iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 **Issue:** XII **Month of publication:** December 2018

DOI:

Boundary Value Problem of First Order Random Differential Inclusions

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Abstract: In this paper, investigate the nonlinear boundary value problem of first order random differential inclusions and prove the existence results through classical fixed point theorem.
Keywords: Multivalued map, random differential inclusions, Nonlinear boundary conditions, Condensing map, random fixed point, Upper and lower random solutions. 2000 Mathematics Subject Classification: 60H25, 47H10, 34A60.

I. DESCRIPTION OF THE PROBLEM

Consider the first order random differential inclusions with boundary condtions

$$y'(t,\omega) \in F(t,\omega,y(t,\omega)),\ for\ a.e. t \in J=[0,T], \omega \in \Omega$$
$$L(y(0,\omega), y(T,\omega)) = 0 \tag{1.1}$$

Where $F: J \times \Omega \times R \to 2^R$ is a compact and convex valued multivalued map and $L: R^2 \times \Omega \to R$ is a continuous single-valued map.

The method of upper and lower solutions has been successfully applied to study the existence of multiple solutions for initial and boundary value problems of first and second order differential inclusions. This method has been used in Bernfeld-Lakshmikantham [10], Heikkila-Laksmikanthan [8], Carl-Heikkila-Kumpulainen [5], Cabada [4], Frigon [7], Lakshmikanthan-Leela [17], Nkashama [13].

The problem(1.1)is new to the differential inclusions. In this paper, we establish an existence result for (1.1) through the existence of upper and lower random solutions and random fixed point theorem for condensing maps of Martelli[19].

II. AUXILIARY RESULTS

We will need some basic definitions and Lemmas from multivalued analysis.

1) *Definition 2.1.*A multivalued map $F: J \times R \times \Omega \to 2^R$ is said to be an L^1- random caratheodory , if
a) $t \to F(t,y,\omega)$ is measurable for each $y \in R, \omega \in \Omega$.
b) $y \to F(t,y,\omega)$ is upper semicontinuous for almost all $t \in J$, $\omega \in \Omega$.
c) For each $k > 0$, there exists $h_k \in L^1(J, R_+, \Omega)$ such that

$\| F(t,y,\omega) \| = \sup\{|v| : v \in F(t,y,\omega)\} \le h_k(t,\omega)$ for all $|y| \le k$ and for almost all $\omega \in \Omega$, $t \in J$.

So let us start by defining what we mean by a solution of problem (1.1).

2) *Definition 2.2.* A function $y \in AC(J,R,\Omega)$ is said to be a solution of (1.1)) if there exists a function $v \in L^1(J,R,\Omega)$ such that $v(t,\omega) \in F(t, y(t,\omega),\omega)$ a.e. on $J_1 y'(t,\omega) = v(t,\omega)$ a.e. on J, $\omega \in \Omega$ and $L(y(0,\omega), y(T,\omega)) = 0$.

Thefollowing concept of lower and upper solutions for (1.1) has been introduced by Halidias and Papageorgiou in [14] for second order multivalued boundary value problems.

3) *Definition 2.3.* A function $\alpha \in AC(J,R,\Omega)$ is said to be a lower solution of (1.1) if there exists $v_1 \in L^1(J,R,\Omega)$ such that $v_1(t,\omega) \in F(t,\alpha(t,\omega),\omega)$ on $J, \alpha'(t,\omega) \le v_1(t,\omega)$ a.e. on J, $\omega \in \Omega$ and $L(\alpha(0,\omega),\alpha(T,\omega)) \le 0$.

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue XII, Dec 2018- Available at www.ijraset.com

Similarly, a function $\beta \in AC(J,R,\Omega)$ is said to be upper solution of (1.1) if there exists $v_2 \in L^1(J,R,\Omega)$ such that $v_2(t,\omega) \in F(t,\beta(t,\omega),\omega)$ a.e. on J, $\beta'(t,\omega) \geq v_2(t,\omega)$ a.e. on J, $\omega \in \Omega$ and $L(\beta(0,\omega),\beta(T,\omega)) \geq 0$.

