



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 7 Issue: III Month of publication: March 2019

DOI: http://doi.org/10.22214/ijraset.2019.3011

www.ijraset.com

Call: © 08813907089 E-mail ID: ijraset@gmail.com

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887

Volume 7 Issue III, Mar 2019- Available at www.ijraset.com

A Connection between Pythagorean Triangle and Sphenic Numbers

S. Mallika

Assistant professor, Department of Mathematics, Shrimati Indira Gandhi College, Trichy- 620 002

Abstract: This paper concerns with the problem of obtaining many Pythagorean triangles where, in each Pythagorean triangles, the expression $\frac{2*Area}{Perimeter} + H - a$ Leg is represented by a Sphenic number and Sphenic palindrome number respectively. Also, we present the number of primitive and non-primitive Triangles.

Keywords: Pythagorean triangles, Sphenic numbers, Sphenic Palindrome numbers, Primitive and non-primitive triangles.

I. INTRODUCTION

Number theory is the Queen of Mathematics. It is one of the largest and oldest branches of mathematics. We may note that there is a one to one correspondence between the polygonal numbers and the sides of polygon. Apart from the above patterns of numbers, Harshed numbers ,Nasty numbers and Dhuruva numbers have been considered in connections with Pythagorean triangles in [1-12]. In this communication, we search for patterns of Pythagorean triangles such that, in each of which, the expression $\frac{2*Area}{Perimeter} + H - a$ Leg is represented by a Sphenic number and Sphenic palindrome number and they are exhibited in sections A and B.

II. DEFINITION

- 1) Palindrome Number: Palindrome number is one that is the same when the digits are reversed.
- 2) Sphenic Number: A Sphenic number is a positive integer which is the product of exactly three distinct prime numbers.
- 3) Sphenic Palindrome Number: A Sphenic number which is palindrome is called a Sphenic palindrome number.

III.METHOD OF ANALYSIS

Let T(x, y, z) be a Pythagorean triangle where

$$x = m^2 - n^2, y = 2mn, z = m^2 + n^2$$
 (1)

Denote the area, perimeter and hypotenuse of T(x, y, z) by A,P and H respectively.

1) Section A: $\frac{2A}{P} + H - y = \alpha$, a Sphenic number of orders 3 and 4.

The problem under consideration is mathematically equivalent to solving the Diophantine equation

$$m(m-n) = \alpha \tag{2}$$

Given α , it is possible to obtain the values of m and n satisfying (2). Knowing m, n and using (1) one obtains Pythagorean triangles, each satisfying the relation, $\frac{2A}{P} + H - y = \alpha$, a Sphenic number. It is worth to note that there are only four Pythagorean triangles as the Sphenic number is a product of exactly three distinct prime numbers, A few illustrations are presented in Table 1 below.



International Journal for Research in Applied Science & Engineering Technology (IJRASET)

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887 Volume 7 Issue III, Mar 2019- Available at www.ijraset.com

