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A Connection between Pythagorean Triangle and Harshad numbers

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Abstract: This paper concerns with the problem of obtaining many Pythagorean triangles where, in each Pythagorean triangle, the expression $\frac{2 * Area}{Perimeter} + H - a$ Leg is represented by a Harshad number, Multiple Harshad number and Nivenmorphic number respectively. Also, we present the number of primitive and non-primitive Pythagorean triangles.

Keywords: Pythagorean triangles, Harshad numbers, multiple Harshad number, Nivenmorphic number, Primitive and non-primitive triangles

I. INTRODUCTION

Number theory is the Queen of Mathematics. It is one of the largest and oldest branches of mathematics. We may note that there is a one to one correspondence between the polygonal numbers and the sides of polygon. Apart from the above patterns of numbers, Sphenic numbers, Nasty numbers and Dhuruva numbers have been considered in connections with Pythagorean triangles in [1-12]. In this communication, we search for patterns of Pythagorean triangles such that, in each of which, the expression is represented by a Harshad number Multiple Harshad number and Nivenmorphic numbers and they are exhibited in sections A, B and C.

A. Definitions

- 1) Harshad numbers in a given number base, is an integer that is divisible by the sum of its digits when written in that base. Harshad numbers in base n are also known as n-Harshad numbers
- 2) Multiple Harshad number is a Harshad number that, when divided by the sum of its digits, produces another Harshad number.
- 3) It is a Harshad number, the digit sum is equal to the last digits of the Harshad number

II. METHOD OF ANALYSIS

Let $T(x, y, z)$ be a Pythagorean triangle where

$$x = m^2 - n^2, y = 2mn, z = m^2 + n^2 \quad (m > n > 0) \tag{1}$$

Denote the area, perimeter and hypotenuse of $T(x, y, z)$ by A, P and H respectively.

A. Section A

$$\frac{2A}{P} + H - y = \alpha, \text{ a Harshad number of orders 3, 4 and 5.}$$

- 1) Case 1 : Harshad number of order 3

The problem under consideration is mathematically equivalent to

$$m(m - n) = \alpha \tag{2}$$

Given α , it is possible to obtain the values of m and n satisfying (2). Knowing m, n and using (1) one obtains Pythagorean triangles, each satisfying the relation $\frac{2A}{P} + H - y = \alpha$, a Harshad number. A few illustrations are presented in Tables 1, 2 and 3 corresponding to 3rd order, 4th order and 5th order Harshad numbers respectively.

Table 1 : $\frac{2A}{P} + H - y = \alpha$, a 3rd order Harshad number

m	n	x	y	z	$\frac{2A}{P} + H - y$	Remark
27	5	704	270	754	594	There are four primitive and four non-primitive Pythagorean triangles
33	15	864	990	1314	594	
54	43	1067	4644	4765	594	
66	57	1107	7524	7605	594	
99	93	1152	18414	18450	594	
198	195	1179	77220	77229	594	
297	295	1184	175230	175234	594	
594	593	1187	704484	704485	594	
36	17	1007	1224	1585	684	There are four primitive and five non-primitive Pythagorean triangles
38	20	1044	1520	1844	684	
57	45	1224	5130	5274	684	
76	67	1287	10184	10265	684	
114	108	1332	24624	24660	684	
171	167	1352	57114	57130	684	
228	225	1359	102600	102609	684	
342	340	1364	232560	232564	684	
684	683	1367	934344	934345	684	
47	29	1368	2726	3050	846	There are three primitive and three non-primitive Pythagorean triangles
94	85	1611	15980	16061	846	
141	135	1656	38070	38106	846	
282	279	1683	157356	157365	846	
423	421	1688	356166	356170	846	
846	845	1691	1429740	1429741	846	
73	63	1360	9198	9298	730	There are two primitive and two non-primitive Pythagorean triangles
146	141	1435	41172	41197	730	
365	363	1456	264990	264994	730	
730	729	1459	1064340	1064341	730	

