $\omega\in\Omega$ and
b) $(y,\omega)\mapsto F(t,y,\omega)$ is upper semi-continuous for a.e. $t\in[0,1]$ and. $\omega\in\Omega$.

For each $x \in C([0,1],\Omega,R)$, define the set of selections of F by

$$S_F(x,\omega)=\{v\in L^1([0,1],\Omega,R) : v(t,\omega)\in F(t,x(t,\omega),\omega) \text{ for a.e.} t\in[0,1],\omega\in\Omega\}$$

Let $F:[0,1]\times\Omega\times R\to P(R)$ be a mult-ivalued map with nonempty compact values. Assign to F the multi-valued operator $F:C([0,1],\Omega,R\to P(L^1)([0,1],\Omega,R))$

by letting

$$F(x,\omega)=\{w\in L^1([0,1],\Omega,R): w(t,\omega)\in F(t,x(t,\omega),\omega) \text{ for a.e.} t\in[0,1]\omega\in\Omega\}.$$

The operator F is called the Nemytskij operator associated with F.

2) *Definition 2.2.* A multi-valued operator $N:\Omega\times X\to P_{cl}(X)$ is called

a) γ -Lipschitz if and only if there exists $\gamma:\Omega\times R^+\to R^+$, $\gamma(\omega)>0$ such that

$$H_d(N(\omega)x, N(\omega)y)\le \gamma(\omega)d(x,y), \text{ for each } x,y\in X,\ \omega\in\Omega .$$

b) a contraction if and only if it is γ -Lipschitz with $\gamma(\omega)<1$.

We apply the following form of the fixed point theorem of Dhage [8] in the sequel.

3) *Theorem 2.1.*Let $B(0,r)$ and $B[0,r]$ denote respectively the open and the closed ball in a Banach space E centered at the origin and of radius r and let $A:\Omega\times B[0,r]\to P_{cl,cv,bd}(E)$ and $B:\Omega\times B[0,r]\to P_{cp,cv}(E)$ be two multi-valued operators satisfying

a) $A(\omega)$ is a multi-valued contraction, and

b) $B(\omega)$ is compact and upper semi-continuous.

Then either the operator inclusion $x\in A(\omega)x+B(\omega)x$ has a solution in $B[0,r]$ or there exists an u∈ E with||u|| = r such that $\lambda(\omega)u\in A(\omega)u+B(\omega)u$ for some $\lambda(\omega)>1$.

The following lemma will be used ..

Lemma 2.1 .Let X be a Banach space. Let $F:[0,1]\times X\to P_{cp,cv}(X)$ be an L^1-Carathéodory mult-ivalued map with $S_{F,x}\neq\phi$ and let Γ be a linear continuous mapping from $L^1([0,1],X)$ to $C([0,1],X)$,then the operator

$$\Gamma\circ S_F : C([0,1],X)\to P_{cp,cv}(C([0,1],X)),$$
$$x\mapsto(\Gamma\circ S_F)(x):=\Gamma(S_{F,x})$$

is a closed graph operator in C([0,1], X) x C([0,1], X).

III. EXISTENCE RESULTS

We are concerned with the existence of solutions for the problem (1.1)–(1.3). We need the following results.

Definition3.1. A function $x\in AC^1((0,1),\Omega,R)$ is said to be a solution of (1.1)–(1.3) if there exist functions $v_1,v_2\in L^1([0,1],\Omega,R)$ with $v_1(t,\omega)\in F(t,x(t,\omega),\omega)$ for a.e. t ∈[0,1] , $\omega\in\Omega$ and $v_2(t,\omega)\in G(t,x(t,\omega),\omega)$ t)) for a.e. t ∈[0,1] , $\omega\in\Omega$ such that $x''(t,\omega)=v_1(t,\omega)+v_2(t,\omega)$ for a.e. t ∈ [0, 1] , $\omega\in\Omega$ and the function x satisfies the conditions (1.2) and (1.3).

