



# IJRASET

International Journal For Research in  
Applied Science and Engineering Technology



---

# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

---

**Volume: 7      Issue: III      Month of publication: March 2019**

**DOI: <http://doi.org/10.22214/ijraset.2019.3055>**

**[www.ijraset.com](http://www.ijraset.com)**

**Call:  08813907089**

**E-mail ID: [ijraset@gmail.com](mailto:ijraset@gmail.com)**

# Integral Points on the Ternary Quadratic Diophantine Equation $y^2 = 33x^2 + 4^t, \quad t \geq 0$

Dr. G. Sumathi<sup>1</sup>, W. Princy Dona<sup>2</sup>

<sup>1</sup>Assistant Professor, PG and Research Department of Mathematics, Shrimati Indira Gandhi College Trichy-2, Tamil Nadu, India.

<sup>2</sup>M.Phil Scholar, PG and Research Department of Mathematics, Shrimati Indira Gandhi College Trichy-2, Tamil Nadu, India.

**Abstract:** The binary quadratic equation  $y^2 = 33x^2 + 4^t$  representing hyperbola is considered for finding its integer solutions. A few interesting properties among the solutions are presented. Also, we present infinitely many positive integer solutions in terms of Generalized Fibonacci sequences of numbers, Generalized Lucas sequences of numbers.

**Keywords:** Binary quadratic integral solutions, generalized Fibonacci Sequences of numbers, generalized Lucas Sequences of numbers.

AMS Mathematics Subject Classification: 11D09

Notations

$GF_n(k, s)$  : Generalized Fibonacci Sequences of rank n.

$GL_n(k, s)$  : Generalized Lucas Sequences of rank n.

$$t_{m,n} = n \left( 1 + \frac{(n-1)(m-2)}{2} \right)$$

$$P_n^m = \frac{[n(n+1)((m-2)(n+(5-m)))]}{6}$$

$$Pr_n = n(n+1)$$

$$Ct_{m,n} = \frac{mn(n+1)}{2} + 1$$

$$S_n = 6n(n-1) + 1$$

## I. INTRODUCTION

The binary quadratic equation of the form  $y^2 = Dx^2 + 1$ , where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1,2,4]. In [3] infinitely many Pythagorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation  $y^2 = 3x^2 + 1$ . In [5] a, special Pythagorean triangle is obtained by employing the integral solutions of  $y^2 = 182x^2 + 14$ . In [6] different pattern of infinitely many Pythagorean triangles are obtained by employing the non-integral solutions of  $y^2 = 14x^2 + 4$ . In this context one may also refer [7,8]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation  $y^2 = 33x^2 + 4^t$  representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration a few patterns of Pythagorean triangles are obtained.

## II. METHODS OF ANALYSIS

Consider the binary quadratic equation

$$y^2 = 33x^2 + 4^t, \quad t \geq 0 \tag{1}$$

with least positive integer solutions is

$$x_0 = 4(2)^t, \quad y_0 = 23(2)^t$$

To obtain the other solutions of (1),

consider the Pell equation

$$y^2 = 33x^2 + 1 \tag{2}$$

whose general solution  $(\tilde{x}_n, \tilde{y}_n)$  is represented by

$$\tilde{x}_n = \frac{1}{2\sqrt{33}} g_n, \tilde{y}_n = \frac{1}{2} f_n$$

in which,

$$f_n = (23 + 4\sqrt{33})^{n+1} + (23 - 4\sqrt{33})^{n+1}$$

$$g_n = (23 + 4\sqrt{33})^{n+1} - (23 - 4\sqrt{33})^{n+1}, n = -1, 0, 1, 2, \dots$$

Applying Brahmagupta lemma between the solutions of  $(x_0, y_0)$  and  $(\tilde{x}_n, \tilde{y}_n)$ , the general solutions of equation (1) are found to be

$$y_{n+1} = \frac{23(2)^t}{2} f_n + 2(2)^t \sqrt{33} g_n$$

$$x_{n+1} = 2(2)^t f_n + \frac{23(2)^t}{2\sqrt{33}} g_n$$

The recurrence relations satisfied by the values of  $x_{n+1}$  and  $y_{n+1}$  are respectively.

$$x_{n+3} - 46x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 46y_{n+2} + y_{n+1} = 0$$

A few numerical examples are presented in the table 1 below:

Table 1: Numerical examples

$n$	$x_{n+1}$	$y_{n+1}$
-1	$4(2)^t$	$23(2)^t$
0	$184(2)^t$	$1057(2)^t$
1	$8460(2)^t$	$48599(2)^t$
2	$388976(2)^t$	$2234497(2)^t$
3	$17884436(2)^t$	$102738263(2)^t$
4	$822295080(2)^t$	$4723725601(2)^t$

A. A Few Interesting Relations Between The Solutions Are Given Below

- 1)  $x_{n+3} - 1057x_{n+1} - 184y_{n+1} = 0$
- 2)  $23x_{n+2} - x_{n+1} - 4y_{n+2} = 0$
- 3)  $23x_{n+2} - 23x_{n+1} - 184y_{n+2} = 0$
- 4)  $1057x_{n+3} - x_{n+1} - 184y_{n+3} = 0$
- 5)  $1057y_{n+1} + 6072x_{n+1} - y_{n+3} = 0$
- 6)  $1057y_{n+2} + 132x_{n+1} - 23y_{n+3} = 0$
- 7)  $x_{n+1} - 46x_{n+2} - x_{n+3} = 0$
- 8)  $4y_{n+1} + 1057x_{n+2} - 23x_{n+3} = 0$

$$9) 4y_{n+3} + x_{n+2} - 23x_{n+3} = 0$$

$$10) 23x_{n+1} - x_{n+2} + 4y_{n+1} = 0$$

$$11) 23y_{n+2} - 132x_{n+2} + y_{n+1} = 0$$

$$12) x_{n+3} - 23x_{n+2} - 4y_{n+2} = 0$$

$$13) y_{n+3} - 132x_{n+2} - 23y_{n+2} = 0$$

$$14) 23x_{n+1} - 1057x_{n+2} + 4y_{n+3} = 0$$

$$15) y_{n+1} + 132x_{n+2} - 23y_{n+3} = 0$$

$$16) 23y_{n+1} + 132x_{n+3} - 1057y_{n+2} = 0$$

$$17) 23y_{n+3} - 132x_{n+3} - y_{n+2} = 0$$

$$18) y_{n+1} + 6072x_{n+3} - 1057y_{n+3} = 0$$

$$19) 132x_{n+1} + 23y_{n+1} - y_{n+2} = 0$$

$$20) y_{n+3} + y_{n+1} - 46y_{n+2} = 0$$

B. Each Of The Following Expression Represents A Nasty Number

$$1) \frac{1}{4(2)^t} (276x_{2n+3} - 12684x_{2n+2} + 48(2)^t)$$

$$2) \frac{1}{184(2)^t} (276x_{2n+4} - 583188x_{2n+2} + 2208(2)^t)$$

$$3) \frac{1}{2^t} (276y_{2n+2} - 1584x_{2n+2} + 12(2)^t)$$

$$4) \frac{1}{23(2)^t} (276y_{2n+3} - 72864x_{2n+2} + 276(2)^t)$$

$$5) \frac{1}{1057(2)^t} (276y_{2n+4} - 3350160x_{2n+2} + 12684(2)^t)$$

$$6) \frac{1}{4(2)^t} (12684x_{2n+4} - 583188x_{2n+3} + 48(2)^t)$$

$$7) \frac{1}{23(2)^t} (12684y_{2n+2} - 1584x_{2n+3} + 276(2)^t)$$

$$8) \frac{1}{2^t} (12684y_{2n+3} - 72864x_{2n+3} + 12(2)^t)$$

$$9) \frac{1}{23(2)^t} (12684y_{2n+4} - 3350160x_{2n+3} + 276(2)^t)$$

$$10) \frac{1}{1057(2)^t} (583188y_{2n+2} - 1584x_{2n+4} + 12684(2)^t)$$

$$11) \frac{1}{23(2)^t} (583188y_{2n+3} - 72864x_{2n+4} + 276(2)^t)$$

$$12) \frac{1}{2^t} (583188y_{2n+4} - 3350160x_{2n+4} + 12(2)^t)$$

$$13) \frac{1}{4(2)^t} (2208y_{2n+2} - 48y_{2n+3} + 48(2)^t)$$

$$14) \frac{1}{184(2)^t} (101520y_{2n+2} - 48y_{2n+4} + 2208(2)^t)$$

$$15) \frac{1}{4(2)^t} (101520y_{2n+3} - 2208y_{2n+4} + 48(2)^t)$$

C. Each Of The Following Expression Represents A Cubical Integer