2) Case 2: Harshad number of order 4

Table 2 : $\frac{2A}{P} + H - y = \alpha$, a 4th order Harshad number

m	n	x	y	z	$\frac{2A}{P} + H - y$	Remark	
44	20	1536	1760	2336	1056	There are three primitive and eight non-primitive Pythagorean triangles	
48	26	1628	2496	2980	1056		
66	50	1856	6600	6856	1056		
88	76	1968	13376	13520	1056		
96	85	1991	16320	16441	1056		
132	124	2048	32736	32800	1056		
176	170	2076	59840	59876	1056		
264	260	2096	137280	137296	1056		
352	349	2103	245696	245705	1056		
528	526	2108	555456	555460	1056		
1056	1055	2111	2228160	2228161	1056		
42	4	1748	336	1780	1596	There are five primitive and seven non-primitive Pythagorean triangles	
57	29	2408	3306	4090	1596		
76	55	2751	8360	8801	1596		
84	65	2831	10920	11281	1596	All the four are primitive Pythagorean triangles	
114	100	2996	22800	22996	1596		
133	121	3048	32186	32330	1596		
228	221	3143	100776	100825	1596		
266	260	3156	138356	138356	1596		
399	395	3176	315210	315226	1596		
532	529	3183	562856	562865	1596		
798	796	3188	1270416	1270420	1596		
1596	1595	3191	5091240	5091241	1596		
91	72	3097	13104	13465	1729		All the four are primitive Pythagorean triangles
133	120	3289	31920	32089	1729		
247	240	3409	118560	118609	1729		
1729	1728	3457	5975424	5975425	1729		
88	1	7743	176	7745	7656	There are eight primitive and ten non-primitive Pythagorean triangles	
116	50	10956	11600	15956	7656		
132	74	11948	19536	22900	7656		
174	130	13376	45240	47176	7656		
232	199	14223	92336	93425	7656		
264	235	14471	124080	124921	7656		
319	295	14736	188210	188786	7656		
348	326	14828	226896	227380	7656		
638	626	15168	798776	798920	7656		
696	685	15191	953520	953641	7656		
957	949	15248	1816386	1816450	7656		
1276	1270	15276	3241040	3241076	7656		
1914	1910	15296	7311480	7311496	7656		
2552	2549	15303	13010096	13010105	7656		
3828	3826	15308	29291856	29291860	7656		
7653	7652	15305	1.17E+08	1.17E+08	7656		
7654	7653	15307	1.17E+08	1.17E+08	7656		
7656	7655	15311	1.17E+08	1.17E+08	7656		

3) Case 3 Harshad number of order 5

Table 3: $\frac{2A}{P} + H - y = \alpha$, a fifth order Harshad number

m	n	x	y	z	$\frac{2A}{P} + H - y$	Remark
171	110	17141	37620	41341	10431	All the six are primitive Pythagorean triangles
183	126	17613	46116	49365	10431	
549	530	20501	581940	582301	10431	
1159	1150	20781	2665700	2665781	10431	
3477	3474	20853	24158196	24158205	10431	
10431	10430	20861	2.18E+08	2.18E+08	10431	
117	14	13493	3276	13885	12051	All the six are primitive Pythagorean triangles
309	270	22581	166860	168381	12051	
927	914	23933	1694556	1694725	12051	
1339	1330	24021	3561740	3561821	12051	
4017	4014	24093	32248476	32248485	12051	
12051	12050	24101	2.9E+08	2.9E+08	12051	
190	1	36099	380	36101	35910	There are twenty primitive and twelve non-primitive Pythagorean triangles
210	39	44061	16380	44139	35910	
266	131	70625	69692	70887	35910	
270	137	72763	73980	73037	35910	
285	159	81066	90630	81384	35910	
315	201	99024	126630	99426	35910	
342	237	116727	162108	117201	35910	
378	283	142601	213948	143167	35910	
399	309	158892	246582	159510	35910	
513	443	262726	454518	263612	35910	
570	507	324393	577980	325407	35910	
630	573	396327	721980	397473	35910	
665	611	441614	812630	442836	35910	
798	753	636051	1201788	637557	35910	
855	813	730212	1390230	731838	35910	
945	907	892118	1714230	893932	35910	
1026	991	1051685	2033532	1053667	35910	
1197	1167	1431642	2793798	1433976	35910	
1330	1303	1767597	3465980	1770203	35910	
1710	1689	2922411	5776380	2925789	35910	
1890	1871	3570229	7072380	3573971	35910	
1995	1977	3978048	7888230	3982002	35910	
2394	2379	5728857	11390652	5733615	35910	
2565	2551	6576674	13086630	6581776	35910	
3591	3581	12891700	25718742	12898862	35910	
3990	3981	15916119	31768380	15924081	35910	

