INTERNATIONAL JOURNAL
FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

www.ijraset.com

Call: ⟨©⟩08813907089 | E-mail ID: ijraset@gmail.com

Theoretical Approach to Determine Flaw Size in Aircraft Fuselage Panels using Extended Kalman Filters

Dhruv Girish Apte[1], Mohit Dhoriya[2], Nitinchandra Rameshchandra Patel[3]

[1, 2]Undergraduate student, [3]Assistant Professor, Department of Mechanical Engineering, G H Patel College of Engineering & Technology, Vallabh Vidhyanagar, Gujarat-388120

Abstract: The use of steel and aluminum alloys in making of aircraft components including fuselage, plates and flanges has increased since the post-WW II era. However, these materials also experience metal fatigue which results in accumulation of damage in the form of "cracks" due to repetitive application of loads. Repeated pressurization-depressurization cycles during take-off & landing of aircraft result in loading cycles and eventually fatigue crack. Prediction of fatigue crack propagation and remaining aircraft life is a crucial component in aircraft safety. Several approaches have been suggested to determine flaw size in these cases like the S-N curve, Miner's equation and Basquin's law. The most effective method is the Paris Law which determines flaw size with material properties as input and wields highly accurate results.

Keywords: Metal fatigue, Paris Law, aircraft fuselage panel, crack propagation

I. INTRODUCTION

Fatigue is defined as the weakening of a material caused by loads being applied repeatedly. The concept of metal fatigue was not worked well upon in aerodynamic designs, until 1954 when two de Havilland Comets broke up in mid-air within few months. A Court of Inquiry was set up by the British government to investigate its causes. Titled as the Cohen Committee after its chairman Lord Cohen, it ultimately published a report citing metal fatigue as the main cause [1]. The stress concentration caused by stress generation near the windows was nearly three times the stress experienced around the fuselage. The planes were immediately grounded and aerospace firms were instructed to make round-shaped windows instead of the previously square-shaped ones.
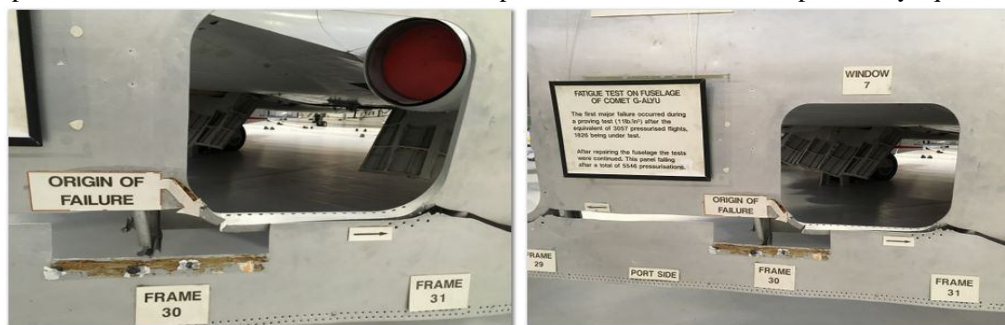

Figure1: Fatigue cracked fuselage of Comet-2 aircraft via repeated cabin pressurization (Photograph Courtesy: Dr. Lin Son, University of Cambridge)

The fatigue process is thought to begin at a surface flaw where stress concentrations occur. It consists of shear flow along slip planes which generate extrusions over a number of cycles, eventually forming a crack. During examination, a region of slow crack growth is observed in the form of a "clamshell" that is concentric around location of the initial flaw. In post-mortem examinations, it is possible to relate the "clamshell" to the initial stress and determine the applied stress at failure just before crack propagation. Previously, engineers had developed quantification of fatigue with empirical formulae. The first model was the S-N curve where constant cyclic stress amplitude is applied to a specimen and number of loading cycles N when the specimen fails is determined. For materials like aluminium, no endurance limit exists. Although, in use in industry, obtaining a full S-N curve is a tedious and expensive process. This problem was solved by Basquin[2] in 1910 when he proposed a law later known as Basquin's Law:

$$\Delta \sigma N_f^\beta = C$$

Where $\Delta\sigma$ is stress range, N_f is number of cycles to failure, a & C are empirically determined constants.

Crack growth will accelerate over life of the airplane part. This was the main shortcoming of the Miner's Law. The Miner's Law states that where there are k different stress magnitude values in a spectrum S_i ($1 \leq i \leq k$), each contributing n_iS_i cycles then if N_iS_i is the number of cycles to failure of a constant stress reversal S_i, failure occurs when:

$$\sum \frac{n_i}{N_i} = C$$

For design purposes, C is assumed to be 1.

Damage accumulation is a combination of several different mechanisms and the Miner's Law assumes only linear damage accumulation. Though this might make calculations easy, this will not be practically correct as when fatigue progresses, parts of the material microstructure become unable to bear the load which increases the stress on the surviving microstructure elements, thus increasing the rate of damage in the final portions of its lifetime. Crack propagation laws in literature treat cracks in infinite sheets subjected to uniform stress perpendicular to the crack. The single form in which all crack-propagation laws are defined is:

$$\frac{da}{dN} = f(\sigma, a, C)$$

Where a is defined as crack length, σ is the stress range and C is used to denote the material properties.

