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Observation on the Binary Quadratic Equation $y^2 = 105x^2 + 4^t, t \geq 0$

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Abstract: The binary quadratic equation is considered and a few interesting properties among the solutions are presented.
Keywords: Binary quadratic, integral solutions, Generalized Fibonacci sequences, Generalized Lucas Sequences.

I. INTRODUCTION

The Binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is a non-square positive integer has been studied by various Mathematicians for its non-trivial integral solutions when D takes different integral values [1-5]. In this context one may also refer [4,5]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation representing a hyperbola. A few interesting properties among the solutions are presented.

II. NOTATIONS

1) $t_{m,n}$: Polygonal number of rank n with size m
2) P_n^m : Pyramidal number of rank n with size m
3) $\Pr_n$: Pronic number of rank n
4) S_n : Star number of rank n
5) $Ct_{m,n}$: Centered Pyramidal number of rank n with size m
6) $GF_n(k,s)$: Generalized Fibonacci sequence number of rank n
7) $GL_n(k,s)$: Generalized Lucas sequence number of rank n

III.METHOD OF ANALYSIS

The binary non-homogeneous quadratic Diophantine equation represents a hyperbola to be solved for its non-zero integral solutions is

$$y^2 = 105x^2 + 4^t, t \geq 0 \tag{1}$$

The smallest positive integer solution (x_0, y_0) of (1) is

$$x_0 = 4(2^t), y_0 = 41(2^t) \tag{2}$$

To obtain the other solutions of (1), consider the pellian equation

$$y^2 = 105x^2 + 1 \tag{3}$$

Applying the Brahmagupta lemma between the solutions (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = \frac{4(2^t)}{2} f_n + \frac{41(2^t)}{2\sqrt{105}} g_n$$

$$y_{n+1} = \frac{41(2^t)}{2} f_n + \frac{420(2^t)}{2\sqrt{105}} g_n$$

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A. A Few Numerical Examples Are Given In The Following Table 1

Tabe1: Examples

n	x_{n+1}	y_{n+1}
-1	$4(2^t)$	$41(2^t)$
0	$328(2^t)$	$3361(2^t)$
1	$26892(2^t)$	$275561(2^t)$
2	$2204816(2^t)$	$22592641(2^t)$

B. A Few Interesting Properties Are Given Below

1) The recurrence relations satisfied by the values of x_{n+1} and y_{n+1} are respectively.

$2x_{n+3} - 164x_{n+2} + 2x_{n+1} = 0$, $n = -1,0,1.......$

$2y_{n+3} - 164y_{n+2} + 2y_{n+1} = 0$, $n = -1,0,1.......$

A few interesting relations among the solutions are given below:

a) $x_{n+3} - 82x_{n+2} + x_{n+1} = 0$

b) $4y_{n+1} - x_{n+2} + 41x_{n+1} = 0$

c) $4y_{n+2} - 41x_{n+2} + x_{n+1} = 0$

d) $4y_{n+3} - 3361x_{n+2} + 41x_{n+1} = 0$

e) $328y_{n+1} - x_{n+3} + 3361x_{n+1} = 0$

f) $8y_{n+2} - x_{n+3} + x_{n+1} = 0$

g) $328y_{n+3} - 3361x_{n+3} + x_{n+1} = 0$

h) $y_{n+2} - 41y_{n+1} - 420x_{n+1} = 0$

i) $y_{n+3} - 3361y_{n+1} - 34440x_{n+1} = 0$

j) $41y_{n+3} - 3361y_{n+2} - 420x_{n+1} = 0$

k) $4y_{n+1} - 41x_{n+3} + 3361x_{n+2} = 0$

l) $4y_{n+2} - x_{n+3} + 41x_{n+2} = 0$

m) $4y_{n+3} - 41x_{n+3} + x_{n+2} = 0$

n) $41y_{n+2} - y_{n+1} - 420x_{n+2} = 0$

o) $y_{n+3} - 41y_{n+2} - 420x_{n+2} = 0$

p) $3361y_{n+2} - 41y_{n+1} - 420x_{n+3} = 0$

q) $3361y_{n+3} - y_{n+1} - 34440x_{n+3} = 0$

r) $y_{n+3} - y_{n+1} - 840x_{n+2} = 0$

s) $41y_{n+3} - y_{n+2} - 420x_{n+3} = 0$

t) $y_{n+1} - y_{n+3} - 82y_{n+2} = 0$

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2) *Each Of The Following Is A Nasty Number*