5130	5123	26311777	52561980	26322023	35910	
5985	5979	35814246	71568630	35826204	35910	
7182	7177	51573947	103090428	51588301	35910	
11970	11967	143268933	286489980	143292867	35910	
17955	17953	322364072	644692230	322399978	35910	
35910	35909	1.289E+09	2578984380	1289564009	35910	
111	9	12240	1998	12402	11322	There are six primitive and six non-primitive Pythagorean triangles
153	79	17168	24174	29650	11322	
222	171	20043	75924	78525	11322	
306	269	21275	164628	165997	11322	
333	299	21488	199134	200290	11322	
629	611	22320	768638	768962	11322	
666	649	22355	864468	864757	11322	
1258	1249	22563	3142484	3142565	11322	
1887	1881	22608	7098894	7098930	11322	
3774	3771	22635	28463508	28463517	11322	
5661	5659	22640	64071198	64071202	11322	
11322	11321	22643	2.56E+08	2.56E+08	11322	

B. Section B

$$\frac{2A}{P} + H - y = \alpha, \text{ a Multiple Harshad number of orders 3, 4 and 5.}$$

1) Case: 4 Multiple Harshad number of order three

The problem under consideration is mathematically equivalent to $m(m - n) = \alpha$ Given α , it is possible to obtain the values of m and n satisfying (2). Knowing m, n and using (1) one obtains Pythagorean triangles, each satisfying the relation

$\frac{2A}{P} + H - y = \alpha$, a Multiple Harshad number. A few illustrations are presented in Tables 4, 5 and 6 corresponding to 3rd order, 4th order and 5th order multiple Harshad numbers respectively.

Table 4 : $\frac{2A}{P} + H - y = \alpha$, a third order Multiple Harshad number

m	n	x	y	z	$\frac{2A}{P} + H - y$	Remarks
27	9	720	486	738	486	There are three primitive and three non-primitive Pythagorean triangles
54	45	2871	4860	2961	486	
81	75	6486	12150	6636	486	
162	159	26085	51516	26403	486	
243	241	58808	117126	59290	486	
486	485	235711	471420	236681	486	
24	15	561	720	591	216	There are three primitive and four non-primitive Pythagorean triangles
27	19	710	1026	748	216	
36	30	1266	2160	1326	216	
54	50	2866	5400	2966	216	
72	69	5115	9936	5253	216	

108	106	11558	22896	11770	216	There are three primitive and seven non-primitive Pythagorean triangles
216	215	46441	92880	46871	216	
27	3	726	162	732	648	
36	18	1278	1296	1314	648	
54	42	2874	4536	2958	648	
72	63	5121	9072	5247	648	
81	73	6488	11826	6634	648	
108	102	11562	22032	11766	648	
162	158	26086	51192	26402	648	
216	213	46443	92016	46869	648	
324	322	104654	208656	105298	648	
648	647	419257	838512	420551	648	

2) Case: 5 Multiple Harshad number of order four

Table 5 : $\frac{2A}{P} + H - y = \alpha$, a fourth order Multiple Harshad number

m	n	x	y	z	$\frac{2A}{P} + H - y$	Remarks
48	21	2283	2016	2325	1296	There are four primitive and eight non-primitive Pythagorean triangles
54	30	2886	3240	2946	1296	
72	54	5130	7776	5238	1296	
81	65	6496	10530	6626	1296	
108	96	11568	20736	11760	1296	
144	135	20601	38880	20871	1296	
162	154	26090	49896	26398	1296	
216	210	46446	90720	46866	1296	
324	320	104656	207360	105296	1296	
432	429	186195	370656	187053	1296	
648	646	419258	837216	420550	1296	
1296	1295	1678321	3356640	1680911	1296	
81	27	6534	4374	6588	4374	There are four primitive and four non-primitive Pythagorean triangles
162	135	26109	43740	26379	4374	
243	225	58824	109350	59274	4374	
486	477	235719	463644	236673	4374	
729	723	530718	1054134	532164	4374	
1458	1455	2124309	4242780	2127219	4374	
2187	2185	4780784	9557190	4785154	4374	
4374	4373	19127503	38255004	19136249	4374	
126	72	15804	18144	15948	6804	
162	120	26124	38880	26364	6804	

