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International Journal For Research in  
Applied Science and Engineering Technology



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# **INTERNATIONAL JOURNAL FOR RESEARCH**

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

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**Volume: 7      Issue: III      Month of publication: March 2019**

**DOI: <http://doi.org/10.22214/ijraset.2019.3057>**

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# On the special Diophantine Equation $y^2 = 7x^2 + 9^t, t \geq 0$

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**Abstract:** The binary quadratic equation  $y^2 = 7x^2 + 9^t, t \geq 0$  representing hyperbola is considered for finding its integer solutions. A few interesting properties among the solutions are presented. Also, we present infinitely many positive integer solutions in terms of generalized Fibonacci sequences of numbers, generalized Lucas sequences of numbers.

**Keywords:** Binary quadratic integral solution, generalized Fibonacci sequences of numbers, generalized Lucas sequences of numbers.

AMS Mathematics subject classification: 11D09

Notations

$GF_n(K, S)$ : Generalized Fibonacci sequences of rank n.

$GL_n(K, S)$ : Generalized Lucas sequences of rank n.

$$t_{m,n} = n \left( 1 + \frac{(n-1)(m-2)}{2} \right)$$

$$P_n^m = \frac{[n(n+1)((m-2)(n+(5-m)))]}{6}$$

$$Pr_n = n(n+1)$$

$$Ct_{m,n} = \frac{mn(n+1)}{2} + 1$$

## I. INTRODUCTION

The binary quadratic equation of the form  $y^2 = Dx^2 + 1$  where D non-square positive integer has been studied by various mathematician for its non-trivial integral solutions. When D takes different integral values [1,2,4]. In [3] infinitely many Pythagorean triangles in each of which hypotenuse is four time the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation  $y^2 = 3x^2 + 1$ . In [5] a, special Pythagorean triangle is obtained by employing the integral solution of  $y^2 = 182x^2 + 14$ . In [6] different pattern of infinitely many Pythagorean triangles are obtained by employing the non-integral solutions of  $y^2 = 14x^2 + 4$ . In this context one may also refer [7,8]. These results have motivated us to search for the integral solutions of yet another binary quartic equation is  $y^2 = 7x^2 + 9^t, t \geq 0$  representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration a few patterns of Pythagorean triangles are obtained.

## II. METHOD OF ANALYSIS

Consider the binary quadratic equation

$$y^2 = 7x^2 + 9^t, t \geq 0 \tag{1}$$

with least positive integer solution of (1),

$$x_0 = 3(3^t), y_0 = 8(3^t), D = 7$$

To obtain the other solutions of (1) consider the pell equation

$$y^2 = 7x^2 + 1 \tag{2}$$

whose general solution  $(\tilde{x}_n, \tilde{y}_n)$  is represented by,

$$\tilde{x}_n = \frac{1}{2\sqrt{7}} g_n, \tilde{y}_n = \frac{1}{2} f_n$$

In which,

$$f_n = (8 + 3\sqrt{7})^{n+1} + (8 - 3\sqrt{7})^{n+1}$$

$$g_n = (8 + 3\sqrt{7})^{n+1} - (8 - 3\sqrt{7})^{n+1}$$

Applying Brahmagupta lemma between  $(x_0, y_0)$  and  $(\tilde{x}_n, \tilde{y}_n)$  the general solution of equation (1) are given to be

$$x_{n+1} = \frac{3(3^t)}{2} f_n + \frac{4(3^t)}{\sqrt{7}} g_n \quad ,$$

$$y_{n+1} = 4(3^t) f_n + \frac{3\sqrt{7}(3^t)}{2} g_n \quad ,$$

The recurrence relation satisfied by the values  $x_{n+1}$  and  $y_{n+1}$  are respectively,

$$x_{n+3} - 16x_{n+2} + x_{n+1} = 0 \quad ,$$

$$y_{n+3} - 16y_{n+2} + y_{n+1} = 0 \quad ,$$

A few numerical examples are in the table 1 below:

Table 1: Numerical Examples

n	$x_{n+1}$	$y_{n+1}$
0	$48(3^t)$	$127(3^t)$
1	$765(3^t)$	$2024(3^t)$
2	$12192(3^t)$	$32257(3^t)$
3	$194307(3^t)$	$514088(3^t)$
4	$3096720(3^t)$	$8193151(3^t)$
5	$49353213(3^t)$	$130576328(3^t)$
6	$786554688(3^t)$	$2081028097(3^t)$

A. A Few Interesting Relations Between The Solutions Are Given Below

$$1) x_{n+2} - 8x_{n+1} - 3y_{n+1} = 0$$

$$2) x_{n+1} - 8x_{n+2} + 3y_{n+2} = 0$$

$$3) 8x_{n+1} - 127x_{n+2} + 3y_{n+3} = 0$$

$$4) 48y_{n+1} - x_{n+3} + 127x_{n+1} = 0$$

$$5) x_{n+1} + 6y_{n+2} - x_{n+3} = 0$$

$$6) 48y_{n+3} - 127x_{n+3} + x_{n+1} = 0$$

$$7) 3y_{n+1} - 8x_{n+3} + 127x_{n+2} = 0$$

$$8) 3y_{n+2} + 8x_{n+2} - x_{n+3} = 0$$

$$9) x_{n+2} - 8x_{n+3} + 3y_{n+3} = 0$$

$$10) 127y_{n+1} - y_{n+3} + 336x_{n+1} = 0$$

$$11) 42x_{n+2} - y_{n+3} + y_{n+1} = 0$$

$$12) 336x_{n+3} - 127y_{n+3} + y_{n+1} = 0$$

$$13) y_{n+2} - 21x_{n+1} - 8y_{n+1} = 0$$

$$14) 21x_{n+2} - 8y_{n+2} + y_{n+1} = 0$$

$$15) 21x_{n+3} - 127y_{n+2} + 8y_{n+1} = 0$$

$$16) 127y_{n+2} - 8y_{n+3} + 21x_{n+1} = 0$$

$$17) 8y_{n+2} - y_{n+3} + 21x_{n+2} = 0$$

$$18) y_{n+2} - 8y_{n+3} + 21x_{n+3} = 0$$

$$19) x_{n+1} - 16x_{n+2} + x_{n+3} = 0$$

$$20) y_{n+1} - 16y_{n+2} + y_{n+3} = 0$$

B. Each Of The Following Expressions Represents A Nasty Number

$$1) \frac{1}{3(3^t)} [96x_{2n+3} - 1524x_{2n+2} + 36(3^t)]$$

$$2) \frac{1}{3(3^t)} [6x_{2n+4} - 1518x_{2n+2} + 36(3^t)]$$

$$3) \frac{1}{3^t} [96y_{2n+2} - 252x_{2n+2} + 12(3^t)]$$

$$4) \frac{1}{3^t} [12y_{2n+3} - 504x_{2n+2} + 12(3^t)]$$

$$5) \frac{1}{127(3^t)} [96y_{2n+4} - 64260x_{2n+2} + 1524(3^t)]$$

$$6) \frac{1}{3(3^t)} [1524x_{2n+4} - 24288x_{2n+3} + 36(3^t)]$$

$$7) \frac{1}{8(3^t)} [1524y_{2n+2} - 252x_{2n+3} + 96(3^t)]$$

$$8) \frac{1}{3^t} [1524y_{2n+3} - 4032x_{2n+3} + 12(3^t)]$$

$$9) \frac{1}{8(3^t)} [1524y_{2n+4} - 64260x_{2n+3} + 96(3^t)]$$

$$10) \frac{1}{127(3^t)} [24288y_{2n+2} - 252x_{2n+4} + 1524(3^t)]$$

$$11) \frac{1}{3^t} [3036y_{2n+3} - 504x_{2n+4} + 12(3^t)]$$

$$12) \frac{1}{3^t} [24288y_{2n+4} - 64260x_{2n+4} + 12(3^t)]$$