Lemma 3.1.For any $\sigma(t),\rho_1(t),\rho_2(t)\in L^1([0,1],R)$, the non-homogeneous linear problem

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$$x''(t) = \sigma(t)\ for a.e. t \in [0,1],$$
$$x(0) - k_1 x'(0) = \int_0^1 \rho_1(s)ds,$$
$$x(1) - k_2 x'(1) = \int_0^1 \rho_2(s)ds,$$

has a unique solution $x \in AC^1((0,1),R)$,

$$x(t) = P(t) + \int_0^1 H(t,s)\sigma(s)ds,$$

where

$$P(t) = \frac{1}{1+k_1+k_2}\left\{(1-t+k_2)\int_0^1 \rho_1(s)ds + (k_1+t)\int_0^1 \rho_2(s)ds\right\}$$

is the unique solution of the problem

$$x''(t) = 0\ for a.e. t \in [0,1],$$
$$x(0) - k_1 x'(0) = \int_0^1 p_1(s)ds,$$
$$x(1) - k_2 x'(1) = \int_0^1 p_1(s)ds,$$

and

$$H(t,s) = \frac{-1}{1+k_1+k_2}\begin{cases}(k_1+t)(1-s+k_2), 0 \le t < s \le 1,\\ (k_1+s)(1-t+k_2), 0 \le s < t \le 1,\end{cases}$$

is the Green's function of the corresponding homogeneous problem.

We transform the BVP (1.1)–(1.3) into a random fixed point problem. Consider the operator $N : C([0,1],\Omega,R) \to p(C([0,1],\Omega,R))$ defined by

$$\left\{u \in C([0,1],\Omega,R) : u(t,\omega) = P[x(t,\omega)] + \int_0^1 H(t,\omega,s)(v_1(s,\omega)+v_2(s,\omega))ds, v_1 \in S_F(x,\omega) and\ v_2 \in S_G(x,\omega)\right\}$$ where the operator $P : AC^1(J,\Omega,R) \to R$ is defined by

$$P[x(t,\omega)] = \frac{1}{1+k_1+k_2}\left\{(1-t+k_2)\int_0^1 h_1(x(s,\omega))ds + (k_1+t)\int_0^1 h_2(x(s,\omega))ds\right\}.$$

1) *Remark 3.1.* Clearly, from Lemma 3.1, the random fixed points of N are the random solutions of (1.1)–(1.3).
wes introduce the following hypotheses:

a) The function $t \to F(t, y, \omega)$ is measurable, convex-valued and integrably bounded for each $\omega \in \Omega, y \in R.$

b) $H_d(F(t,y,\omega), F(t,\bar{y},\omega)) \le l(t,\omega)|y - \bar{y}|$ for a.e. $t \in [0,1]$ and all $y, \bar{y} \in R, \omega \in \Omega$.
where $l \in L^1([0,1],\Omega,R)$ and $H_d(0, F(t,\omega,0)) \le l(t,\omega)$ for a.e. $t \in [0,1]$, $\omega \in \Omega$.

c) There exist two nonnegative constants d_1 and d_2 such that

$$|h_1(y)-h_1(\bar{y})|\le d_1|y-\bar{y}| \text{ and } |h_2(y)-h_2(\bar{y})|\le d_2|y-\bar{y}|$$

for all $y,\bar{y}\in R$.

d) The multi-valued map G(t,y,ω)) has compact and convex values for each $(t,y,\omega)\in[0,1]\times R\times\Omega\ \ \omega\in\Omega$.

e) G is random carathéodory.

f) There exist a function $q\in L^1([0,1],\Omega,R)$ with $q(t,\omega)>0$ for a.e. t$\in$ [0, 1], $\omega\in\Omega$ and a continuous non-decreasing function such that

$$\|G(t,y,\omega)\|\,p\le q(t,\omega)\Psi(|y|) \text{ for a.e.t}\in[0,1] \text{ for all } \omega\in\Omega, y\in R.$$

g) There exists a real number r > 0 such that

$$r>\frac{\frac{1}{1+k_1+k_2}\left[(1+k_2)h_1(r)+(1+k_1)h_2(r)\right]+H^*\|l\|_{L^1}+H^*\psi(r)\|q\|_{L^1}}{1-H^*\|l\|_{L^1}}$$

Where H* = $\sup_{(t,s,\omega)\in[0,1]\times[0,1]\times\Omega}|H(t,s,\omega)|$.

2) *Theorem 3.1.*Suppose that hypotheses (A1)–(A7) are satisfied. If

$$\frac{1}{1+k_1+k_2}\left[(1+k_1)d_1+(1+k_1)d_2\right]+H^*\|l\|_{L^1}<1$$

then the BVP (1.1)–(1.3) has at least one solution.