189	153	35568	57834	35874	6804
243	215	58834	104490	59264	6804
252	225	63279	113400	63729	6804
324	303	104673	196344	105279	6804
378	360	142524	272160	143244	6804
486	472	235724	458784	236668	6804
567	555	320934	629370	322044	6804
756	747	570789	1129464	572283	6804
1134	1128	1284828	2558304	1287084	6804
1701	1697	2891704	5773194	2895098	6804
2268	2265	5141559	10274040	5146089	6804
3402	3400	11570204	23133600	11577004	6804
6801	6800	46246801	92493600	46260401	6804
6802	6801	46260403	92520804	46274005	6804
6803	6802	46274007	92548012	46287611	6804
6804	6803	46287613	92575224	46301219	6804

There are seven primitive and eleven non-primitive Pythagorean triangles

3) Case: 6 Multiple Harshad number of order five

Table 6 : $\frac{2A}{P} + H - y = \alpha$, a fifth order Multiple Harshad number

m	n	x	y	z	$\frac{2A}{P} + H - y$	Remarks
162	81	26163	26244	26325	13122	There are five primitive and three non-primitive Pythagorean triangles
243	189	58860	91854	59238	13122	
486	459	235737	446148	236655	13122	
729	711	530730	1036638	532152	13122	
1458	1449	2124315	4225284	2127213	13122	
2184	2181	4767675	9526608	4772037	13122	
6561	6559	43040162	86067198	43053280	13122	
13122	13121	172173763	344347524	172200005	13122	
162	54	26190	17496	26298	17496	There are five primitive and eleven non-primitive Pythagorean triangles
216	135	46521	58320	46791	17496	
243	171	58878	83106	59220	17496	
324	270	104706	174960	105246	17496	
486	450	235746	437400	236646	17496	
648	621	419283	804816	420525	17496	
729	705	530736	1027890	532146	17496	
972	954	943830	1854576	945738	17496	
1458	1446	2124318	4216536	2127210	17496	
1944	1935	3777201	7523280	3781071	17496	



2187	2179	4780790	9530946	4785148	17496
2916	2910	8500146	16971120	8505966	17496
4374	4370	19127506	38228760	19136246	17496
5832	5829	34006395	67989456	34018053	17496
8748	8746	76518758	153020016	76536250	17496
17496	17495	306092521	612185040	306127511	17496
162	2	26242	648	26246	25920
180	36	32364	12960	32436	25920
192	57	36807	21888	36921	25920
216	96	46560	41472	46752	25920
240	132	57468	63360	57732	25920
270	174	72726	93960	73074	25920
288	198	82746	114048	83142	25920
320	239	102161	152960	102639	25920
324	244	104732	158112	105220	25920
360	288	129312	207360	129888	25920
405	341	163684	276210	164366	25920
432	372	186252	321408	186996	25920
480	426	229974	408960	230826	25920
540	492	291108	531360	292092	25920
576	531	331245	611712	332307	25920
648	608	419296	787968	420512	25920
720	684	517716	984960	519084	25920
810	778	655322	1260360	656878	25920
864	834	745662	1441152	747330	25920
960	933	920667	1791360	922533	25920
1080	1056	1165344	2280960	1167456	25920
1296	1276	1678340	3307392	1680892	25920
1440	1422	2072178	4095360	2075022	25920
1620	1604	2622796	5196960	2626004	25920
1728	1713	2984271	5920128	2987697	25920
2160	2148	4663452	9279360	4667748	25920
2592	2582	6715882	13385088	6721046	25920
2880	2871	8291529	16536960	8297271	25920
3240	3232	10494368	20943360	10500832	25920
4320	4314	18658086	37272960	18666714	25920
5184	5179	26868677	53695872	26879035	25920
6480	6476	41983924	83928960	41996876	25920
8640	8637	74640963	149247360	74658237	25920
12960	12958	167948642	335871360	167974558	25920
25920	25919	671820481	1343640960	671872319	25920

There are nine primitive and twenty six non-primitive Pythagorean triangles

C. Section C

$$\frac{2A}{P} + H - y = \alpha, \text{ a Nivenmerphic number of order 3, 4 and 5}$$

The problem under consideration is mathematically equivalent to $m(m - n) = \alpha$. Given α , it is possible to obtain the values of m and n satisfying (2). Knowing m, n and using (1) one obtains Pythagorean triangles, each satisfying the relation

$\frac{2A}{P} + H - y = \alpha$, a Nivenmerphic number. A few illustrations are presented in Tables 7,8 and 9 corresponding to 3rd order 4th order and 5th order Nivenmerphic numbers respectively.