Table 1: $\frac{2A}{P} + H - y = a$ sphenic number

m			I			T	D 1
46 41 435 3772 3797 230 are primitive and two are non-primitive triangles. 115 113 456 25990 25994 230 230 229 459 105,340 105,341 230 370 370 27 640 1998 2098 370 370 370 469 715 10212 10237 370 370 369 739 273060 273061 370 370 369 739 273060 273061 370 43 28 1065 2408 2633 645 45 41l the four triangles are primitive triangles are primitive triangles 41l the four triangles are primitive triangles 4215 212 1281 91160 91169 645 45 45 45 45 45 45 464 45 464 45 464 45 464 45 464 45 464 45 464 45 464 45 464 45 464 45 464 4	m	n	X	у	Z	$\frac{2A}{P} + H - y$	Remark
46 41 435 3772 3797 230 are primitive and two are non-primitive triangles. 115 113 456 25990 25994 230 230 229 459 105,340 105,341 230 370 370 27 640 1998 2098 370 370 370 469 715 10212 10237 370 370 369 739 273060 273061 370 370 369 739 273060 273061 370 43 28 1065 2408 2633 645 45 41l the four triangles are primitive triangles are primitive triangles 41l the four triangles are primitive triangles 4215 212 1281 91160 91169 645 45 45 45 45 45 45 464 45 464 45 464 45 464 45 464 45 464 45 464 45 464 45 464 45 464 45 464 4							
46 41 435 3772 3797 230 115 113 456 25990 25994 230 230 229 459 105,340 105,341 230 37 27 640 1998 2098 370 Two of the triangles are primitive and two are non-primitive triangles. 185 183 736 67710 67714 370 370 369 739 273060 273061 370 370 369 739 273060 273061 370 43 28 1065 2408 2633 645 431 45 1265 31992 32017 645 45 431 the four triangles are primitive triangles 464 1289 830760 830761 645 464 1289 830760 830761 645 45<	23	13	360	598	698	230	=
230 229 459 105,340 105,341 230 37 27 640 1998 2098 370 Two of the triangles are primitive and two are non-primitive triangles. 74 69 715 10212 10237 370 370 185 183 736 67710 67714 370 370 370 369 739 273060 273061 370 370 43 28 1065 2408 2633 645 All the four triangles are primitive triangles are primitive triangles 215 212 1281 91160 91169 645 All the four triangles are primitive triangles 645 644 1289 830760 830761 645 Two of the triangles are primitive triangles 131 121 2520 31702 31802 1310 Two of the triangles are primitive and two are non-primitive triangles 655 653 2616 855430 855434 1310 310 All the four triangles are primitive triangles	46	41	435	3772	3797	230	_
37 27 640 1998 2098 370 Two of the triangles are primitive and two are non-primitive triangles. 74 69 715 10212 10237 370 370 369 739 273060 273061 370 370 369 739 273060 273061 370 370 369 739 273060 273061 370 370 369 739 273060 273061 370 370 369 739 273060 273061 370 370 369 739 273060 273061 370 370 370 369 739 273060 273061 370 370 43 28 1065 2408 2633 645 4645	115	113	456	25990	25994	230	
74 69 715 10212 10237 370 are primitive and two are non-primitive triangles. 185 183 736 67710 67714 370 And the four triangles. 370 369 739 273060 273061 370 And the four triangles. 43 28 1065 2408 2633 645 All the four triangles are primitive triangles. 215 212 1281 91160 91169 645 All the four triangles are primitive triangles. 645 644 1289 830760 830761 645 All the four triangles are primitive and two are non-primitive triangles. 262 257 2595 134668 134693 1310 Two of the triangles are primitive are non-primitive triangles. 655 653 2616 855430 855434 1310 All the four triangles are primitive triangles. 61 28 2937 3416 4505 2013 All the four triangles are primitive triangles. 671 668 4017 896456 89646	230	229	459	105,340	105,341	230	