Proof. Let X =C([0; 1]; R) and define an open ball $B(0,r)$ in X centered at the origin and of radius r, where the real number r satisfies the inequality in hypothesis (A7). Define two multi-valued maps on $B[0,r]$ by

$$A(\omega)(x)=\left\{u\in X: u(t,\omega)=Px(t,\omega)+\int_0^1 H(t,s,\omega)v_1(s,\omega)ds, v_1\in S_F(x,\omega)\right\} \quad (4)$$

and

$$B(\omega)(x)=\left\{u\in X: u(t,\omega)=\int_0^1 H(t,s,\omega)v_2(s,\omega)ds, v_2\in S_G(x,\omega)\right\} \quad (5)$$

We shall show that the operators $A(\omega)$ and $B(\omega)$ satisfy all the conditions of Theorem 2.1. The proof will be given in several steps.

a) *Step 1:* First we show that $A(\omega)(x)$ is a closed convex and bounded subsetof X for each $x\in B[0,r]$. This follows easily if we show that the values of the Nemytskij operator associated are closed in $L^1([0,r],\Omega,R)$ .Let $\{w_n\}$ be a sequence in $L^1([0,r],\Omega,R)$ converging to a point w. Then $w_n\to w$ in measure and thus there exists a subset S of positive integers with $\{w_n\}$ converging a.e. to w as $n\to\infty$ through S. Now, since (A1) holds, the values of $S_F(x,\omega)$ are closed in $L^1([0,r],\Omega,R)$. Thus, for each $x\in B[0,r]$, we have that $A(\omega)(x)$ is a nonempty and closed subset of X.

We prove that $A(\omega)(x)$ is a convex subset of X for each $x\in B[0,r]$. Let $u_1, u_2\in A(\omega)(x)$. Then there exist u_1, $u_2\in S_F(x,\omega)$ such that for each $\omega\in\Omega$,t$\in$[0,1] ,we have

$$u_i(t,\omega)=Px(t,\omega)+\int_0^1 H(t,s,\omega)v_i(s,\omega)ds \quad (i=1,2).$$

Let $0\le d\le 1$. Then, for each $t\in[0,1]$, $\omega\in\Omega$ .we obtain

$$\left(du_1+(1-d)u_2\right)(t,\omega) = P\,x(t,\omega)+\int_0^1 H(t,s,\omega)\left[dv_1(s,\omega)+(1-d)v_2(s,\omega)\right]ds.$$

Since $S_F(x,\omega)$ is convex .we have

$$du_1+ (1-d)u_2 \in A(\omega)(x).$$

b) *Step 2:* We show that A is a multi-valued contraction on B[0, r]. Let $x,\overline{x}\in B[0,r]$ and $u_1 \in A(\omega)(x)$. Then there exists $v_1(t,\omega)\in F(t,x(t,\omega),\omega)$ such that for each $t\in[0,1], \omega\in\Omega$.

$$u_1(t,\omega)=Px(t,\omega)+\int_0^1 H(t,s,\omega)v_1(s,\omega)ds.$$

From (A2) it follows that

$$H_d\left(F(t,x(t,\omega),\omega),\, F((t,\overline{x}(t,\omega),\omega))\right)\le l(t,\omega)\left|x(t,\omega)-\overline{x}(t,\omega)\right|.$$

Hence there exists $w\in F\left(t,\overline{x}(t,\omega)\right)$ such that

$$\left|v_1(t,\omega)\text{-} w\right|\le l(t,\omega)\left|x(t,\omega)-\overline{x}(t,\omega)\right|, t\in[0,1], \omega\in\Omega.$$

Consider $U:[0,1]\times\Omega\to P(R)$ given by

$$U(t,\omega)=\left\{w\in R:\left|v_1(t,\omega)-w\right|\le l(t,\omega)\left|x(t,\omega)-\overline{x}(t,\omega)\right|\right\}.$$

Since the multi-valued operator $V(t,\omega)=U(t,\omega)F\left(t,\overline{x}(t,\omega)\right)$ is measurable, there exists a function $v_2(t,\omega)$ which is a measurable selection for V . So, $v_2(t,\omega)\in F(t,\overline{x}(t,\omega),\omega)$ and for each t$\in$[0, 1] , $\omega\in\Omega$.