a) $\frac{1}{(2^t)}\left[123x_{2n+3} - 10083x_{2n+2} + 12(2^t)\right]$

b) $\frac{1}{2(2^t)}\left[3x_{2n+4} - 20163x_{2n+2} + 8(2^t)\right]$

c) $\frac{1}{(2^t)}\left[492y_{2n+2} - 5040x_{2n+2} + 12(2^t)\right]$

d) $\frac{1}{(2^t)}\left[12y_{2n+3} - 10080x_{2n+2} + 12(2^t)\right]$

e) $\frac{1}{3361(2^t)}\left[492y_{2n+4} - 33883920x_{2n+2} + 40332(2^t)\right]$

f) $\frac{1}{(2^t)}\left[10083x_{2n+4} - 826683x_{2n+3} + 12(2^t)\right]$

g) $\frac{1}{41(2^t)}\left[40332y_{2n+2} - 5040x_{2n+3} + 492(2^t)\right]$

h) $\frac{1}{(2^t)}\left[40332y_{2n+3} - 68880x_{2n+3} + 12(2^t)\right]$

i) $\frac{1}{41(2^t)}\left[40332y_{2n+4} - 33883920x_{2n+3} + 492(2^t)\right]$

j) $\frac{1}{3361(2^t)}\left[3306732y_{2n+2} - 5040x_{2n+4} + 40332(2^t)\right]$

k) $\frac{1}{41(2^t)}\left[3306732y_{2n+3} - 413280x_{2n+4} + 492(2^t)\right]$

l) $\frac{1}{(2^t)}\left[3306732y_{2n+4} - 33883920x_{2n+4} + 12(2^t)\right]$

m) $\frac{1}{35(2^t)}\left[34440y_{2n+2} - 420y_{2n+3} + 420(2^t)\right]$

n) $\frac{1}{41(2^t)}\left[40338y_{2n+2} - 6y_{2n+4} + 492(2^t)\right]$

o) $\frac{1}{(2^t)}\left[80676y_{2n+3} - 164y_{2n+4} + 12(2^t)\right]$

3) *Each Of The Following Is A Cubical Integer*

a) $\frac{1}{2(2^t)}\left[41x_{3n+4} - 3361x_{3n+3} + 123x_{n+2} - 10083x_{n+1}\right]$

b) $\frac{1}{4(2^t)}\left[x_{3n+5} - 6721x_{3n+3} + 3x_{n+3} - 20163x_{n+1}\right]$

c) $\frac{1}{(2^t)}\left[82y_{3n+3} - 840x_{3n+3} + 246y_{n+1} - 2520x_{n+1}\right]$

d) $\frac{1}{(2^t)}\left[2y_{3n+4} - 1680x_{2n+2} + 6y_{n+2} - 5040x_{n+1}\right]$

e) $\frac{1}{3361(2^t)}\left[82y_{3n+5} - 5647320x_{3n+3} + 246y_{n+3} - 16941960x_{n+1}\right]$

f) $\frac{1}{2(2^t)}[3361x_{3n+5} - 275561x_{3n+4} + 10083x_{n+3} - 826683x_{n+2}]$

g) $\frac{1}{41(2^t)}[6722y_{3n+3} - 840x_{3n+4} + 20166y_{n+1} - 2520x_{n+2}]$

h) $\frac{1}{2(2^t)}[6722y_{3n+4} - 68880x_{3n+4} + 20166y_{n+2} - 206640x_{n+2}]$

i) $\frac{1}{41(2^t)}[6722y_{3n+5} - 647320x_{3n+4} + 20166y_{n+3} - 16941960x_{n+2}]$

j) $\frac{1}{3361(2^t)}[551122y_{3n+3} - 840x_{3n+5} + 1653366y_{n+1} - 2520x_{n+3}]$

k) $\frac{1}{41(2^t)}[551122y_{3n+4} - 68880x_{3n+5} + 1653366y_{n+2} - 206640x_{n+3}]$