For the multivalued map F and for each $y \in C(J,R,\Omega)$ we define $S\dfrac{1}{F,y}$ by

$$S\frac{1}{F,y} = \{v \in L^1(J,R,\Omega) : v(t,\omega) \in F(t,y(t,\omega),\omega) \text{ for a.e. } t \in J, \omega \in \Omega\}.$$

Our main result is based on the following :

a) *Lemma 2.1.* [18]. Let I be a compact real interval and X be a Banch space. Let $F : I \times X \to CC(X); (t,y) \to F(t,y)$ measurable with respect to t for any $y \in X$ and u.s.c. with respect to y for almost each $t \in I$ and $S\dfrac{1}{F,y} \neq \emptyset$ for any $y \in C(I,X)$ and let Γ be a linear continuous mapping from $L^1(I,X)$ to $C(I,X)$ then the operator

$$\Gamma \circ S\frac{1}{F} : C(I,X) \to CC(C(I,X)), y1 \to (\Gamma \circ S\frac{1}{F})(y) := \Gamma\left(S\frac{1}{F,y}\right)$$

is a closed graph operator in $C(I,X) \times C(I,X)$.

b) *Lemma 2.2.* [19] Let $G : X \to CC(X)$ be anu.s.c. condensing map. If the set

$$M := \{v \in X : \lambda v \in G(v) \text{ for some } \lambda > 1\}$$

is bounded, then G has a fixed point.

III. MAIN RESULT

We are now in a position to state and prove our existence result for the problem (1.1).

A. *Theorem 3.1*

Suppose $F : J \times R \times \Omega \to CC(R)$ is an L^1-random Caratheodory multivalued map.

And assume the following conditions.

there exist α and β in $W^{1,1}(J,R,\Omega)$ lower and upper random solutions respectively for the problem (1.1) such that, $\alpha \leq \beta$,

L is a continuous single-valued map in $(x,y) \in [a(0),\beta(0)] \times [a(T),\beta(T)]$ and nonincreasing in $y \in [\alpha(T),\beta(T)]$

are satisfied. Then the problem (1.1) has at least one random solution $y \in W^{1,1}(J,R,\Omega)$ such that

$$a(t,\omega) \leq y(t,\omega) \leq \beta(t,\omega) \text{ for all } t \in J, \omega \in \Omega.$$

Proof. Transform the problem into a fixed point problem. Consider the following modified problem

$$y'(t,\omega) + y(t,\omega) \in F_1(t,y(t,\omega),\omega), \text{ a.e. } t \in J. \quad (3.1)$$

$$y(0,\omega) = \tau((0,y(0,\omega),\omega) - L(\bar{y}(0,\omega),\bar{y}(T,\omega))) \quad (3.2)$$

Where $\omega \in \Omega$, $F_1(t,y,\omega) = F(t,\omega,\tau(t,y,\omega)) + \tau(t,y,\omega) = \max\{a(t,\omega),\min\{y,\beta(t,\omega)\}\}$ and $\bar{y}(t,\omega) = \tau(t,\omega,y(t,\omega))$.

Remark(i) Notice that F_1 is an L^1-random Caratheodory multivalued map with compact convex values and there exists $\emptyset \in L^1(J,R_+,\Omega)$ such that

$\| F_1(t, y(t,\omega),\omega) \| \leq \emptyset(t,\omega) + \max(\sup_{t\in J} |a(t,\omega)|, \sup_{t\in J} |\beta(t,\omega)|)$ and all $y \in C(J,R,\Omega)$.

By the definition of τ it is clear that $\alpha(0,\omega) \leq y(0,\omega) \leq \beta(0,\omega)$.

Clearly a solution to (3.1) – (3.2) is a fixed point of the operator $N : C(J,R,\Omega) \to 2^{C(J,R,\Omega)}$ defined by

$$N(y) := \left\{ h \in C(J,R,\Omega) : h(t,\omega) = y(0,\omega) + \int_0^t [v(s,\omega) + \overline{y}(s,\omega) - y(s,\omega)]ds, v \in \overset{-1}{S} \frac{1}{F,y} \right\}$$

Where

$$\overset{-1}{S}_{F,\overline{y}} = \{ v \in S^1_{F.\overline{y}} : v(t,\omega) \geq v_1(t,\omega) \text{ a.e. on } A_1 \text{ and } v(t,\omega) \leq v_2(t,\omega) \text{ a.e. on } A_2 \}$$

$$S^1_{F,\overline{y}} = \{ v \in L^1(J,R,\Omega) : v(t,\omega) \in F(t,\overline{y}(t,\omega),\omega) \text{ for a.e. } t \in J \},$$

$$A_1 = \{ t \in J : y(t,\omega) < \alpha(t,\omega) \leq \beta(t,\omega) \},\ A_2 = \{ t \in J : a(t,\omega) \leq \beta(t,\omega) < y(t,\omega) \}.$$

Since (i) For each $y \in C(J,R,\Omega)$. the set $S^1_{F,y}$ is nonempty..

