# The Study of Automotive Dynamics Caught up in Car Crashes 

Eng. Sava Daniel ${ }^{1}$, Eng. Darie Catalin-Dumitru ${ }^{2}$, Univ. Prof. Eng. Copae Ion ${ }^{3}$, Eng. Barothi Laszlo ${ }^{4}$<br>${ }^{1,2}$ S.A.-R City Insurance, Bucharest, Romania<br>${ }^{3}$ PhD Military Technical Academy "Ferdinand I", Bucharest, Romania<br>${ }^{4}$ Military Technical Academy "Ferdinand I", Bucharest, Romania


#### Abstract

The paper presents some viewpoints regarding mathematical models used in the analysis and reconstruction of automotives car crashes. There are highlighted the uncertainties that appear when we adopt parameters needed in car crashes theoretical studies, with consequences on theoretical approaches. The PC-Crash specialised programme is used for the study of automotive dynamics and kinematics which are involved in car crashes. There are presented examples based on real car crashes. Keywords: Motor vehicle, crash models, vehicle dynamics, uncertainty


## I. INTRODUCTION

The process of analysis and reconstruction of car crashes is characterized by a large complexity and by the existence of multiple parametrical uncertainties, functional and exploitation. This complexity is due to some difficulties, inclusive regarding the mathematical description, in describing, as real as possible, the automotive dynamics and kinematics. Currently there are some mathematical models (Brach, Ishikawa, PC-Crash etc.) which describe the phenomenon of car crash $[1 ; 2 ; 3 ; 4 ; 5 ; 6]$, but without taking into consideration various uncertainties that appear in sizes apreciation. In order to give some examples from car crashes, there are uncertainties regarding weight of moving vehicles, mass moments of inertia, rolling radius, road resistance, the aerodynamic coefficient, the frontal area, the adhesion coefficient, the position of centre of gravity, the position of crash centre, the restitution coefficient, the coefficient of tangential friction between vehicles, reaction time and driver action etc. As it can be seen, uncertainties are related to those three factors participant to the car crash: the vehicle, the environment and the driver.
To talk about uncertainties means to use intervals analysis (when there are statistical data) and uncertainty theory (when statistical data do not exist, in this case the most important role has the technical expert).

## II. AUTOMOTIVE DYNAMICS CAUGHT UP IN CAR CRASHES

PC-Crash programme will be used to establish car crash kinematics and dynamics. The programme uses Euler equations because of the rigid with a fixed point, and second order differential equations system (1) for translation [3; 6]:

$$
\left\{\begin{array} { l } 
{ m \ddot { x } = \sum F _ { x } }  \tag{1}\\
{ m \ddot { y } = \sum F _ { y } } \\
{ m \ddot { z } = \sum F _ { z } }
\end{array} \Rightarrow \left\{\begin{array}{l}
m a_{x}=\sum F_{x} \\
m a_{y}=\sum F_{y} \\
m a_{z}=\sum F_{z}
\end{array}\right.\right.
$$

and first order differential equation (2) for revolutions:

$$
\left\{\begin{array}{l}
J_{x} \dot{\omega}_{x}+\left(J_{z}-J_{y}\right) \omega_{y} \omega_{z}=\sum M_{x}  \tag{2}\\
J_{y} \dot{\omega}_{y}+\left(J_{x}-J_{z}\right) \omega_{x} \omega_{z}=\sum M_{y} \\
J_{z} \dot{\omega}_{z}+\left(J_{y}-J_{x}\right) \omega_{x} \omega_{y}=\sum M_{z}
\end{array}\right.
$$

In these expressions intervine mass $m$, linear accelerations $a_{(\cdot)}$, exterior forces $F_{(.)}$, exterior moments $M_{(\cdot)}$, angular velocities $\square_{(.)}$, angular accelerations $\dot{\omega}_{(\cdot)}$ and mass moments of inertia $J_{(.)}$.
In formulas (2) there are relations between angular velocities and angles for rolling movement, pitching motion and yaw:

$$
\begin{equation*}
\omega_{x}=\dot{\Phi}_{1} ; \omega_{y}=\dot{\Phi}_{2} ; \omega_{z}=\dot{\Phi}_{3} \tag{3}
\end{equation*}
$$

Follow-up is reconstructed the car crash of two automotives, Citroen C4 and Mazda Premacy, noted in fig. 1 with A1 and A2 [3]. In fig. 1 is shown the car crash scene (according to measurements and statements of drivers and witnesses), where automotive's initial positions are noted with A1i and A2i, same with those from the moment of collision (A1c and A2c), and final positions/stopped with A1o and A20. In scheme are also presented measured distances, automotives incline angles for longitudinal axes as well as centres of gravity CG1 and CG2. Also, there are given calculus initial values for sizes, according to established notation from specialty literature $[1 ; 2 ; 3 ; 4 ; 5 ; 6]$.


Figure 1: Car crash scene
Before presenting obtained results, in fig. 2 are shown 6 sequences obtained from tridimensional car crash simulation, possibility given by PC-Crash programme; sequences show the existence of a car crash with tilt but no rolling in case of Citroen C 4 automotive.