74 69 715 10212 10237 370 non-primitive triangles. 185 183 736 67710 67714 370 anon-primitive triangles. 370 369 739 273060 273061 370 anon-primitive triangles. 43 28 1065 2408 2633 645 anon-primitive triangles are primitive triangles are primitive triangles. 215 212 1281 91160 91169 645 anon-primitive triangles. 645 644 1289 830760 830761 645 anon-primitive triangles. 131 121 2520 31702 31802 1310 Two of the triangles are primitive and two are non-primitive triangles. 655 653 2616 855430 855434 1310 310 310 All the four triangles are primitive triangles are primitive triangles are primitive triangles. 61 28 2937 3416 4505 2013 All the four triangles are primitive triangles. 671 668 4017 <td< td=""><td>37</td><td>27</td><td>640</td><td>1998</td><td>2098</td><td>370</td><td>_</td></td<>	37	27	640	1998	2098	370	_
370 369 739 273060 273061 370 43 28 1065 2408 2633 645 129 124 1265 31992 32017 645 215 212 1281 91160 91169 645 645 644 1289 830760 830761 645 131 121 2520 31702 31802 1310 Two of the triangles are primitive and two are non-primitive triangles. 655 653 2616 855430 855434 1310 1310 1300 2619 3429580 3429581 1310 1310 1310 1310 1310 All the four triangles are primitive triangles are primitive triangles are primitive triangles are primitive triangles 183 172 3905 62952 63073 2013 All the four triangles are primitive triangles 2013 2012 4025 8100312 8100313 2013 All the four triangles are primitive triangles 205 192 5161 78720 78889	74	69	715	10212	10237	370	=
43 28 1065 2408 2633 645 129 124 1265 31992 32017 645 All the four triangles are primitive triangles 215 212 1281 91160 91169 645 645 644 1289 830760 830761 645 131 121 2520 31702 31802 1310 Two of the triangles are primitive and two are non-primitive triangles. 655 653 2616 855430 855434 1310 1310 1309 2619 3429580 3429581 1310 1310 1310 1309 2619 3429580 3429581 1310 All the four triangles are primitive triangles are primitive triangles 183 172 3905 62952 63073 2013 All the four triangles are primitive triangles 2013 2012 4025 8100312 8100313 2013 All the four triangles are primitive triangles 205 192 5161 78720 78889 2665 All the four triangles <	185	183	736	67710	67714	370	
129 124 1265 31992 32017 645 All the four triangles are primitive triangles 215 212 1281 91160 91169 645 645 644 1289 830760 830761 645 131 121 2520 31702 31802 1310 Two of the triangles are primitive and two are non-primitive triangles. 655 653 2616 855430 855434 1310 1310 1309 2619 3429580 3429581 1310 1310 All the four triangles are primitive are primitive triangles. 61 28 2937 3416 4505 2013 All the four triangles are primitive triangles are primitive triangles. 671 668 4017 896456 896465 2013 2013 2012 4025 8100312 8100313 2013 65 24 3649 3120 4801 2665 205 192 5161 78720 78889 2665 833 528	370	369	739	273060	273061	370	
129 124 1265 31992 32017 645 primitive triangles 215 212 1281 91160 91169 645 780 645 780 645 780 645 780 645 780 780 645 780 645 780 645 780 65 653 2616 855430 855434 1310 781 781 781 781 781 781 781 781 781 781 781 781 781 781 781 781 781 <	43	28	1065	2408	2633	645	All the form trien also one
645 644 1289 830760 830761 645 131 121 2520 31702 31802 1310 Two of the triangles are primitive and two are non-primitive triangles. 262 257 2595 134668 134693 1310 Two of the triangles are primitive and two are non-primitive triangles. 655 653 2616 855430 855434 1310 1310 1309 2619 3429580 3429581 1310 1310 1310 All the four triangles are primitive triangles are primitive triangles 183 172 3905 62952 63073 2013 All the four triangles are primitive triangles 2013 2012 4025 8100312 8100313 2013 All the four triangles are primitive triangles 205 192 5161 78720 78889 2665 All the four triangles are primitive triangles 833 528 415105 879648 972673 2665 All the four triangles	129	124	1265	31992	32017	645	_