For each $y \in C(J,R,\Omega)$. the set $S^1_{F,\overline{y}}$ is nonempty. Indeed, by (i) there exists $v \in S^1_{F,y.}$ Set

$$w = v_1\chi_{A_1} + v_2\chi_{A_2} + v\chi_{A_3},$$

Where

$$A_3 = \{ t \in J : \alpha(t,\omega) \leq \beta(t,\omega) \}$$

Then by decomposability $w \in S^1_{F,\overline{y}}$

We shall show that N is a completey continuous multivalued map, u.s.c. with convex closed values. The proof will be given in several steps.

1) *Step 1 :* $N(y)$ is convex for each $y \in C(J,R,\Omega)$.

Indeed, if $h, \overline{h}$ belong to $N(y)$, then there exist $v \in S^1_{F,\overline{y}}$ and $\overline{v} \in S^1_{F,\overline{y}}$ such that

$$h(t,\omega) = y(0,\omega) + \int_0^t [v(s,\omega) + \overline{y}(s,\omega) - y(s,\omega)]ds, \quad t \in J,\ \omega \in \Omega$$

And

$$\overline{h}(t,\omega) = y(0,\omega) + \int_0^t [\overline{v}(s,\omega) + \overline{y}(s,\omega) - y(s,\omega)]ds \quad ,t \in J, \omega \in \Omega.$$

Let $0 \leq k \leq 1$. Then for each $t \in J$, $\omega \in \Omega$. we have

$$\left[kh + (1-k)\overline{h} \right](t,\omega) = y(0,\omega) + \int_0^t \left[kv(s,\omega) + (1-k)\overline{v}(s,\omega) + \overline{y}(s,\omega) - y(s,\omega) \right] ds.$$

Since $S^1_{F,\overline{y}}$ is convex (because F has convex values) then

$$kh + (1-k)\overline{h} \in G(y).$$

2) *Step 2:* N sends bounded sets into bounded sets in $C(J,R,\Omega)$

Let $B_r := \{ y \in C(J,R,\Omega) : \| y \|_\infty \leq r \}$, $(\| y \|_\infty := \sup\{ | y(t,\omega) | : t \in J, \omega \in \Omega \})$ be a bounded set in

$C(J,R,\Omega)$ and $y \in B_r$, then for each $h \in N(y)$ there exists $v \in S^1_{F,\overline{y}}$ such that

$$h(t,\omega)=y(0,\omega)+\int_0^t[v(s,\omega)+\bar{y}(s,\omega)-y(s,\omega)]ds, t\in J,\omega\in\Omega.$$

Thus for each $t\in J$ we get

$$|h(t,\omega)|\le|y(0,\omega)|+\int_0^t[|v(s,\omega)|+|\bar{y}(s,\omega)|+|y(s,\omega)|]ds$$
$$\le\max(\alpha(0,\omega),\beta(0,\omega))+\|\phi_r\|_{L^1}+T\max(\gamma,\sup_{t\in J}|\alpha(t,\omega)\sup_{t\in J}|\beta(t,\omega)|)+Tr.$$

3) *Step 3:* N sends bounded sets in $C(J,R,\Omega)$ into equicontinuous sets.

Let $u_1,u_2\in J,u_1<u_2,B_r:=\{y\in C(J,R,\Omega):\|y\|_\infty\le r\}$ be a bounded set in $C(J,R,\Omega)$ and $y\in B_r$. For each $h\in N(y)$ there exists $v\in S^1_{F,\bar{y}}$ such that

$$h(t,\omega)=y(0,\omega)+\int_0^t[v(s,\omega)+\bar{y}(s,\omega)-y(s,\omega)]ds \quad ,t\in J,\omega\in\Omega.$$

We then have

$$|h(u_2)-h(u_1)\le\int_{u_1}^{u_2}[|v(s,\omega)+\bar{y}(s,\omega)|+|y(s,\omega)|]ds$$
$$\le\int_{u_1}^{u_2}|\phi_r(s,\omega)|ds+(u_2-u_1)\max(r,\sup_{t\in J}|\alpha(t,\omega))|,\sup_{t\in J}|\beta(t,\omega)|)+r(u_2-u_1)$$

As a consequence of Step2, Step 3 together with the Ascoli-Arzela theorem we can conclude that $N:C(J,R,\Omega)\to 2^{C(J,R,\Omega)}$ is a compact multivalued map, and therefore, a condensing map.