1) Case: 7 Nivenmerphic number of order three

Table 7: $\frac{2A}{P} + H - y = \alpha$, a third order Nivenmerphic number

m	n	x	y	z	$\frac{2A}{P} + H - y$	Remarks
38	14	1430	1064	1458	912	There are three primitive and seven non-primitive Pythagorean triangles
48	29	2275	2784	2333	912	
57	41	3208	4674	3290	912	
76	64	5712	9728	5840	912	
114	106	12890	24168	13102	912	
152	146	22958	44384	23250	912	
228	224	51760	102144	52208	912	
304	301	92115	183008	92717	912	
456	454	207482	414048	208390	912	
912	911	830833	1661664	832655	912	
61	46	3675	5612	3767	915	All the four are primitive Pythagorean triangles
183	178	33311	65148	33667	915	
305	302	92723	184220	93327	915	
915	914	836311	1672620	838139	915	
34	7	1149	476	1163	918	There are four primitive and two non-primitive Pythagorean triangles
51	33	2568	3366	2634	918	
54	37	2879	3996	2953	918	
102	93	10311	18972	10497	918	
153	147	23262	44982	23556	918	
918	917	841807	1683612	843641	918	

2) Case: 8 Nivenmerphic number of order four

Table 8 : $\frac{2A}{P} + H - y = \alpha$, a fourth order Nivenmerphic number

m	n	x	y	z	$\frac{2A}{P} + H - y$	Remarks
82	5	6719	820	6729	6314	
154	11	23705	3388	23727	6314	
287	265	82104	152110	82634	6314	
51	437	202964	394174	203838	6314	There are six primitive and two non-primitive Pythagorean triangles
574	563	328913	646324	330039	6314	
902	895	812709	1614580	814499	6314	
2105	2102	4428923	8849420	4433127	6314	
3157	3155	9963494	19920670	9969804	6314	
6314	6313	39860283	79720564	39872909	6314	
124	44	15332	10912	15420	9920	There are four primitive and eleven non-primitive Pythagorean triangles
155	91	23934	28210	24116	9920	
160	98	25502	31360	25698	9920	
248	208	61296	103168	61712	9920	
310	278	95822	172360	96378	9920	
320	289	102111	184960	102689	9920	
496	476	245540	472192	246492	9920	
620	604	383796	748960	385004	9920	
992	982	983082	1948288	985046	9920	
1240	1232	1536368	3055360	1538832	9920	
1984	1979	3934277	7852672	3938235	9920	
2480	2476	6147924	12280960	6152876	9920	
3307	3304	10932945	21852656	10939553	9920	
4960	4958	24596642	49183360	24606558	9920	
9920	9919	98396481	196792960	98416319	9920	

3) Case: 9 Nivenmerphic number of order five

Table 9: $\frac{2A}{P} + H - y = \alpha$, a fifth order Nivenmerphic number

m	n	x	y	z	$\frac{2A}{P} + H - y$	Remarks
289	236	83285	136408	83757	15317	All are primitive Pythagorean triangles
901	884	810917	1592968	812685	15317	
15317	15316	234595173	469190344	234625805	15317	
323	270	104059	174420	104599	17119	All are primitive Pythagorean triangles
901	882	810919	1589364	812683	17119	
1007	990	1013059	1993860	1015039	17119	
17119	17118	293043043	586086084	293077279	17119	
159	40	25241	12720	25321	18921	All are primitive Pythagorean triangles
357	304	127145	217056	127753	18921	
371	320	137321	237440	137961	18921	
901	880	810921	1585760	812681	18921	
1113	1096	1237673	2439696	1239865	18921	
18921	18920	357985321	715970640	358023161	18921	

III.CONCLUSION

In this paper, we have made an attempt to find Pythagorean triangles in connection with Harshad numbers Multiple Harshad numbers and Nivenmorphic numbers. To conclude one may search for other choices of Pythagorean triangles for any other Special numbers.

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