131 121 2520 31702 31802 1310 Two of the triangles are primitive and two are non-primitive triangles. 262 257 2595 134668 134693 1310 Two of the triangles are primitive and two are non-primitive triangles. 655 653 2616 855430 855434 1310 1310 1309 2619 3429580 3429581 1310 61 28 2937 3416 4505 2013 183 172 3905 62952 63073 2013 671 668 4017 896456 896465 2013 2013 2012 4025 8100312 8100313 2013 65 24 3649 3120 4801 2665 205 192 5161 78720 78889 2665 833 528 415105 879648 972673 2665	215	212	1281	91160	91169	645	
262 257 2595 134668 134693 1310 are primitive and two are non-primitive triangles. 655 653 2616 855430 855434 1310 1310 1310 1309 2619 3429580 3429581 1310	645	644	1289	830760	830761	645	
262 257 2595 134668 134693 1310 non-primitive triangles. 655 653 2616 855430 855434 1310 1310 1309 2619 3429580 3429581 1310 61 28 2937 3416 4505 2013 183 172 3905 62952 63073 2013 671 668 4017 896456 896465 2013 2013 2012 4025 8100312 8100313 2013 65 24 3649 3120 4801 2665 205 192 5161 78720 78889 2665 833 528 415105 879648 972673 2665 All the four triangles are primitive triangles are primitive triangles	131	121	2520	31702	31802	1310	
1310 1309 2619 3429580 3429581 1310 61 28 2937 3416 4505 2013 183 172 3905 62952 63073 2013 671 668 4017 896456 896465 2013 2013 2012 4025 8100312 8100313 2013 65 24 3649 3120 4801 2665 205 192 5161 78720 78889 2665 833 528 415105 879648 972673 2665 All the four triangles are primitive triangles are primitive triangles	262	257	2595	134668	134693	1310	_
61 28 2937 3416 4505 2013 All the four triangles are primitive triangles 183 172 3905 62952 63073 2013 All the four triangles are primitive triangles 671 668 4017 896456 896465 2013 2013 2013 2012 4025 8100312 8100313 2013 65 24 3649 3120 4801 2665 205 192 5161 78720 78889 2665 833 528 415105 879648 972673 2665	655	653	2616	855430	855434	1310	
183 172 3905 62952 63073 2013 All the four triangles are primitive triangles 671 668 4017 896456 896465 2013 2013 2012 4025 8100312 8100313 2013 65 24 3649 3120 4801 2665 205 192 5161 78720 78889 2665 833 528 415105 879648 972673 2665 All the four triangles are primitive triangles are primitive triangles	1310	1309	2619	3429580	3429581	1310	
183 172 3905 62952 63073 2013 are primitive triangles 671 668 4017 896456 896465 2013 2013 2012 4025 8100312 8100313 2013 65 24 3649 3120 4801 2665 205 192 5161 78720 78889 2665 833 528 415105 879648 972673 2665 All the four triangles are primitive triangles triangles	61	28	2937	3416	4505	2013	All the form triengles
671 668 4017 896436 896463 2013 2013 2012 4025 8100312 8100313 2013 65 24 3649 3120 4801 2665 205 192 5161 78720 78889 2665 All the four triangles are primitive triangles 833 528 415105 879648 972673 2665	183	172	3905	62952	63073	2013	are primitive
65 24 3649 3120 4801 2665 205 192 5161 78720 78889 2665 All the four triangles are primitive triangles 833 528 415105 879648 972673 2665	671	668	4017	896456	896465	2013	triangles
205 192 5161 78720 78889 2665 All the four triangles are primitive triangles 833 528 415105 879648 972673 2665	2013	2012	4025	8100312	8100313	2013	
205 192 5161 78720 78889 2665 are primitive triangles 833 528 415105 879648 972673 2665 triangles	65	24	3649	3120	4801	2665	All the four triangles
833 328 413103 879648 972673 2663	205	192	5161	78720	78889	2665	are primitive
2665 2664 5329 14199120 14199121 2665	833	528	415105	879648	972673	2665	triangles
	2665	2664	5329	14199120	14199121	2665	