4) *Step.4:* N has a closed graph.

Let $y_n\to y_0,h_n\in N(y_n)$ and $h_n\to h_0$. We shall prove that $h_0\in N(y_0)$.

$h_n\in N(y_n)$ means that there exists $v_n\in S_{F,\bar{y}_n}$ such that

$$h_n(t,\omega)=y(0,\omega)+\int_0^t[v_n(s,\omega)+\bar{y}_n(s,\omega)-y_n(s,\omega)]ds, t\in J,\omega\in\Omega.$$

We must prove that there exists $v_0\in S^1_{F,\bar{y}_0}$ such that

$$h_0(t,\omega)=y(0,\omega)+\int_0^t[v_0(s,\omega)+\bar{y}_0(s,\omega)-y_0(s,\omega)]ds, t\in J,\omega\in\Omega.$$

Consider the linear constinuous operator $\Gamma:L^1(J,R,\Omega)\to C(J,R,\Omega)$ defined by

$$(\Gamma v)(t,\omega)=\int_0^t v(s,\omega)ds.$$

We have

$$\left\|\left(h_n-y(0,\omega)-\int_0^t[\bar{y}_n(s,\omega)-y_n(s,\omega)]ds\right)-\left(h_0-y(0,\omega)+\int_0^t[\bar{y}_0(s,\omega)-y_0(s,\omega)ds\right)\right\|_\infty\to 0.$$

From Lemma 2.1, it follows that $\Gamma \circ \overset{-1}{S}_{F}$ is a closed graph operator.

Also from the definition of Γ we have that

$$\left(h_n(t,\omega) - y(0,\omega) - \int_0^t [\bar{y}_n(s,\omega) - y_0(s,\omega)]ds \right) \in \Gamma\left(\overset{-1}{S}_{F,\bar{y}_n} \right)$$

Since $y_n \to y_0$ it follows from Lemma 2.1 that

$$h_0(t,\omega) = y(0,\omega) + \int_0^t [v_0(s,\omega) + \bar{y}_0(s,\omega) - y_0(s,\omega)]ds\,, t \in J\,, \omega \in \Omega.$$

For some $v_0 \in \overset{1}{S}_{F,\bar{y}_0}$

Next we shall show that N has a fixed point, by proving that

5) *Step 5:* The set $M := \{v \in C(J,R,\Omega) : \lambda v \in N(v)$, for some $\omega \in \Omega\,, \lambda > 1\}$

is bounded. Let $y \in M$ then $\lambda y \in N(y)$ for some $\lambda > 1$. Thus there exists $v \in \overset{1}{S}_{F,\bar{y}}$ such that

$$y(t,\omega) = \lambda^{-1} y(0,\omega) + \lambda^{-1} \int_0^t [v(s,\omega) + \bar{y}(s,\omega) - y(s,\omega)ds,\ t \in J.$$

Thus

$$| y(t,\omega) | \leq | y(0,\omega) | + \int_0^t | v(s,\omega) + \bar{y}(s,\omega) - y(s,\omega) | \, ds, \ \ ,\omega \in \Omega \ ,t \in J.$$

From the definition of τ there exists $\emptyset \in L^1(J, R^+ , \Omega^+)$ such that

$$\| F(t, \bar{y}(t,\omega),\omega) \| = \sup\{| v | : v \in F(t, \bar{y}(t,\omega),\omega)\} \leq \emptyset(t,\omega) \text{ for each } y \in C(J,R,\Omega).$$

$$y(t,\omega) | \leq \max(\alpha(0,\omega), \beta(0,\omega)) + \| \emptyset \|_{L^1} + T \max(\sup_{t\in J} |\alpha(t,\omega)|, \sup_{t\in J} |\beta(t,\omega)|) + \int_0^t | y(s,\omega) | \, ds.$$

Set

$$z_0 = \max(a(0,\omega), \beta(0,\omega)) + \| \emptyset \|_{L^1} + T \max(\sup_{t\in J} |\alpha(t,\omega)|, \sup_{t\in J} |\beta(t,\omega)|).$$

Using the Gronwall's Lemma [11], we get for each $t \in J$, $\omega \in \Omega$.

$$| y(t,\omega) | \leq z_0 + z_0 \int_0^t e^{t-s} ds$$

$$\leq z_0 + z_0(e^t - 1).$$

Thus

This shows that M is bounded.