$$13) \frac{1}{3^t} [192y_{2n+2} - 12y_{2n+3} + 12(3^t)]$$

$$14) \frac{1}{16(3^t)} [3060y_{2n+2} - 12y_{2n+4} + 192(3^t)]$$

$$15) \frac{1}{3(3^t)} [9180y_{2n+3} - 576y_{2n+4} + 36(3^t)]$$

C. Each Of The Following Expressions Is A Cubical Integer

$$1) \frac{1}{3(3^t)} [16x_{3n+4} - 254x_{3n+3} + 48x_{n+2} - 762x_{n+1}]$$

$$2) \frac{1}{3^t} [16y_{3n+3} - 42x_{3n+3} + 48y_{n+1} - 126x_{n+1}]$$

$$3) \frac{1}{3^t} [2y_{3n+4} - 84x_{3n+3} + 6y_{n+2} - 252x_{n+1}]$$

$$4) \frac{1}{127(3^t)} [16y_{3n+5} - 10710x_{3n+3} + 48y_{n+3} - 32130x_{n+1}]$$

$$5) \frac{1}{3(3^t)} [254x_{3n+5} - 4048x_{3n+4} + 762x_{n+3} - 12144x_{n+2}]$$

$$6) \frac{1}{8(3^t)} [254y_{3n+3} - 42x_{3n+4} + 762y_{n+1} - 126x_{n+2}]$$

$$7) \frac{1}{3^t} [254y_{3n+4} - 672x_{3n+4} + 762y_{n+2} - 2016x_{n+2}]$$

$$8) \frac{1}{8(3^t)} [254y_{3n+5} - 10710x_{3n+4} + 762y_{n+3} - 10710x_{n+2}]$$

$$9) \frac{1}{127(3^t)} [4048y_{3n+3} - 42x_{3n+5} + 12144y_{n+1} - 126x_{n+3}]$$

$$10) \frac{1}{3^t} [506y_{3n+4} - 84x_{3n+5} + 1518y_{n+2} - 252x_{n+3}]$$

$$11) \frac{1}{3^t} [4048y_{3n+5} - 10710x_{3n+5} + 12144y_{n+3} - 32130x_{n+3}]$$

$$12) \frac{1}{3^t} [32y_{3n+3} - 2y_{3n+4} + 96y_{n+1} - 6y_{n+2}]$$

$$13) \frac{1}{16(3^t)} [510y_{3n+3} - 2y_{3n+5} + 1530y_{n+1} - 6y_{n+3}]$$

$$14) \frac{1}{3(3^t)} [1530y_{3n+4} - 96y_{3n+5} + 4590y_{n+2} - 288y_{n+3}]$$

D. Each Of The Following Expressions Represent A Bi-Quadratic Integer

$$1) \frac{1}{3(3^t)} [16x_{4n+5} - 254x_{4n+4} + 64x_{2n+3} - 1016x_{2n+2} + 18(3^t)]$$

$$2) \frac{1}{3(3^t)} [x_{4n+6} - 253x_{4n+4} + 4x_{2n+4} - 1012x_{2n+2} + 18(3^t)]$$

$$3) \frac{1}{3^t} [16y_{4n+4} - 42x_{4n+4} + 64y_{2n+2} - 168x_{2n+2} + 3^t]$$

$$4) \frac{1}{3^t} [2y_{4n+5} - 84x_{4n+4} + 8y_{2n+3} - 336x_{2n+2} + 3^t]$$

$$5) \frac{1}{127(3^t)} [16y_{4n+6} - 10710x_{4n+4} + 64y_{2n+4} - 42480x_{2n+2} + 762(3^t)]$$

$$6) \frac{1}{3(3^t)} [254x_{4n+6} - 4048x_{4n+5} + 1016x_{2n+4} - 16192x_{2n+3} + 18(3^t)]$$

$$7) \frac{1}{8(3^t)} [254y_{4n+4} - 42x_{4n+5} + 1016y_{n+1} - 168x_{n+2} + 48(3^t)]$$