$$\left|v_1(t,\omega)\text{-} v_2(t,\omega)\right|\le l(t,\omega)\left|x(t,\omega)-\overline{x}(t,\omega)\right|.$$

Let us define for each $t\in[0,1]$, $\omega\in\Omega$.

$$u_2(t,\omega)=P\overline{x}(t,\omega)+\int_0^1 H(t,s,\omega)v_2(s,\omega)ds$$

where

$$P\overline{x}(t,\omega)=\frac{1}{1+k_1+k_2}\left[(1-t+k_2)\int_0^1 h_1(\overline{x}(s,\omega))ds+(1+k_1)\int_0^1 h_2(\overline{x}(s,\omega))ds\right]$$

We have

$$\begin{aligned}\left|u_1(t,\omega)-u_2(t,\omega)\right| &\le \left|Px(t,\omega)-P\overline{x}(t,\omega)\right|+\int_0^1\left|H(t,s,\omega)\right|\left|v_1(s,\omega)-v_2(s,\omega)\right|ds\\ &\le\frac{1}{1+k_1+k_2}[(1+k_1)d_1+(1+k_2)d_2]\left\|x-\overline{x}\right\|_\infty\\ &+\int_0^1\left|H(t,s,\omega)\right|l(s,\omega)\left|x(s,\omega)-\overline{x}(s,\omega)\right|ds\end{aligned}$$

Thus

$$\|u_1-u_2\| \le \left(\frac{1}{1+k_1+k_2}\left[(1+k_1)d_1+(1+k_2)d_2\right]+H^*\|l\|_{L^1}\right)\|x-\overline{x}\|_\infty$$

From an analogous relation obtained by interchanging x and $\overline{x}$, it follows that on

$$H_d(A(\omega)(x), A(\omega)(\overline{x})) \le \left(\frac{1}{1+k_1+k_2}\left[(1+k_1)d_1+(1+k_2)d_2\right]+H^*\|l\|_{L^1}\right)\|x-\overline{x}\|_\infty .$$

So, $A(\omega)$ is a multi-valued random contraction X.

c) *Step 3:* Now we show that the multi-valued random operator $B(\omega)$ is compact and upper semi-continuous on $B[0,r]$. First we show that $B(\omega)$ is compact on $B[0,r]$. Let $x \in B[0,r]$ be arbitrary. Then for each $u \in B(\omega)(x)$ there exists $v \in S_G(\omega, x)$ such that for each t∈[0, 1], $\omega \in \Omega$ .we have

$$u(t,\omega)=\int_0^1 H(t,s,\omega)v(s,\omega)ds.$$

From (A5) we have

$$|u(t,\omega)|=\int_0^1 |H(t,s,\omega)||v(s,\omega)|ds \le H^*\int_0^1 |v(s,\omega)|ds.$$

$$|u(t,\omega)|=\int_0^1 |H(t,s,\omega)||v(s,\omega)|ds \le H^*\int_0^1 |v(s,\omega)|ds.$$

$$\le H^*\int_0^1 q(s,\omega)\Psi(\|x\|_\infty)ds \le H^*\|q\|_{L^1}\Psi(r).$$

Next, we show that B maps the bounded sets into the equi-continuous sets of X. Let t,τ ∈[0,1],and $x \in B[0,r]$. For each $u \in B(\omega)(x)$.

$$|u(t,\omega)-u(\tau,\omega)| \le \int_0^1 |H(t,s,\omega)-H(\tau,\omega)||v(s,\omega)|ds$$

$$\le \int_0^1 |H(t,s,\omega)-H(\tau,\omega)|q(t,\omega)\psi(\|x\|_\infty)ds$$

$$\le \int_0^1 |H(t,s,\omega)-H(\tau,s,\omega)|q(s,\omega)\psi(r)ds.$$

The right-hand side tends to zero as $|t-\tau| \to 0$ .An application of the Arzel´a– Ascoli theorem implies that the operator $B: B[0,r]\times\Omega \to P(X)$ is compact.