Hence, Lemma 2.2 applies and N has a random fixed point which is a solution to problem (3.1)- (3.2).

6) *Step 6:* We shall show that the solution y of (3.1)-(3.2) satisfies

$$\alpha(t,\omega) \leq y(t,\omega) \leq \beta(t,\omega) \text{ for all } t \in J, \omega \in \Omega.$$

Let y be a solution to (3.1) – (3.2). We prove that

$$\alpha(t,\omega) \leq y(t,\omega) \text{ for all } t \in J, \omega \in \Omega.$$

Suppose not. Then there exist $t_1, t_2 \in J, t_1 < t_2$ such that $\alpha(t_1) = y(t_1)$ and

$$\alpha(t,\omega) > y(t,\omega) \text{ for all } t \in (t_1, t_2).$$

In view of the definition of τ one has

$$y'(t,\omega)+y(t,\omega)\in F(t,\alpha(t,\omega),\omega)+\alpha(t,\omega) \text{ a.e. on } (t_1,t_2).$$

Thus there exists $v(t,\omega)\in F(t,\alpha(t,\omega),\omega)$ a.e. on J with $v(t,\omega)\geq v_1(t,\omega)$ a.e. on (t_1,t_2) such that

$$y'(t,\omega)+y(t,\omega)=v(t,\omega)+\alpha(t,\omega) \text{ a.e. on } (t_1,t_2).$$

An integration on (t_1,t), with $t\in(t_1,t_2)$ yields

$$y(t,\omega)-y(t,\omega)=\int_{t_1}^{t}[v(s,\omega)+(a-y)(s,\omega)]ds$$

$$>\int_{t_1}^{t}v(s,\omega)ds.$$

Since α is a lower solution to (1.1), then

$$\alpha(t,\omega)-\alpha(t_1,\omega)\leq\int_{t_1}^{t}v_1(s,\omega)ds \ , t\in(t_1,t_2), \omega\in\Omega.$$

It follows from the facts $y(t_1,\omega), v(t,\omega)\geq v_1(t,\omega)$ that

$$\alpha(t,\omega)<y(t,\omega) \text{ for all } t\in(t_1,t_2), \ \omega\in\Omega.$$

Which is a contradiction, since $y(t,\omega)<\alpha(t,\omega)$ for all $t\in(t_1,t_2)$. Consequently

$$\alpha(t,\omega)\leq y(t,\omega) \text{ for all } \omega\in\Omega, t\in J.$$

Analogously, we can prove that

$$y(t,\omega)\leq\beta(t,\omega) \text{ for all } \omega\in\Omega, \ t\in J.$$

This shows that the problem (3.1) – (3.2) has a random solution in the interval $[\alpha,\beta]$.

Finally, we prove that every solution of (3.1) – (3.2) is also a solution to (1.1). We only need to show that

$$\alpha(0,\omega)\leq y(0,\omega)-L(\overline{y}(0,\omega),\overline{y}(T,\omega))\leq\beta(0,\omega).$$

Notice first that we can prove that

$$\alpha(T,\omega)\leq y(T,\omega)\leq\beta(T,\omega).$$

Suppose now that $y(0,\omega)-L(\overline{y}(0,\omega),(T),\omega)<\alpha(0,\omega)$. Then $y(0,\omega)=\alpha(0,\omega)$ and

$$y(0,\omega)-L(\alpha(0,\omega),\overline{y}(T,\omega)<\alpha(0,\omega)).$$

Since L is nonincreasingin y, we have

$$\alpha(0,\omega)\leq\alpha(0,\omega)-L(\alpha(0,\omega),\alpha(T,\omega))\leq\alpha(0,\omega)-L(\alpha(0,\omega),\overline{y}(T,\omega))<\alpha(0,\omega).$$

Which is a contradiction.

Analogously we can prove that

$$y(0,\omega)-L(\tau(0,\omega),\tau(T,\omega))\leq\beta(0,\omega).$$

Then y is a solution to (1.1).