$$8) \frac{1}{3^t} [254y_{4n+5} - 672x_{4n+5} + 1016y_{2n+3} - 672x_{2n+3} + 3^t]$$

$$9) \frac{1}{8(3^t)} [254y_{4n+6} - 10710x_{4n+5} + 1016y_{2n+4} - 4284x_{2n+3} + 48(3^t)]$$

$$10) \frac{1}{127(3^t)} [4048y_{4n+4} - 42x_{4n+6} + 16192y_{2n+2} - 168x_{2n+4} + 762(3^t)]$$

$$11) \frac{1}{3^t} [506y_{4n+5} - 84x_{4n+6} + 2024y_{2n+3} - 336x_{2n+4} + 3^t]$$

$$12) \frac{1}{3^t} [4048y_{4n+6} - 10710x_{4n+6} + 16192y_{2n+4} - 42840x_{2n+4} + 3^t]$$

$$13) \frac{1}{3^t} [32y_{4n+4} - 2y_{4n+5} + 128y_{2n+2} - 8y_{2n+3} + 3^t]$$

$$14) \frac{1}{16(3^t)} [510y_{4n+4} - 2y_{4n+6} + 2040y_{2n+2} - 8y_{2n+4} + 96(3^t)]$$

$$15) \frac{1}{3(3^t)} [1530y_{4n+5} - 96y_{4n+6} + 6120y_{n+2} - 384y_{n+3} + 18(3^t)]$$

E. Each Of The Following Expression Represents A Quantic Integer

$$1) \frac{1}{3(3^t)} [16x_{5n+6} - 254x_{5n+5} + 80x_{3n+4} - 1270x_{3n+3} + 160x_{n+2} - 2540x_{n+1}]$$

$$2) \frac{1}{3(3^t)} [x_{5n+7} - 253x_{5n+5} + 5x_{3n+5} - 1265x_{3n+3} + 10x_{n+3} - 2530x_{n+1}]$$

$$3) \frac{1}{3^t} [16y_{5n+5} - 21x_{5n+5} + 80y_{3n+3} - 210x_{3n+3} + 160y_{n+1} - 420x_{n+1}]$$

$$4) \frac{1}{3^t} [2y_{5n+6} - 84x_{5n+5} + 10y_{3n+4} - 420x_{3n+3} + 20y_{n+2} - 840x_{n+1}]$$

$$5) \frac{1}{127(3^t)} [16y_{5n+7} - 10710x_{5n+5} + 80y_{3n+5} - 53550x_{3n+3} + 160y_{n+3} - 107100x_{n+1}]$$

$$6) \frac{1}{3(3^t)} [254x_{5n+7} - 4048x_{5n+6} + 1270x_{3n+5} - 20240x_{3n+4} + 2540x_{n+3} - 40480x_{n+2}]$$

$$7) \frac{1}{8(3^t)} [254y_{5n+5} - 42x_{5n+6} + 1270y_{3n+3} - 210x_{3n+4} + 2540y_{n+1} - 420x_{n+2}]$$

$$8) \frac{1}{3^t} [254y_{5n+6} - 672x_{5n+7} + 1270y_{3n+4} - 3360x_{3n+4} + 2540y_{n+2} - 10020x_{n+2}]$$

$$9) \frac{1}{8(3^t)} [254y_{5n+7} - 10710x_{5n+6} + 1270y_{3n+5} - 53550x_{3n+4} - 460y_{n+3} - 107100x_{n+2}]$$

$$10) \frac{1}{127(3^t)} [4048y_{5n+5} - 42x_{5n+7} + 20240y_{3n+3} - 210x_{3n+5} + 40480y_{n+1} - 420x_{n+3}]$$

$$11) \frac{1}{3^t} [506y_{5+6} - 84x_{5n+7} + 2530y_{3n+4} - 420x_{3n+5} + 5060y_{n+2} - 840x_{n+3}]$$

$$12) \frac{1}{3^t} [4048y_{5n+7} - 10710x_{5n+7} + 20240y_{3n+5} - 53550x_{3n+5} + 40480y_{n+3} - 107100x_{n+3}]$$