l) $\frac{1}{(2^t)}[551122y_{3n+5} - 5647320x_{3n+5} + 1653366y_{n+3} - 16941960x_{n+3}]$

m) $\frac{1}{210(2^t)}[34440y_{3n+3} - 420y_{3n+4} + 103320y_{n+1} - 1260y_{n+2}]$

n) $\frac{1}{41(2^t)}[6723y_{3n+3} - y_{3n+5} + 20169y_{n+1} - 3y_{n+3}]$

o) $\frac{1}{(2^t)}[13446y_{3n+4} - 164x_{3n+5} + 40338y_{n+2} - 492y_{n+3}]$

4) Each of the following expression is a biquadratic integer

a) $\frac{1}{2(2^t)}[41x_{4n+5} - 3361x_{4n+4} + 164x_{2n+3} - 13444x_{2n+4} + 12(2^t)]$

b) $\frac{1}{4(2^t)}[x_{4n+6} - 6721x_{4n+4} + 4x_{2n+4} - 26884x_{2n+2} + 24(2^t)]$

c) $\frac{1}{(2^t)}[82y_{4n+4} - 840x_{4n+4} + 328y_{2n+2} - 3360x_{2n+2} + 6(2^t)]$

d) $\frac{1}{(2^t)}[2y_{4n+5} - 1680x_{4n+4} + 8y_{2n+3} - 6720x_{2n+2} + 6(2^t)]$

e) $\frac{1}{3361(2^t)}[82y_{4n+6} - 5647320x_{4n+4} + 328y_{2n+4} - 22589280x_{2n+2} + 20166(2^t)]$

f) $\frac{1}{2(2^t)}[3361x_{4n+6} - 275561x_{4n+5} + 13444x_{2n+4} - 1102244x_{2n+3} + 12(2^t)]$

g) $\frac{1}{41(2^t)}[6722y_{4n+4} - 840x_{4n+5} + 26888y_{2n+2} - 3360x_{2n+3} + 246(2^t)]$

h) $\frac{1}{(2^t)}[6722y_{4n+5} - 68880x_{4n+5} + 26888y_{2n+3} - 275520x_{2n+3} + 6(2^t)]$

i) $\frac{1}{41(2^t)}[6722y_{4n+6} - 56473270x_{4n+5} + 26888y_{2n+4} - 22589280x_{2n+3} + 246(2^t)]$

j) $\frac{1}{3361(2^t)}[551122y_{4n+4} - 840x_{4n+6} + 2204488y_{2n+2} - 33604x_{2n+4} + 20166(2^t)]$

k) $\frac{1}{41(2^t)}[551122y_{4n+5} - 68880x_{4n+6} + 2204488y_{2n+3} - 275520x_{2n+4} + 246(2^t)]$

l) $\frac{1}{\left(2^t\right)}\left[551122y_{4n+6} - 5647320x_{4n+6} + 2204488y_{2n+4} - 22589280x_{2n+4} + 6\left(2^t\right)\right]$

m) $\frac{1}{210\left(2^t\right)}\left[34440y_{4n+4} - 420y_{4n+5} + 137760y_{2n+2} - 1680y_{2n+3} + 1260\left(2^t\right)\right]$

n) $\frac{1}{41\left(2^t\right)}\left[6723y_{4n+4} - y_{4n+6} + 26892y_{2n+2} - 4y_{2n+4} + 246\left(2^t\right)\right]$

o) $\frac{1}{\left(2^t\right)}\left[13446y_{4n+5} - 164y_{4n+6} + 53784y_{2n+3} - 656y_{2n+4} + 6\left(2^t\right)\right]$

5) Each of the following expression is a quintic integer

a) $\frac{1}{2\left(2^t\right)}\begin{bmatrix} 41x_{5n+6} - 3361x_{5n+5} + 205x_{3n+4} - 16805x_{3n+3} + 615x_{n+2} \\ - 50415x_{n+1} - 205x_{n+2} + 16805x_{n+1} \end{bmatrix}$

b) $\frac{1}{4\left(2^t\right)}\begin{bmatrix} x_{5n+7} - 6721x_{5n+5} + 5x_{3n+5} - 33605x_{3n+3} + 15x_{n+3} \\ -100815x_{n+1} - 5x_{n+3} + 33605x_{n+1} \end{bmatrix}$