$$1) \frac{1}{2(2)^t} [69x_{n+2} - 3171x_{n+1} + 23x_{3n+4} - 1057x_{3n+3}]$$

$$2) \frac{1}{4(2)^t} [3x_{n+3} - 6339x_{n+1} + x_{3n+5} - 2113x_{3n+3}]$$

$$3) \frac{1}{2^t} [138y_{n+1} - 792x_{n+1} + 46y_{3n+3} - 264x_{3n+3}]$$

$$4) \frac{1}{2^t} [6y_{n+2} - 1584x_{n+1} + 2y_{3n+4} - 528x_{3n+3}]$$

$$5) \frac{1}{1057(2)^t} [138y_{n+3} - 1675080x_{n+1} + 46y_{3n+5} - 558360x_{3n+3}]$$

$$6) \frac{1}{2(2)^t} [3171x_{n+3} - 145797x_{n+2} + 1057x_{3n+5} - 48599x_{3n+4}]$$

$$7) \frac{1}{23(2)^t} [6342y_{n+1} - 792x_{n+2} + 2114y_{3n+3} - 264x_{3n+4}]$$

$$8) \frac{1}{2^t} [6342y_{n+2} - 36432x_{n+2} + 2114y_{3n+4} - 12144x_{3n+4}]$$

$$9) \frac{1}{23(2)^t} [6342y_{n+3} - 1675080x_{n+2} + 2114y_{3n+5} - 558360x_{3n+4}]$$

$$10) \frac{1}{1057(2)^t} [291594y_{n+1} - 792x_{n+3} + 97198y_{3n+3} - 264x_{3n+5}]$$

$$11) \frac{1}{2^t} [12678y_{n+2} - 1584x_{n+3} + 4226y_{3n+4} - 528x_{3n+5}]$$

$$12) \frac{1}{2^t} [291594y_{n+3} - 1675080x_{n+3} + 97198y_{3n+5} - 1558360x_{3n+5}]$$

$$13) \frac{1}{2^t} [276y_{n+1} - 6y_{n+2} + 92y_{3n+3} - 2y_{3n+4}]$$

$$14) \frac{1}{23(2)^t} [6345y_{n+1} - 3y_{n+3} + 2115y_{3n+3} - y_{3n+5}]$$

$$15) \frac{1}{2^t} [12690y_{n+2} - 276y_{n+3} + 4230y_{3n+4} - 92y_{3n+5}]$$

D. Each Of The Following Expression Represents A Bi-Quadratic Integer

$$1) \frac{1}{2(2)^t} [92x_{2n+3} - 4228x_{2n+2} + 23x_{4n+5} - 1057x_{4n+4} + 12(2)^t]$$

$$2) \frac{1}{4(2)^t} [23x_{2n+4} - 8452x_{2n+2} + x_{4n+6} - 2113x_{4n+4} + 24(2)^t]$$

$$3) \frac{1}{2^t} [184y_{2n+2} - 1056x_{2n+2} + 46y_{4n+4} - 264x_{4n+4} + 6(2)^t]$$

$$4) \frac{1}{2^t} [8y_{2n+3} - 2112x_{2n+2} + 2y_{4n+5} - 528x_{4n+4} + 6(2)^t]$$

$$5) \frac{1}{1057(2)^t} [1184y_{2n+4} - 2233440x_{2n+2} + 46y_{4n+6} - 558360x_{4n+4} + 6342(2)^t]$$

$$6) \frac{1}{2(2)^t} [4228x_{2n+4} - 194396x_{2n+3} + 1057x_{4n+6} - 48599x_{4n+5} + 12(2)^t]$$

$$7) \frac{1}{23(2)^t} [8456y_{2n+2} - 1056x_{2n+3} + 2114y_{4n+4} - 264x_{4n+5} + 138(2)^t]$$

$$8) \frac{1}{2^t} [8456y_{2n+3} - 48576x_{2n+3} + 2114y_{4n+5} - 12144x_{4n+5} + 6(2)^t]$$

$$9) \frac{1}{23(2)^t} [8456y_{2n+4} - 2233440x_{2n+3} + 2114y_{4n+6} - 558360x_{4n+5} + 138(2)^t]$$

$$10) \frac{1}{1057(2)^t} [388792y_{2n+2} - 1056x_{2n+4} + 97198y_{4n+4} - 264x_{4n+6} + 6342(2)^t]$$

$$11) \frac{1}{2^t} [16904y_{2n+3} - 2112x_{2n+4} + 4226y_{4n+5} - 528x_{4n+6} + 6(2)^t]$$