Figure 2: Car crash tridimensional simulation

Moreover, from fig. 3 and fig. 4 is determined the existance of a car crash with repeated collisions namely 15 , first one at $t=0 \mathrm{~s}$, last one at $t=0.46 \mathrm{~s}$.
In fig. 3 are shown the positions obtained by reconstruction of automotives, noted with A1r and A2r, which basically superpose with final positions A1o and A2o.


Figure 3: Repeated collisions phenomenon

In fig. 4 are presented impact speeds $\left(v_{1}, v_{2}\right)$ and separation speeds $\left(V_{1}, V_{2}\right)$ of two automotives during the 15 collisions. From fig. 4 a is determined that in case of Citroen C 4 automotive, separation speeds are less than impact speeds ( $V_{1}<v_{1}$ ), and from fig. 4 b is noticed that at Mazda Premacy automotive, separation speeds are bigger than impact speeds $\left(V_{2}>v_{2}\right)$.


Figure 4: Automotive speeds (impact and separation) during repeated collisions
In fig. 5 are presented speeds, and in fig. 6 covered distance, in both graphs are also given some parameters of first collision (at $t=0$ s). From fig. 5 is observed the existence of some harmonics in range $t=0-0.46 \mathrm{~s}$, which confirms the existance of repeated collisions with sudden fluctuation of speed. Also, from fig. 5 is determined first collision lasting ( $T_{c}=2.95 \mathrm{~ms}$ ), as well as impact speeds ( $v_{1}$ and $v_{2}$ ) and separation ( $V_{1}$ and $V_{2}$ ). Likewise, relative separation speed is $V_{s}=8 \mathrm{~km} / \mathrm{h}$. In addition is observed that Mazda Premacy automotive is fully stopped after 1.38 s , and Citroen C4 after 4.32 s . Finally, from fig. 5 we can observe that on certain sections Citroen's speed has small accesions, due to overturning phenomenon.
From fig. 6 is observed that the distance covered by Citroen is 16.44 m , and the distance covered by Mazda is 5.09 m , the meaning of the other sizes is known from specialty literature $[1 ; 2 ; 3 ; 4 ; 5 ; 6]$.


Figure 5: Automotive speeds (of centres of gravity)


Figure 6: Covered distance by the two automotives (positions of centres of gravity)
In fig. 7 are presented the rolling angle, pitching angle and yaw angle for the two automotives. From the two graphs is determined that pitching motion is reduced for both vehicles and rolling movement is reduced for Mazda automotive. Instead, from fig.7a is observed that Citroen rolling movement is pronounced, sudden sign change of $\square_{3}$ angle from time $t=2.47 \mathrm{~s}$ confirms it's overturning; forward tooks place another sudden change of sign at $t=3.7 \mathrm{~s}$ and another one near the stop point, which confirms a backwards harmonic in phase of overturning and not a rolling. The two graphs also confirms automotives direction angles from crash scene (fig.1) through initial and final values of yaw angle.
From fig. 7 a is also determined that Citroen automotive has a negative initial value for yaw angle, after which increases to positive values; this means that this automotive has a trigonometrical rolling movement, as it can be infered from fig.3. Instead, from fig.7b is determined that Mazda automotive has a negative initial value for yaw angle, after which still increases towards negative values; this means that this automotive has a clockwise rolling movement, as it can also be infered from fig. 3 .


Figure 7: Rolling angles around coordinate axes
From fig. 8 can be seen that Citroen automotive had an overturning movement, but also normal forces to automotive wheels. Indeed, from upper graphs is determined that for all four wheels of Citroen automotive normal forces suddenly become null without stopping the car, which means to lose contact with rolling track, meaning to overturn; as it can be seen, first, left wheels lose contact with the rolling track (before Mazda automotive stopps at $t=1,38 \mathrm{~s}$ ) and then the right ones (after Mazda automotive stopps).

Wheels normal forces of the two automotives, Citroen C4 and Mazda Premacy


Figure 8: Normal forces to automotive wheels

In fig. 9 are presented the two automotive speeds derivative, so their acceleration and decelerations (of centres of gravity). In graphs are also shown some extreme values for accelerations and decelerations. As it can be seen from fig.9, in case of Citroen C4 automotive there are very high extreme values for accelerations and decelerations due to overturning phenomenon; so, major accelerations and decelerations are during overturn and not during the 15 collisions. In case of Mazda automotive there are smaller values for accelerations and decelerations, including the extreme ones (fig.9b).
In fig. 10 are presented angular velocities around the three coordinate axes. As it can be seen from fig. 10b, in case of Mazda automotive there is a sudden variation of yaw angular velocity, clockwise movement ( $\square_{z}<0$ ), due to the collision with Citroen at first impact (at $t=0 \mathrm{~s}$ ). From fig.10a is determined that in case of Citroen automotive, biggest values belong to rolling movement because of it's overturn, in graph are also shown extreme values of $\square_{x}$ angular velocity.


Figure 9: The derivative of automotive speeds (accelerations/decelerations centres of gravity)

Rolling, pitching and yawing angular velocity of the two automotives, Citroen C4 and Mazda Premacy


Figure 10: Rotational angular velocities around coordinate axes

## III. CONCLUSIONS

The study of automotives dynamics caught up car crashes allow establishing other sizes beside the ones presented in paper, for example exterior forces with formulas (1), exterior moments with expressions (2), braking forces and side forces that apply to wheels etc. Also, a more realistic study needs operating with different uncertainties, which implies using interval analysis and uncertainty theory.

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