c) $\frac{1}{\left(2^t\right)}\begin{bmatrix} 82y_{5n+5} - 840x_{5n+5} + 410y_{3n+3} - 4200x_{3n+3} + 1230y_{n+1} \\ -12600x_{n+1} - 410y_{n+1} + 4200x_{n+1} \end{bmatrix}$

d) $\frac{1}{\left(2^t\right)}\begin{bmatrix} 2y_{5n+6} - 1680x_{5n+5} + 10y_{3n+4} - 8400x_{2n+2} + 30y_{n+2} \\ - 25200x_{n+1} - 10y_{n+2} + 8400x_{n+1} \end{bmatrix}$

e) $\frac{1}{3361\left(2^t\right)}\begin{bmatrix} 82y_{5n+7} - 5647320x_{5n+5} + 410y_{3n+5} - 28236600x_{3n+3} + 1230y_{n+3} \\ - 84709800x_{n+1} - 410y_{n+3} + 28236600x_{n+1} \end{bmatrix}$

f) $\frac{1}{2\left(2^t\right)}\begin{bmatrix} 3361x_{5n+7} - 275561x_{5n+6} + 16805x_{3n+5} - 1377805x_{3n+4} + 50415x_{n+3} \\ - 4133415x_{n+2} - 16805x_{n+3} + 1377805x_{n+2} \end{bmatrix}$

g) $\frac{1}{41\left(2^t\right)}\begin{bmatrix} 6722y_{5n+5} - x_{5n+6} + 19355y_{3n+3} - 3225x_{3n+4} + 58065y_{n+1} \\ - 9675x_{n+2} - 193555y_{n+1} + 3225x_{n+2} \end{bmatrix}$

h) $\frac{1}{\left(2^t\right)}\begin{bmatrix} 6722y_{5n+6} - 68880x_{5n+6} + 33610y_{3n+4} - 344400x_{3n+4} + 100830y_{n+2} \\ -100830y_{n+2} - 33610y_{n+2} + 344400x_{n+2} \end{bmatrix}$

i) $\frac{1}{41\left(2^t\right)}\begin{bmatrix} 6722y_{5n+7} - 5647320x_{5n+6} + 33610y_{3n+5} - 28236600x_{3n+4} + 100830y_{n+3} \\ - 84709800x_{n+2} - 33610y_{n+3} + 28236600x_{n+2} \end{bmatrix}$

j) $\frac{1}{3361\left(2^t\right)}\begin{bmatrix} 551122y_{5n+5} - 840x_{5n+7} + 2755610y_{3n+3} - 4200x_{3n+5} + 8266830y_{n+1} \\ -12600x_{n+3} - 2755610y_{n+1} + 4200x_{n+3} \end{bmatrix}$

k) $\frac{1}{41\left(2^t\right)}\begin{bmatrix} 551122y_{5n+6} - 68880x_{5n+7} + 2755610y_{3n+4} - 344400x_{3n+5} + 8266830y_{n+2} \\ -1033200x_{n+3} - 2755610y_{n+2} + 34440x_{n+3} \end{bmatrix}$

l) $\frac{1}{\left(2^t\right)}\begin{bmatrix} 551122y_{5n+7} - 5647320x_{5n+7} + 27556100y_{3n+5} - 28236600x_{3n+5} + 8266830y_{n+3} \\ -84709800x_{n+3} - 2755610y_{n+3} + 2823660x_{n+3} \end{bmatrix}$

m) $\frac{1}{210\left(2^t\right)}\begin{bmatrix} 34440y_{5n+5} - 420y_{5n+6} + 172200y_{3n+3} - 2100y_{3n+4} + 516600y_{n+1} \\ - 6300y_{n+2} - 172200y_{n+1} + 2100y_{n+2} \end{bmatrix}$

n) $\frac{1}{41\left(2^t\right)}\begin{bmatrix} 6723y_{5n+5} - y_{5n+7} + 33615y_{3n+3} - 5y_{3n+5} + 100845y_{n+1} \\ -15y_{n+3} - 33615y_{n+1} + 5y_{n+3} \end{bmatrix}$

o) $\frac{1}{\left(2^t\right)}\begin{bmatrix} 13446y_{5n+6} - 164y_{5n+7} + 67230y_{3n+4} - 820y_{3n+5} + 201690y_{n+2} \\ - 2460y_{n+3} - 67230y_{n+2} + 820y_{n+3} \end{bmatrix}$

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6) Remarkable Observations

a) Employing linear combinations among the solutions of (1.1), one may generate integer solutions for other choices of hyperbola which are presented in table below.