$$12) \frac{1}{2^t} [388792y_{2n+4} - 2233440x_{2n+4} + 97198y_{4n+6} - 558360x_{4n+6} + 6(2)^t]$$

$$13) \frac{1}{2^t} [368 y_{2n+2} - 8 y_{2n+3} + 92 y_{4n+4} - 2 y_{4n+5} + 6(2)^t]$$

$$14) \frac{1}{23(2)^t} [8460 y_{2n+2} - 4 y_{2n+4} + 2115 y_{4n+4} - y_{4n+6} + 138 (2)^t]$$

$$15) \frac{1}{2^t} [16920y_{2n+3} - 368y_{2n+4} + 4230y_{4n+5} - 92y_{4n+6} + 6(2)^t]$$

E. Each Of The Following Expression Represents A Quintic Integer

- 1)  $\frac{1}{4(2)^t} [10x_{n+3} - 21130x_{n+1} + 5x_{3n+5} - 10565x_{3n+3} + x_{5n+7} - 2113x_{5n+5}]$
- 2)  $\frac{1}{8(2)^t} [20x_{n+3} - 42260x_{n+1} + 10x_{3n+5} - 21130x_{3n+3} + 2x_{5n+7} - 4226x_{5n+5}]$
- 3)  $\frac{1}{2^t} [460y_{n+1} - 2640x_{n+1} + 230y_{3n+3} - 1320x_{3n+3} + 46y_{5n+5} - 264x_{5n+5}]$
- 4)  $\frac{1}{2^t} [20y_{n+2} - 5280x_{n+1} + 10y_{3n+4} - 2640x_{3n+3} + 2y_{5n+6} - 528x_{5n+5}]$
- 5)  $\frac{1}{1057(2)^t} [460y_{n+3} - 5583600x_{n+1} + 230y_{3n+5} - 2791800x_{3n+3} + 46y_{5n+7} - 558360x_{5n+5}]$
- 6)  $\frac{1}{2(2)^t} [10570x_{n+3} - 485990x_{n+2} + 5285x_{3n+5} - 242995x_{3n+4} + 1057x_{5n+7} - 48599x_{5n+6}]$
- 7)  $\frac{1}{23(2)^t} [21140y_{n+1} - 2640x_{n+2} + 10570y_{3n+3} - 1320x_{3n+4} + 2114y_{5n+5} - 264x_{5n+6}]$
- 8)  $\frac{1}{2^t} [21140y_{n+2} - 121440x_{n+2} + 10570y_{3n+4} - 60720x_{3n+4} + 2114y_{5n+6} - 12144x_{5n+6}]$
- 9)  $\frac{1}{23(2)^t} [21140y_{n+3} - 5583600x_{n+2} + 10570y_{3n+5} - 2791800x_{3n+4} + 2114y_{5n+7} - 558360x_{5n+6}]$
- 10)  $\frac{1}{1057(2)^t} [971980y_{n+1} - 2640x_{n+3} + 485990y_{3n+3} - 1320x_{3n+5} + 97198y_{5n+5} - 264x_{5n+7}]$
- 11)  $\frac{1}{2^t} [84520y_{n+2} - 5280x_{n+3} + 21130y_{3n+4} - 2640x_{3n+5} + 4226y_{5n+6} - 528x_{5n+7}]$
- 12)  $\frac{1}{2^t} [971980y_{n+3} - 5583600x_{n+3} + 485990y_{3n+5} - 2791800x_{3n+5} + 97198y_{5n+7} - 558360x_{5n+7}]$
- 13)  $\frac{1}{2^t} [920y_{n+1} - 20y_{n+2} + 460y_{3n+3} - 10y_{3n+4} + 92y_{5n+5} - 2y_{5n+6}]$
- 14)  $\frac{1}{8(2)^t} [21150 y_{n+1} - 10 y_{n+3} + 10575 y_{3n+3} - 5 y_{3n+5} + 2115 y_{5n+5} - y_{5n+7}]$
- 15)  $\frac{1}{2^t} [42300 y_{n+2} - 920 y_{n+3} + 21150 y_{3n+4} - 460 y_{3n+5} + 4230 y_{5n+6} - 92 y_{5n+7}]$

### III. REMARKABLE OBSERVATIONS

1) The solutions of (1) in terms of special integers namely, generalized Fibonacci  $GF_n$  and

Lucas  $GL_n$  are exhibited below.

