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A Connection between Pythagorean Triangle and Sphenic Numbers

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Abstract: ***This paper concerns with the problem of obtaining many Pythagorean triangles where, in each Pythagorean triangles, the expression*** $\frac{2*\text{Area}}{\text{Perimeter}}+H-a\ \text{Leg}$ ***is represented by a Sphenic number and Sphenic palindrome number respectively. Also, we present the number of primitive and non-primitive Triangles.***

Keywords: Pythagorean triangles, Sphenic numbers, Sphenic Palindrome numbers, Primitive and non-primitive triangles.

I. INTRODUCTION

Number theory is the Queen of Mathematics. It is one of the largest and oldest branches of mathematics. We may note that there is a one to one correspondence between the polygonal numbers and the sides of polygon. Apart from the above patterns of numbers, Harshed numbers ,Nasty numbers and Dhuruva numbers have been considered in connections with Pythagorean triangles in [1-12]. In this communication, we search for patterns of Pythagorean triangles such that, in each of which, the expression $\frac{2*\text{Area}}{\text{Perimeter}}+H-a\ \text{Leg}$ is represented by a Sphenic number and Sphenic palindrome number and they are exhibited in sections A and B.

II. DEFINITION

1) *Palindrome Number:* Palindrome number is one that is the same when the digits are reversed.
2) *Sphenic Number:* A Sphenic number is a positive integer which is the product of exactly three distinct prime numbers.
3) *Sphenic Palindrome Number:* A Sphenic number which is palindrome is called a Sphenic palindrome number.

III.METHOD OF ANALYSIS

Let $T(x,y,z)$ be a Pythagorean triangle where

$$x=m^2-n^2, y=2mn, z=m^2+n^2 \quad (1)$$

Denote the area, perimeter and hypotenuse of $T(x,y,z)$ by A,P and H respectively.

1) *Section A:* $\frac{2A}{P}+H-y=\alpha$, a Sphenic number of orders 3 and 4.

The problem under consideration is mathematically equivalent to solving the Diophantine equation

$$m(m-n)=\alpha \quad (2)$$

Given α, it is possible to obtain the values of m and n satisfying (2). Knowing m, n and using (1) one obtains Pythagorean triangles, each satisfying the relation , $\frac{2A}{P}+H-y=\alpha$, a Sphenic number. It is worth to note that there are only four Pythagorean triangles as the Sphenic number is a product of exactly three distinct prime numbers, A few illustrations are presented in Table 1 below.

Table 1 : $\frac{2A}{P} + H - y = $ a sphenic number

m	n	x	y	z	$\frac{2A}{P} + H - y$	Remark
23	13	360	598	698	230	Two of the triangles are primitive and two are non-primitive triangles.
46	41	435	3772	3797	230	
115	113	456	25990	25994	230	
230	229	459	105,340	105,341	230	
37	27	640	1998	2098	370	Two of the triangles are primitive and two are non-primitive triangles.
74	69	715	10212	10237	370	
185	183	736	67710	67714	370	
370	369	739	273060	273061	370	
43	28	1065	2408	2633	645	All the four triangles are primitive triangles
129	124	1265	31992	32017	645	
215	212	1281	91160	91169	645	
645	644	1289	830760	830761	645	
131	121	2520	31702	31802	1310	Two of the triangles are primitive and two are non-primitive triangles.
262	257	2595	134668	134693	1310	
655	653	2616	855430	855434	1310	
1310	1309	2619	3429580	3429581	1310	
61	28	2937	3416	4505	2013	All the four triangles are primitive triangles
183	172	3905	62952	63073	2013	
671	668	4017	896456	896465	2013	
2013	2012	4025	8100312	8100313	2013	
65	24	3649	3120	4801	2665	All the four triangles are primitive triangles
205	192	5161	78720	78889	2665	
833	528	415105	879648	972673	2665	
2665	2664	5329	14199120	14199121	2665	

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2) Section B: $\frac{2A}{P}+H-y=\alpha$, a Sphenic palindrome number of order 3 and 4.

The problem under consideration is mathematically equivalent to solving the Diophantine equation $m(m-n)=\alpha$ .Given α , it is possible to obtain the values of m and n satisfying (2). Knowing m, n and using (1) one obtains Pythagorean triangles, each satisfying the relation $\frac{2A}{P}+H-y=\alpha$, a sphenic palindrome number. A few illustrations are presented in Table 2 below.