S.No	Hyperbola	(X_n, Y_n)
1	$X_n^2 - 420Y_n^2 = 16(4^t)$	$\begin{pmatrix} 41x_{n+2} - 3361x_{n+1} \\ 164x_{n+1} - 2x_{n+2} \end{pmatrix}$
2	$1681X_n^2 - 1680Y_n^2 = 107584(4^t)$	$\begin{pmatrix} x_{n+3} - 6721x_{n+1} \\ 6723x_{n+1} - x_{n+3} \end{pmatrix}$
3	$X_n^2 - 105Y_n^2 = 4(4^t)$	$\begin{pmatrix} 82y_{n+1} - 840x_{n+1} \\ 82x_{n+1} - 8y_{n+1} \end{pmatrix}$
4	$1681X_n^2 - 105Y_n^2 = 6724(4^t)$	$\begin{pmatrix} 2y_{n+2} - 1680x_{n+1} \\ 6722x_{n+1} - 8y_{n+2} \end{pmatrix}$
5	$X_n^2 - 105Y_n^2 = 45185284(4^t)$	$\begin{pmatrix} 82y_{n+3} - 5647320x_{n+1} \\ 551122x_{n+1} - 8y_{n+3} \end{pmatrix}$
6	$X_n^2 - 420Y_n^2 = 16(4^t)$	$\begin{pmatrix} 3361x_{n+3} - 275561x_{n+2} \\ 13446x_{n+2} - 164x_{n+3} \end{pmatrix}$
7	$X_n^2 - 105Y_n^2 = 6724(4^t)$	$\begin{pmatrix} 6722y_{n+1} - 840x_{n+2} \\ 82x_{n+2} - 656y_{n+1} \end{pmatrix}$
8	$X_n^2 - 105Y_n^2 = 4(4^t)$	$\begin{pmatrix} 6722y_{n+2} - 68880x_{n+2} \\ 6722x_{n+2} - 656y_{n+2} \end{pmatrix}$
9	$X_n^2 - 105Y_n^2 = 6724(4^t)$	$\begin{pmatrix} 6722y_{n+3} - 5647320x_{n+2} \\ 551122x_{n+2} - 656y_{n+3} \end{pmatrix}$
10	$X_n^2 - 105Y_n^2 = 45185284(4^t)$	$\begin{pmatrix} 551122y_{n+1} - 840x_{n+3} \\ 82x_{n+3} - 53784y_{n+1} \end{pmatrix}$
11	$X_n^2 - 105Y_n^2 = 6724(4^t)$	$\begin{pmatrix} 551122y_{n+2} - 68880x_{n+3} \\ 6722x_{n+3} - 53784y_{n+2} \end{pmatrix}$
12	$X_n^2 - 105Y_n^2 = 4(4^t)$	$\begin{pmatrix} 551122y_{n+3} - 5647320x_{n+3} \\ 551122x_{n+3} - 53784y_{n+3} \end{pmatrix}$
13	$X_n^2 - 105Y_n^2 = 176400(4^t)$	$\begin{pmatrix} 34440y_{n+1} - 420y_{n+2} \\ 41y_{n+2} - 3361y_{n+1} \end{pmatrix}$
14	$35280X_n^2 - 35301Y_n^2 = 237222720(4^t)$	$\begin{pmatrix} 6723y_{n+1} - y_{n+3} \\ y_{n+3} - 6721y_{n+1} \end{pmatrix}$
15	$8820X_n^2 - 21Y_n^2 = 35280(4^t)$	$\begin{pmatrix} 13446y_{n+2} - 164y_{n+3} \\ 3361y_{n+3} - 275561y_{n+2} \end{pmatrix}$

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b) Employing linear combinations among the solutions of (1.1), one may generate integer solutions for other choices of Parabola which are presented in table below.