IV. CONCLUSION

The fixed point theorems are useful for proving the existence theorems as well as for characterizing the solutions of different types of functional random differential equations on bounded or unbounded intervals of real line. The choice of the fixed point theorems depends upon the situations and the circumstances of the nonlinearities involved the problems. The selection of the fixed point theorems yields very powerful existence results as well as different characterizations of the nonlinear function differential equations. In this article, considered the nonlinear boundary value problem of first order random differential inclusions and prove the existence results through classical fixed point theorem.

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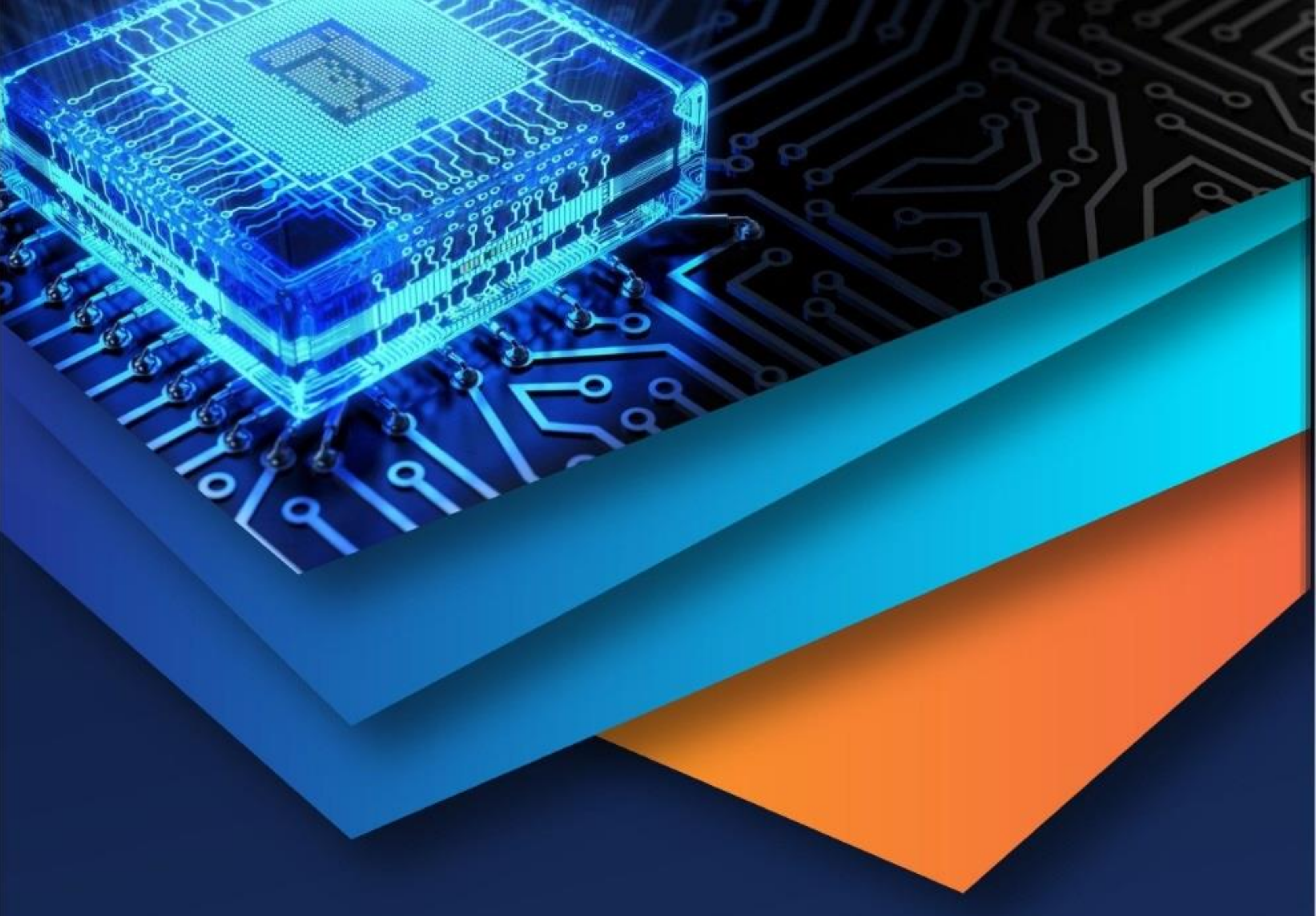

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