$$x_{n+1} = 2(2)^t GL_{n+1}(46, -1) + 92(2)^t GF_{n+1}(46, -1)$$

$$y_{n+1} = \frac{23(2)^t}{2} GL_{n+1}(46, -1) + 528(2)^t GF_{n+1}(46, -1)$$

2) Employing The Linear Combinations Among The Solutions Of (1), One May Generate Integer Solutions For Other Choices Of Hyperbola Which Are Presented In The Table 2 Below

Table: 2 Hyperbolas

S. No	Hyperbola	(X, Y)
1	$X^2 - 33Y^2 = 64(2^t)^2$	$(46x_{n+2} - 2114x_{n+1}, 368x_{n+1} - 8x_{n+2})$
2	$X^2 - 33Y^2 = 135424(2^t)^2$	$(46x_{n+3} - 97198x_{n+1}, 16920x_{n+1} - 8x_{n+3})$
3	$X^2 - 33Y^2 = 4(2^t)^2$	$(46y_{n+1} - 264x_{n+1}, 46x_{n+1} - 8y_{n+1})$
4	$X^2 - 33Y^2 = 2116(2^t)^2$	$(46y_{n+2} - 12144x_{n+1}, 2114x_{n+1} - 8y_{n+2})$
5	$X^2 - 33Y^2 = 4468996(2^t)^2$	$(46y_{n+3} - 558360x_{n+1}, 97198x_{n+1} - 8y_{n+3})$
6	$X^2 - 33Y^2 = 64(2^t)^2$	$(2114x_{n+3} - 97198x_{n+2}, 16920x_{n+2} - 368x_{n+3})$
7	$X^2 - 33Y^2 = 2116(2^t)^2$	$(2114y_{n+1} - 264x_{n+2}, 46x_{n+2} - 368y_{n+1})$
8	$X^2 - 33Y^2 = 4(2^t)^2$	$(2114y_{n+2} - 12144x_{n+2}, 2114x_{n+2} - 368y_{n+2})$
9	$X^2 - 33Y^2 = 2116(2^t)^2$	$(2114y_{n+3} - 558360x_{n+2}, 97198x_{n+2} - 368y_{n+3})$
10	$X^2 - 33Y^2 = 4468996(2^t)^2$	$(97198y_{n+1} - 264x_{n+3}, 46x_{n+3} - 16920y_{n+1})$
11	$X^2 - 33Y^2 = 2116(2^t)^2$	$(97198y_{n+2} - 12144x_{n+3}, 2114x_{n+3} - 16920y_{n+2})$
12	$X^2 - 33Y^2 = 4(2^t)^2$	$(97198y_{n+3} - 558360x_{n+3}, 97198x_{n+3} - 16920y_{n+3})$
13	$33X^2 - Y^2 = 2112(2^t)^2$	$(368y_{n+1} - 8y_{n+2}, 46y_{n+2} - 2114y_{n+1})$
14	$33X^2 - Y^2 = 4468992(2^t)^2$	$(16920y_{n+1} - 8y_{n+3}, 46y_{n+3} - 97198y_{n+1})$
15	$33X^2 - Y^2 = 2112(2^t)^2$	$(16920y_{n+2} - 368y_{n+3}, 2114y_{n+3} - 97198y_{n+2})$

3) Employing Linear Combinations Among The Solutions Of (1), One May Generate Integer Solutions For Other Choices Of Parabola Which Are Presented In The Table 3 Below