S.No	Parabola	(X_n, Y_n)
1	$(2^t)X_n - 210Y_n^2 = 4(4^t)$	$\begin{pmatrix} 41x_{2n+3} - 3361x_{2n+2} \\ 164x_{2n+2} - 2x_{2n+3} \end{pmatrix}$
2	$1681(2^t)X_n - 420Y_n^2 = 13448(4^t)$	$\begin{pmatrix} x_{2n+4} - 6721x_{2n+2} \\ 6723x_{2n+2} - x_{2n+4} \end{pmatrix}$
3	$(2^t)X_n - 105Y_n^2 = 4(4^t)$	$\begin{pmatrix} 82y_{2n+2} - 840x_{2n+2} \\ 82x_{2n+2} - 68y_{2n+2} \end{pmatrix}$
4	$1681(2^t)X_n - 105Y_n^2 = 3362(4^t)$	$\begin{pmatrix} 2y_{2n+3} - 1680x_{2n+2} \\ 6722x_{2n+2} - 8y_{2n+3} \end{pmatrix}$
5	$3361(2^t)X_n - 105Y_n^2 = 22592642(4^t)$	$\begin{pmatrix} 82y_{2n+4} - 5647320x_{2n+2} \\ 551122x_{2n+2} - 8y_{2n+4} \end{pmatrix}$
6	$(2^t)X_n - 210Y_n^2 = 4(4^t)$	$\begin{pmatrix} 6722y_{2n+2} - 840x_{2n+3} \\ 82x_{2n+3} - 656x_{2n+2} \end{pmatrix}$
7	$41(2^t)X_n - 105Y_n^2 = 3362(4^t)$	$\begin{pmatrix} 6722y_{2n+2} - 840x_{2n+3} \\ 82x_{2n+3} - 656y_{2n+2} \end{pmatrix}$
8	$(2^t)X_n - 105Y_n^2 = 2(4^t)$	$\begin{pmatrix} 6722y_{2n+3} - 68880x_{2n+3} \\ 6722x_{2n+3} - 656y_{2n+3} \end{pmatrix}$
9	$41(2^t)X_n - 105Y_n^2 = 3362(4^t)$	$\begin{pmatrix} 6722y_{2n+4} - 5647320x_{2n+3} \\ 551122x_{2n+3} - 656y_{2n+4} \end{pmatrix}$
10	$3361(2^t)X_n - 105Y_n^2 = 22592642(4^t)$	$\begin{pmatrix} 551122y_{2n+2} - 840x_{2n+4} \\ 82x_{2n+4} - 53784y_{2n+2} \end{pmatrix}$
11	$41(2^t)X_n - 105Y_n^2 = 3362(4^t)$	$\begin{pmatrix} 551122y_{2n+3} - 68880x_{2n+4} \\ 6722x_{2n+4} - 53784y_{2n+3} \end{pmatrix}$
12	$(2^t)X_n - 105Y_n^2 = 2(4^t)$	$\begin{pmatrix} 551122y_{2n+4} - 5647320x_{2n+4} \\ 551122x_{2n+4} - 53784y_{2n+4} \end{pmatrix}$
13	$210(2^t)X_n - 105Y_n^2 = 176400(4^t)$	$\begin{pmatrix} 34440y_{2n+2} - 420y_{2n+3} \\ 41y_{2n+3} - 3361y_{2n+2} \end{pmatrix}$
14	$35280(2^t)X_n - 861Y_n^2 = 2892960(4^t)$	$\begin{pmatrix} 6723y_{2n+2} - y_{2n+4} \\ y_{2n+4} - 6721y_{2n+2} \end{pmatrix}$
15	$8820(2^t)X_n - 21Y_n^2 = 17640(4^t)$	$\begin{pmatrix} 13446y_{2n+3} - 164y_{2n+4} \\ 3361y_{2n+4} - 275561y_{2n+3} \end{pmatrix}$

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c) Employing the linear combination among the solutions of (1), one may generate integer solutions for other choices of straight line which are presented in the table below:

S.No.	Straight line	(Y, X)
1	$Y = 2X$	$Y = x_{n+3} - 6721x_{n+1}$ $X = 41x_{n+2} - 3361x_{n+1}$
2	$Y = X$	$Y = 82y_{n+1} - 840x_{n+1}$ $X = 2y_{n+2} - 1680x_{n+1}$
3	$Y = 41X$	$Y = 6722y_{n+1} - 840x_{n+2}$ $X = 6722y_{n+2} - 68880x_{n+2}$
4	$Y = 3361X$	$Y = 82y_{n+3} - 1680x_{n+1}$ $X = 551122y_{n+3} - 5647320x_{n+3}$
5	$Y = X$	$Y = 41x_{n+2} - 3361x_{n+1}$ $X = 3361x_{n+3} - 275561x_{n+2}$
6	$Y = 210X$	$Y = 34440y_{n+1} - 420y_{n+2}$ $X = 82y_{n+1} - 840x_{n+1}$
7	$Y = 210X$	$Y = 34440y_{n+1} - 420y_{n+2}$ $X = 13446y_{n+2} - 164y_{n+3}$
8	$Y = X$	$Y = 6722y_{n+3} - 5647320x_{n+2}$ $X = 6723y_{n+1} - y_{n+3}$
9	$Y = 2X$	$Y = 3361x_{n+3} - 275561x_{n+2}$ $X = 2y_{n+2} - 1680x_{n+1}$
10	$Y = 2X$	$Y = 176y_{n+1} - 2y_{n+2}$ $X = 3361x_{n+3} - 275561x_{n+2}$
11	$Y = 2X$	$Y = x_{n+3} - 6721x_{n+1}$ $X = 551122y_{n+3} - 5647320x_{n+3}$
12	$Y = X$	$Y = 82y_{n+1} - 840x_{n+1}$ $X = 13446y_{n+2} - 164y_{n+3}$
13	$Y = 41X$	$Y = 6723y_{n+1} - y_{n+3}$ $X = 2y_{n+2} - 1680x_{n+1}$
14	$Y = 3361X$	$Y = 82y_{n+3} - 5647320x_{n+1}$ $X = 13446y_{n+2} - 164y_{n+3}$
15	$Y = X$	$Y = 551122y_{n+2} - 68880x_{n+3}$ $X = 6723y_{n+1} - y_{n+3}$

d) The solutions in terms of special integer sequence namely, generalized Fibonacci sequence $GF_n(k,s)$ and Lucas sequence $GL_n(k,s)$ are exhibited below.

$$x_{n+1} = 2(2^t)GL_{n+1}(82,-1) + 164(2^t)GF_{n+1}(82,-1)$$

$$y_{n+1} = \frac{41}{2}(2^t)GL_{n+1}(82,-1) + 1680(2^t)GF_{n+1}(82,-1)$$

e) Employing the solutions (1) ,each of the following among the special polygonal, pyramidal, star numbers, pronic numbers is a congruent to under modulo 4

i) $$\left(\frac{12p_y^5}{s_{y+1}-1}\right)^2-105\left(\frac{3p_x^3}{t_{3,x+1}}\right)^2$$

ii) $$\left(\frac{4p_y^5}{ct_{4,y}-1}\right)^2-105\left(\frac{12p_x^5}{s_{x+1}-1}\right)^2$$

iii) $$\left(\frac{3p_{y-2}^3}{t_{3,y-2}}\right)^2-105\left(\frac{4p_x^5}{ct_{4,x}-1}\right)^2$$

iv) $$\left(\frac{6p_y^3}{pr_{y+1}}\right)^2-105\left(\frac{p_x^5}{t_{3,x}}\right)^2$$

v) $$\left(\frac{4p_y^5}{ct_{4,x}-1}\right)^2-105\left(\frac{6p_{x-1}^4}{t_{3,2x-1}}\right)^2$$

vi) $$\left(\frac{36p_{y-2}^3}{s_{y+1}-1}\right)^2-105\left(\frac{3p_{x-2}^3}{t_{3,y-2}}\right)^2$$

vii) $$\left(\frac{12p_y^5}{s_{y+1}-1}\right)^2-105\left(\frac{3p_{x-2}^3}{t_{3,x-2}}\right)^2$$

viii) $$\left(\frac{4pt_{y-3}}{p_{y-3}^3}\right)^2-105\left(\frac{2p_{x-1}^5}{t_{4,x-1}}\right)^2$$

ix) $$\left(\frac{6p_y^5}{ct_{6,y}-1}\right)^2-105\left(\frac{3p_{x+1}^4-p_{x+1}^3}{t_{4,x+1}}\right)^2$$

x) $$\left(\frac{6p_{y-1}^4}{t_{3,2y-1}}\right)^2-105\left(\frac{p_x^5}{t_{3,x}}\right)^2$$