Table 2: $\frac{2A}{P}+H-y$ = Sphenic palindrome number.

m	n	x	y	z	$\frac{2A}{P}+H-y$	Remarks
47	41	528	3854	3890	282	One is non-primitive.and all the other triangles are primitive triangles
94	91	555	17108	17117	282	
141	139	560	39198	39202	282	
282	281	563	158484	158485	282	
31	17	672	1054	1250	434	Two of the triangles are primitive and two are non-primitive triangles.
62	55	819	6820	6869	434	
217	215	864	93310	93314	434	
434	433	867	375844	375845	434	
51	32	1577	3264	3625	969	All the triangles are primitive triangles
57	40	1649	4560	4849	969	
323	320	1929	206720	206729	969	
969	968	1937	1875984	1875985	969	
187	168	6745	62832	63193	3553	All the triangles are primitive triangles
209	192	6817	80256	80545	3553	
323	312	6985	201552	201673	3553	
3553	3552	7105	25240512	25240513	3553	
239	206	14685	98468	99557	7887	All the triangles are primitive triangles
717	706	15653	1012404	1012525	7887	
2629	2626	15765	13807508	13807517	7887	
7887	7886	15773	124393764	124393765	7887	
319	288	18817	183744	184705	9889	All the triangles are primitive triangles
341	312	18937	212784	213625	9889	
899	888	19657	1596624	1596745	9889	
9889	9888	19777	195564864	195564865	9889	

IV.CONCLUSION

In this paper, we have made an attempt to find Pythagorean triangles in connection with Sphenic numbers and Sphenic palindrome numbers. To conclude, one may search for other choices of Pythagorean triangles for other Sphenic numbers of higher orders.

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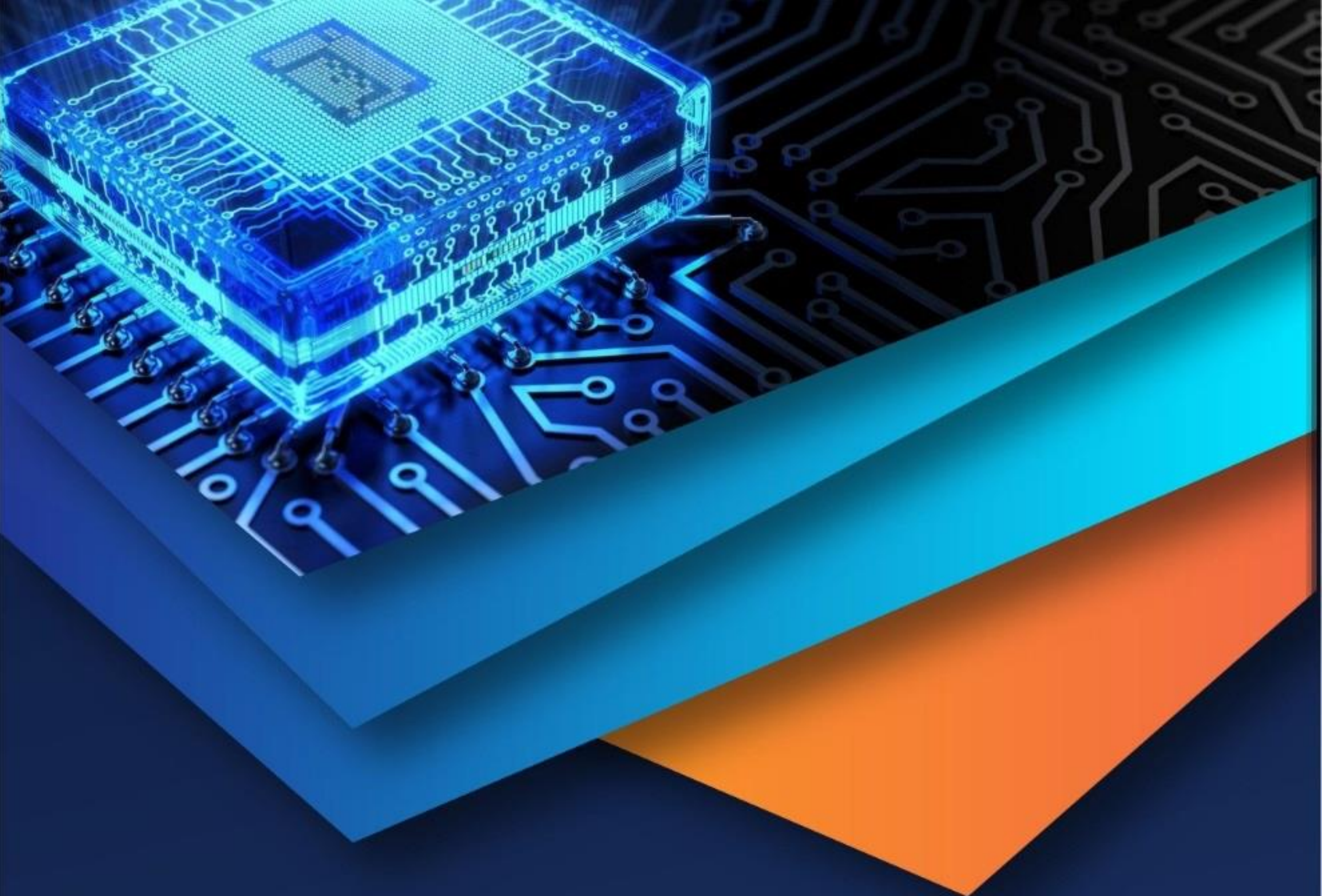

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