Table: 3 Parabolas

S. No	Parabola	(X, Y)
1	$2^t X - 132Y^2 = 8(4)^t$	$(46x_{2n+3} - 2114x_{2n+2}, 368x_{n+1} - 8x_{n+2})$
2	$184(2)^t X - 33Y^2 = 67712(4)^t$	$(46x_{2n+4} - 97198x_{2n+2}, 16920x_{n+1} - 8x_{n+3})$
3	$2^t X - 33Y^2 = 2(4)^t$	$(46y_{2n+2} - 264x_{2n+2}, 46x_{n+1} - 8y_{n+1})$
4	$23(2)^t X - 33Y^2 = 1058(4)^t$	$(46y_{2n+3} - 12144x_{2n+2}, 2114x_{n+1} - 8y_{n+2})$
5	$1057(2)^t X - 33Y^2 = 2234498(4)^t$	$(46y_{2n+4} - 558360x_{2n+2}, 97198x_{n+1} - 8y_{n+3})$
6	$4(2)^t X - 33Y^2 = 32(4)^t$	$(2114x_{2n+4} - 97198x_{2n+3}, 16920x_{n+2} - 368x_{n+3})$
7	$23(2)^t X - 33Y^2 = 1058(2)^t$	$(2114y_{2n+2} - 264x_{2n+3}, 46x_{n+2} - 368y_{n+1})$
8	$2^t X - 33Y^2 = 2(4)^t$	$(2114y_{2n+3} - 12144x_{2n+3}, 2114x_{n+2} - 368y_{n+2})$
9	$23(2)^t X - 33Y^2 = 1058(4)^t$	$(2114y_{2n+4} - 558360x_{2n+3}, 97198x_{n+2} - 368y_{n+3})$
10	$1057(2)^t X - 33Y^2 = 2234498(4)^t$	$(97198y_{2n+2} - 264x_{2n+4}, 46x_{n+3} - 16920y_{n+1})$
11	$23(2)^t X - 33Y^2 = 1058(4)^t$	$(97198y_{2n+3} - 12144x_{2n+4}, 2114x_{n+3} - 16920y_{n+2})$
12	$2^t X - 33Y^2 = 2(4)^t$	$(97198y_{2n+4} - 558360x_{2n+4}, 97198x_{n+3} - 16920y_{n+3})$
13	$132(2)^t X - Y^2 = 1056(4)^t$	$(368y_{2n+2} - 8y_{2n+3}, 46y_{n+2} - 2114y_{n+1})$
14	$6072(2)^t X - Y^2 = 2234496(4)^t$	$(16920y_{2n+2} - 8y_{2n+4}, 46y_{n+3} - 97198y_{n+1})$
15	$132(2)^t X - Y^2 = 1056(4)^t$	$(16920y_{2n+3} - 368y_{2n+4}, 2114y_{n+3} - 97198y_{n+2})$

- 4) Employing the following solution  $(x, y)$ , each of the following expressions among the special polygonal, pyramidal, star, pronic and centered polygonal number is congruent to under modulo 4.

$$\left(\frac{3P_{y-2}^3}{t_{3,y-2}}\right)^2 - \left(\frac{198P_x^3}{Pr_{x+1}}\right)^2$$

$$\left(\frac{12P_y^5}{S_{y+1}-1}\right)^2 - \left(\frac{1188P_{x-2}^3}{S_{x-2}-1}\right)^2$$

$$\left(\frac{P_y^5}{t_{3,y}}\right)^2 - \left(\frac{396P_x^5}{S_{x+1}-1}\right)^2$$

$$\left(\frac{2P_y^5}{Ct_{4,y}-1}\right)^2 - \left(\frac{198P_x^5}{Ct_{6,x}}\right)^2$$

$$\left(\frac{4Pt_{n-3}}{P_{n-3}^3}\right)^2 - \left(\frac{132P_x^5}{Ct_{4,x}-1}\right)^2$$

#### IV. CONCLUSION

To conclude, one may search for other choices of positive Pell equations for finding their integer solutions with suitable properties.

#### REFERENCES

- [1] L.E.Dickson, History of Theory of numbers, Vol.2, Chelsea Publishing Company, New York, (1952).
- [2] L.J.Mordell, Diophantine Equations, Academic Press, New York, (1970).
- [3] M.A.Gopalan and G.Janaki, Observation On  $y^2 = 3x^2 + 1$ , Acta Ciancia Indica, XXXIVM(2) (2008) 639-696.
- [4] L.E.Dickson, History of theory of numbers and Diophantine analysis, Vol 2, Dover publications, New York,(2005).
- [5] Dr.G.Sumathi "Observations on the hyperbola"  $y^2 = 182x^2 + 14$ , Journal of mathematics and Informatics, Vol 11, 73-81, 2017.
- [6] Dr.G.Sumathi "Observations on the Pell Equation  $y^2 = 14x^2 + 4$ " International Journal of Creative Research Thoughts, Vol 6, Issue 1, Pp1074-1084, March 2018.
- [7] Dr.G.Sumathi "Observations on the Equation  $y^2 = 312x^2 + 1$ " International Journal of Mathematics Trends and Technology, Vol 50, Issue4, 31-34, Oct 2017.
- [8] Dr.G.Sumathi "Observations on the hyperbola  $y^2 = 150x^2 + 16$ " International Journal of recent Trends in Engineering and Research, Vol 3, Issue 9, Pp198-206, sep 2017.



10.22214/IJRASET



45.98



IMPACT FACTOR:  
7.129



IMPACT FACTOR:  
7.429



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24\*7 Support on Whatsapp)