d) *Step 4:* Next we prove that $B(\omega)$ has a closed graph. Let $(x_n,\omega) \to (x^*,\omega)$,, $(y_n,\omega) \in B(\omega)(y_n)$.
Andy $(y_n,\omega) \to (y^*,\omega)$ .We need to show that $(y^*,\omega) \in B(x^*,\omega)$.
$(y_n,\omega) \in B(x_n,\omega)$ means that there exists $v_n \in S_G(\omega,x)$ such that for each t ∈[0,1] and $\omega \in \Omega$.

$$y_n(t,\omega)=\int_0^1 H(t,s,\omega)v_n(s,\omega)ds.$$

We must show that there exists $y_n \in S_G(\omega,x)$ such that for each $t \in [0,1]$, $\omega \in \Omega$.

$$y_*(t,\omega)=\int_0^1 H(t,s,\omega)v_*(s,\omega)ds.$$

Clearly, we have

$$\|(y_n,\omega)-(y_*,\omega)\|_\infty \to 0 \text{ as } n\to\infty.$$

Consider the continuous linear operator $\Gamma : L^1([0,1],\Omega,R)\to X$
given by

$$v\mapsto(\Gamma v)(t,\omega)=\int_0^1 H(t,s,\omega)v(s,\omega)ds$$

From Lemma 2.1 it follows that $\Gamma\circ S_F$ is a closed graph operator. Moreover, we have

$$y_n(t,\omega)\in\Gamma(S_G(\omega,x_n).\ y_n(t,\omega)\in\Gamma(S_G(x_n,\omega).$$

Since $(x_n,\omega)\to(x^*,\omega)$ it follows from Lemma 2.1 that

$$y_*(t,\omega)=\int_0^1 H(t,s,\omega)v_*(s,\omega)ds$$

for some $v_*\in S_G(x_*,\omega)$.

e) *Step5:* Now we show that the second assertion of Theorem 2.1 is not true.Let $u\in X$ be a possible solution of $\lambda(\omega)u\in A(\omega)(u)+B(\omega)(u)$ for some real number, $\lambda(\omega)>1$ with $\|u\|_\infty=r$. Then there exist $v_1\in S_F(u,\omega)$ and $v_2\in S_G(u,\omega)$ such that for each $t\in[0,1]$, $\omega\in\Omega$ .we have

$$u(t,\omega)=\lambda^{-1}Px(t,\omega)+\lambda^{-1}\int_0^1 H(t,s,\omega)v_1(s,\omega)ds+\lambda^{-1}\int_0^1 H(t,s,\omega)v_2(s,\omega)ds.$$

Then

$$|u(t,\omega)|\le|P\,x(t,\omega)|+H^*\int_0^1|v_1(s,\omega)|ds+H^*\int_0^1|v_2(s,\omega)|ds$$

$$\le\frac{1}{1+k_1+k_2}\left[(1+k_2)\int_0^1 h_1(u(s,\omega))ds+(1+k_1)\int_0^1 h_2(u(s,\omega))ds\right]$$

$$+H^*\int_0^1(l(s,\omega)|u(s,\omega)|+l(s,\omega))ds+H^*\int_0^1 q(s,\omega)\psi\left(|u(s,\omega)|\right)ds$$

$$\le\frac{1}{1+k_1+k_2}\left[(1+k_2)\int_0^1 h_1\left(\|u\|_\infty\right)ds+(1+k_1)\int_0^1 h_2\left(\|u\|_\infty\right)ds\right]$$

$$+H^*\int_0^1(l(s,\omega)\|u\|_\infty+l(s,\omega))ds+H^*\int_0^1 q(s,\omega)\psi\left(\|u\|_\infty\right)ds.$$

Taking the supremum over t , ω .we get

$$\|u\|_\infty\le\frac{1}{1+k_1+k_2}\left[(1+k_2)\int_0^1 h_1\left(\|u\|_\infty\right)ds+(1+k_1)\int_0^1 h_2\left(\|u\|_\infty\right)ds\right]$$

$$+H^*\int_0^1(l(s,\omega)\|u\|_\infty+l(s,\omega))ds+H^*\int_0^1 q(s,\omega)\psi\left(\|u\|_\infty\right)ds.$$

Substituting $\|u\|_\infty = r$ into the above inequality yields

$$r \leq \frac{\frac{1}{1+k_1+k_2}\left[\left(1+k_2\right)h_1\left(r\right)+\left(1+k_1\right)h_2\left(r\right)\right]+H^*\|l\|_{L^1}+H^*\psi(r)\|q\|_{L^1}}{1-H^*\|l\|_{L^1}}$$

which is a contradiction to (A7). As a result, the conclusion (ii) of Theorem 2.1 does not hold. Hence the conclusion (i) holds and consequently the BVP(1.1)–(1.3) has a solution x on [0, 1]. This completes the proof.