International Journal for Research in Applied Science & Engineering Technology (IJRASET)

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887

Volume 7 Issue III, Mar 2019- Available at www.ijraset.com

Section B: $\frac{2A}{P} + H - y = \alpha$, a Sphenic palindrome number of order 3 and 4.

The problem under consideration is mathematically equivalent to solving the Diophantine equation $m(m-n) = \alpha$. Given α , it is possible to obtain the values of m and n satisfying (2). Knowing m, n and using (1) one obtains Pythagorean triangles, each satisfying the relation $\frac{2A}{P} + H - y = \alpha$, a sphenic palindrome number. A few illustrations are presented in Table 2 below.

Table 2:
$$\frac{2A}{P} + H - y =$$
Sphenic palindrome number.

m	n	X	у	Z	$\frac{2A}{P} + H - y$	Remarks
47	41	528	3854	3890	282	One is
94	91	555	17108	17117	282	non-primitive.and all the
141	139	560	39198	39202	282	other triangles are primitive triangles
282	281	563	158484	158485	282	primuve trangles
31	17	672	1054	1250	434	Two of the triangles are primitive and two are non-primitive triangles.
62	55	819	6820	6869	434	
217	215	864	93310	93314	434	
434	433	867	375844	375845	434	
51	32	1577	3264	3625	969	All the triangles are primitive triangles
57	40	1649	4560	4849	969	
323	320	1929	206720	206729	969	
969	968	1937	1875984	1875985	969	
187	168	6745	62832	63193	3553	All the triangles are primitive triangles
209	192	6817	80256	80545	3553	
323	312	6985	201552	201673	3553	
3553	3552	7105	25240512	25240513	3553	
239	206	14685	98468	99557	7887	All the triangles are primitive triangles
717	706	15653	1012404	1012525	7887	
2629	2626	15765	13807508	13807517	7887	
7887	7886	15773	124393764	124393765	7887	
319	288	18817	183744	184705	9889	All the triangles are primitive triangles
341	312	18937	212784	213625	9889	
899	888	19657	1596624	1596745	9889	
9889	9888	19777	195564864	195564865	9889	

IV.CONCLUSION

In this paper, we have made an attempt to find Pythagorean triangles in connection with Sphenic numbers and Sphenic palindrome numbers. To conclude, one may search for other choices of Pythagorean triangles for other Sphenic numbers of higher orders.



International Journal for Research in Applied Science & Engineering Technology (IJRASET)

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887 Volume 7 Issue III, Mar 2019- Available at www.ijraset.com

REFERENCES

- [1] W.Sierpinski Pythagorean triangles, Dover publications, INC, Newyork, 2003.
- [2] M.A.Gopalan and A.Vijaysankar, "Observations on a Pythagorean problem", Acta Ciencia Indica, Vol. XXXVI M.No.4,517-520,2010.
- [3] M.A.Gopalan, A.Gnanam and G.Janaki, 'A Remarkable Pythagorean problem' Acta Ciencia Indica, Vol. XXXIII M, No, 4, 1429- 1434, 2007.
- [4] M.A.Gopalan and A.Gnanam, 'Pythagorean triangles and Polygonal numbers International Journal of Mathematical Sciences, Vol 9, No 1-2, 211-215, 2010.
- [5] M.A.Gopalan and G.Janaki, 'Pythagorean triangle with Area Perimeter as a special number' Bulletin of pure and Applied sciences, Vol 27(2), 393-402, 2008.
- [6] M.A.Gopalan and G.Janaki, 'Pythagorean triangle with nasty number as a leg' Journal of Applied Analysis and Applications, Vol 4, No 1-2, 13-17, 2008.
- [7] G.Janaki and R.Radha, 'Special Pythagorean triangle and six digit Harshad numbers' IJIRSET, Vol. 5, Issue 3, 3931-3933, March 2016.
- [8] G.Janaki and R.Radha, 'Special pairs of Pythagorean triangle and Harshad numbers' Asian Journal of Science and Technology, volume .7, Issue. 8, 3397-3399, August 2016.
- [9] G.Janaki and P.Saranya, 'Pythagorean Triangle with Area/Perimeter as a Jarasandha numbers of orders 2 and 4' IRJET, Volume .3, Issue .7, 1259-1264, July 2016
- [10] G.Janaki and R.Radha, 'Pythagorean Triangle with Area/Perimeter as a Harshad number of digits 4,5 and 6' IJRASET, Volume. 5 ,Issue. 12, 1754-1762, December 2017.
- [11] G.Janaki and P.Saranya, 'Special Pythagorean triangles in connection with the Narcissistic Numbers of order 3 and 4' AIJRSTEM, 14(2), 150-153, 2016.
- [12] G.Janaki and P.Saranya, 'Special pairs of Pythagorean triangles and Narcissistic numbers', IJMRD, Vol. 3, Issue. 4, 106-108, April 2016.









45.98



IMPACT FACTOR: 7.129



IMPACT FACTOR: 7.429



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call: 08813907089 🕓 (24*7 Support on Whatsapp)