$$13) \frac{1}{3^t} [32y_{5n+5} - 2y_{5n+6} + 160y_{3n+3} - 10y_{3n+4} + 320y_{n+1} - 25y_{n+2}]$$

$$14) \frac{1}{16(3^t)} [510y_{5n+5} - 2y_{5n+7} + 2550y_{n+3} - 10y_{3n+5} + 5100y_{n+1} - 25y_{n+3}]$$

$$15) \frac{1}{3(3^t)} [1530y_{5n+6} - 96y_{5n+7} + 7650y_{3n+4} - 480y_{3n+5} + 15300y_{n+2} - 960y_{n+3}]$$

F. The solution of (1) in terms of special integers namely , generalized fibonacci sequence and generalized Lucas sequence are exhibited below

$$x_{n+1} = \frac{3(3^t)}{2} GL_{n+1}(16,-1) + 24(3^t)GF_{n+1}(16,-1)$$

$$y_{n+1} = 4(3^t)GL_{n+1}(16,-1) + 63(3^t)GF_{n+1}(16,-1)$$

### III. REMARKABLE OBSERVATION

Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbolas which are presented in table 2 below

Table 2: Hyperbola

S.NO	Hyperbola	(X,Y)
1	$4X^2 - 63Y^2 = 36(3^{2t})$	$(8x_{n+2} - 127x_{n+1}, 32x_{n+1} - 2x_{n+2})$
2	$64X^2 - 63Y^2 = 2304(3^{2t})$	$(x_{n+3} - 253x_{n+1}, 255x_{n+1} - x_{n+3})$
3	$4X^2 - 14Y^2 = 4(3^{2t})$	$(8y_{n+1} - 21x_{n+1}, 8x_{n+1} - 3y_{n+1})$
4	$16X^2 - 7Y^2 = 64(3^{2t})$	$(2y_{n+2} - 84x_{n+1}, 127x_{n+1} - 3y_{n+2})$
5	$X^2 - 7Y^2 = 64516(3^{2t})$	$(16y_{n+3} - 10710x_{n+1}, 4048x_{n+1} - 6y_{n+3})$
6	$X^2 - 63Y^2 = 36(3^{2t})$	$(254x_{n+3} - 4048x_{n+2}, 510x_{n+2} - 32x_{n+3})$
7	$X^2 - 448Y^2 = 256(3^{2t})$	$(254y_{n+1} - 42x_{n+2}, 2x_{n+2} - 12y_{n+1})$
8	$4X^2 - 28Y^2 = 4(3^{2t})$	$(127y_{n+2} - 336x_{n+2}, 127x_{n+2} - 48y_{n+2})$
9	$X^2 - 7Y^2 = 256(3^{2t})$	$(254y_{n+3} - 10710x_{n+2}, 4048x_{n+2} - 96y_{n+3})$
10	$X^2 - 7Y^2 = 64516(3^{2t})$	$(4048y_{n+1} - 42x_{n+3}, 16x_{n+3} - 1530y_{n+1})$
11	$64X^2 - 7Y^2 = 256(3^{2t})$	$(506y_{n+2} - 84x_{n+3}, 254x_{n+3} - 1530y_{n+2})$
12	$X^2 - 28Y^2 = 4(3^{2t})$	$(4048y_{n+3} - 1530y_{n+2}, 2024x_{n+3} - 765y_{n+3})$
13	$63X^2 - Y^2 = 252(3^{2t})$	$(32y_{n+1} - 2y_{n+2}, 16y_{n+2} - 254y_{n+1})$
14	$252X^2 - 256Y^2 = 258048(3^{2t})$	$(510y_{n+1} - 2y_{n+3}, 2y_{n+3} - 506y_{n+1})$
15	$63X^2 - 9Y^2 = 2268(3^{2t})$	$(1530y_{n+2} - 96y_{n+3}, 254y_{n+3} - 4048y_{n+2})$

- 1) Employing linear combination among the solutions of (1), one may generate integer solutions for other choices of parabolas which are presented in table 3 below