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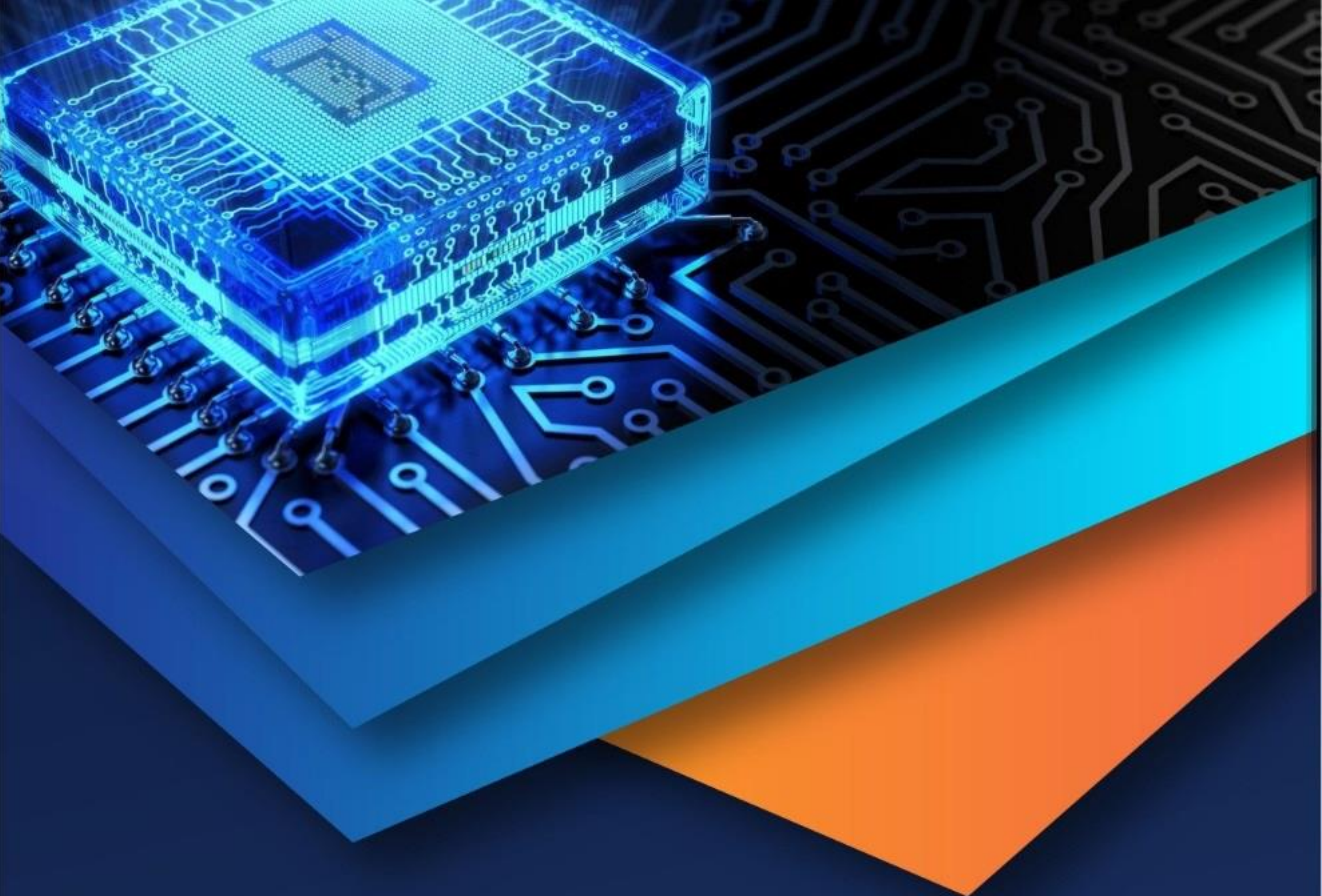

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