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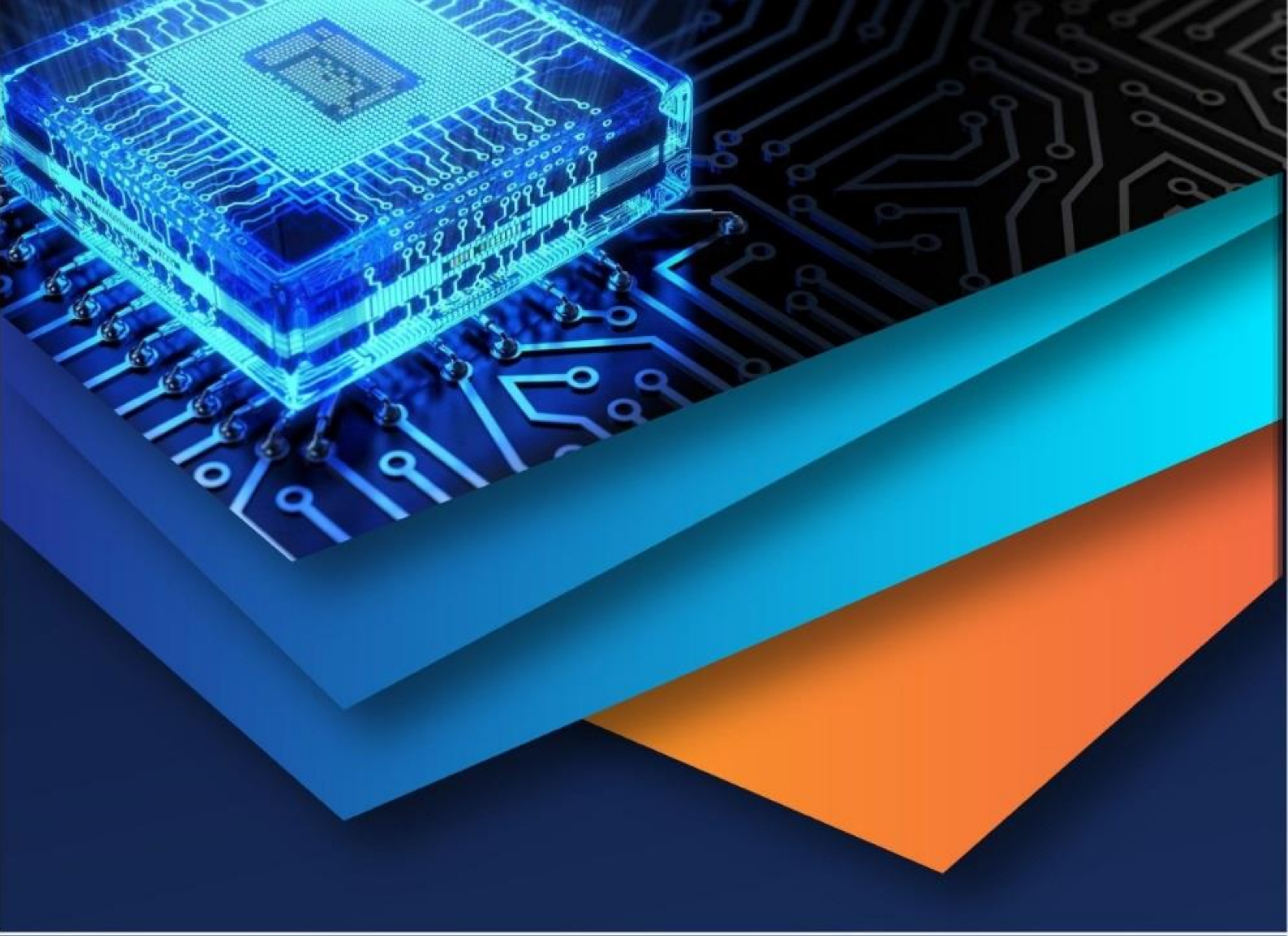

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