Table 3: Parabola

S.NO	Parabola	(X,Y)
1	$2(3^t)X - 7Y^2 = 6(3^{2t})$	$(8x_{n+2} - 127x_{n+1}, 32x_{n+1} - 2x_{n+2})$
2	$64(3^t)X - 7Y^2 = 384(3^{2t})$	$(x_{n+3} - 253x_{n+1}, 255x_{n+1} - x_{n+3})$
3	$2(3^t)X - 28Y^2 = 2(3^{2t})$	$(8y_{n+1} - 21x_{n+1}, 8x_{n+1} - 3y_{n+1})$
4	$16(3^t)X - 7Y^2 = 32(3^{2t})$	$(2y_{n+2} - 84x_{n+1}, 127x_{n+1} - 3y_{n+2})$
5	$127(3^t)X - 7Y^2 = 32258(3^{2t})$	$(16y_{n+3} - 10710x_{n+1}, 4048x_{n+1} - 6y_{n+3})$
6	$3(3^t)X - 7Y^2 = 18(3^{2t})$	$(254x_{n+3} - 4048x_{n+2}, 510x_{n+2} - 32x_{n+3})$
7	$(3^t)X - 7Y^2 = 16(3^{2t})$	$(254y_{n+1} - 42x_{n+2}, 2x_{n+2} - 12y_{n+1})$
8	$2(3^t)X - 28Y^2 = 2(3^{2t})$	$(127y_{n+2} - 336x_{n+2}, 127x_{n+2} - 48y_{n+2})$
9	$8(3^t)X - 7Y^2 = 128(3^{2t})$	$(254y_{n+3} - 10710x_{n+2}, 4048x_{n+2} - 96y_{n+3})$
10	$127(3^t)X - 7Y^2 = 32258(3^{2t})$	$(4048y_{n+1} - 42x_{n+3}, 16x_{n+3} - 1530y_{n+1})$
11	$64(3^t)X - 7Y^2 = 128(3^{2t})$	$(506y_{n+2} - 84x_{n+3}, 254x_{n+3} - 1530y_{n+2})$
12	$(3^t)X - 28Y^2 = 2(3^{2t})$	$(4048y_{n+3} - 10710x_{n+3}, 2024x_{n+3} - 765y_{n+3})$
13	$63(3^t)X - Y^2 = 126(3^{2t})$	$(32y_{n+1} - 2y_{n+2}, 16y_{n+2} - 254y_{n+1})$
14	$252(3^t)X - 16Y^2 = 8064(3^{2t})$	$(510y_{n+1} - 2y_{n+3}, 2y_{n+3} - 506y_{n+1})$
15	$63(3^t)X - 3Y^2 = 378(3^{2t})$	$(1530y_{n+2} - 96y_{n+3}, 254y_{n+3} - 4048y_{n+2})$

- 2) Employing the following solutions(x,y), each of the following expressions among the special polygonal, star, pyramidal, pronic and centered polygonal number id congruent under modulo 9

$$\left(\frac{3p^3_{y-2}}{t_{3,y-2}}\right)^2 - \left(\frac{42p^3_x}{pr_{x+1}}\right)^2$$

$$\left(\frac{12p^5_y}{s_{y+1} - 1}\right) - \left(\frac{252p^3_{x-2}}{s_{x-2} - 1}\right)^2$$

$$\left(\frac{p^5_y}{t_{3,y}}\right)^2 - \left(\frac{84p^5_x}{s_{x+1} - 1}\right)^2$$

$$\left(\frac{2p_y^5}{ct_{4,y}-1}\right)^2 - \left(\frac{42p_x^5}{ct_{6,x}}\right)^2$$

$$\left(\frac{4pt_{n-3}}{p_{n-3}^3}\right)^2 - \left(\frac{28p_x^5}{ct_{4,x}-1}\right)^2$$

#### IV. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the negative Pell Equations  $y^2 = 7x^2 + 9t$ ,  $t \geq 0$ . As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of Pell Equations and determine their integer solutions along with suitable properties.

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