The first chronological crack-propagation law was defined by Head[3] which employed a mechanical model considering rigid plastic work hardening elements ahead of a crack tip and elastic elements over remaining part of the infinite sheet. The law was defined as:

$$\frac{da}{dN} = \frac{C_1 \sigma^3 a^{3/2}}{(C_2 - \sigma)\omega_0^{1/2}} \quad \text{(Head's Law)}$$

Where C_1 depends on strain-hardening modulus and C_2 is material yield strength. Head defined ω_0 as the size of plastic zone near crack tip assuming it constant during crack propagation. But Frost[4] noted that plastic zone size was directly proportional to crack length. Based on his studies, Irwin[5] devised an analytical solution where:

$$\omega_0 \approx \sigma^2 a$$

Head's Law was thus corrected to:

$$\frac{da}{dN} = \frac{C_1 \sigma^2 a}{(C_2 - \sigma)} \quad \text{(Head's Corrected Law)}$$

Frost and Dugdale[4] presented a new approach by dimensional analysis incremental increase in crack length was directly proportional to an applied stress function, B:

$$\frac{da}{dN} = \frac{\sigma^3 a}{C}$$

Later, McEvily and Illg[6] devised a theory where presuming radius of a crack tip ρ_1 for stress ρ_0, the stress is expressed as:

$$\sigma_0 = K_n \sigma_{net}$$

Where K_n is stress concentration factor and σ_{net} is net area stress at the cracked section. They also tested their theory for aluminum alloys 2024T3 and 7075T6. Drawing knowledge from the theories of Head, Frost & Dugdale and McEvily and Illg, Paris & Erdogan introduced the Paris Law[7] where the conflicting constants from the above-mentioned theories were substituted by material properties thus; setting a law that could be applied over a wide range of data.

II. PARIS LAW

It was proved previously that speed of crack propagation was dependent on time. But it could not be understood as how. It was Paris et al.[7][8] who suggested using the stress intensity factor range, there are three fracture regimes with respect to variation of crack growth rate per loading cycle due to fatigue. Paris Law analyses the fatigue crack growth mechanism when $10^{-8} \leq (da/dN) \leq 10^{-5}$ m/cycle. For a given load ration $R = \sigma_{min}/\sigma_{max}$, there exists a linear relationship between log (da/dN) and log ΔK_t where ΔK_t is the range of the stress intensity factor.

$$\text{Log}_{10}(da/dN) = \log_{10}A + m\log_{10}(\Delta K_t)$$
$$= \log_{10}A + \log_{10}\{(\Delta K_t)^m\}$$
$$= \log_{10}\{A(\Delta K_t)^m\}$$

Removing log, we obtain:

$$\frac{da}{dN} = A(\Delta K_t^m)$$

Here A and m are constants that depend on the material, environment and stress ratio.

The Paris Law was different from S-N curve as it took considerably different components than the latter. This caused slower damage propagation in uncracked surfaces which was expected. The Paris Law's long crack propagation theory further implied that the dependence on the initial size of the crack was different from that calculated by crack propagation threshold and toughness value. The law often gives fairly accurate results and which may be termed as 'beautiful'. But when it does not work, it gives a serious limitation of having stress intensity factor on the x-axis. If the factor range cannot correctly predict the crack, we are left with a multivariate function with sparse data points making it more difficult for analysis.

Although there has been significant development and modifications regarding Paris Law, most design processes use the empirical approaches of the Pre-Paris Law era for high cycle fatigue. Since cracks develop slowly and are undetectable until the final stages, it is very difficult to process a damage-tolerant approach. Crack propagation starts from the initiation phase, continuing with the propagation phase (where the Paris Law is supposed to be applied) and then fast crack propagation that leads to failure of the structure.

Scientists have therefore modified the Paris Law and published the variations. These variations include only a single factor removal or departure from the ideal conditions. Modifications worth mentioning in this paper include crack closure [9] and short cracks [10][11][12][13]. Short cracks; however don't have a single type of deviation and thus some authors have suggested a classification of short cracks:

A. Microscopic short crack where micro structural fracture mechanics is applied as in case of Hobsen et al [14] and Navarro and de los Rios [15].
B. Physically small crack for which Elastic-Plastic Fracture Mechanics (EPFM) is needed where a general relation between *da/dN* and crack tip decohesion is established under high strain fatigue.
C. Macroscopic long crack described by Linear Elastic Fracture Mechanics (LEFM)

The Paris Law as modified by Paris and Erdogan in 1963 gives the advancement of fatigue crack per unit cycle as a function of the stress intensity factor [7]:

$$v^a = \frac{da}{dN} = C\Delta K^m, \Delta K_{th} < \Delta K < K_{tc}$$

Where K_{th} = fatigue threshold and K_{tc} = fracture toughness with C and a being constants
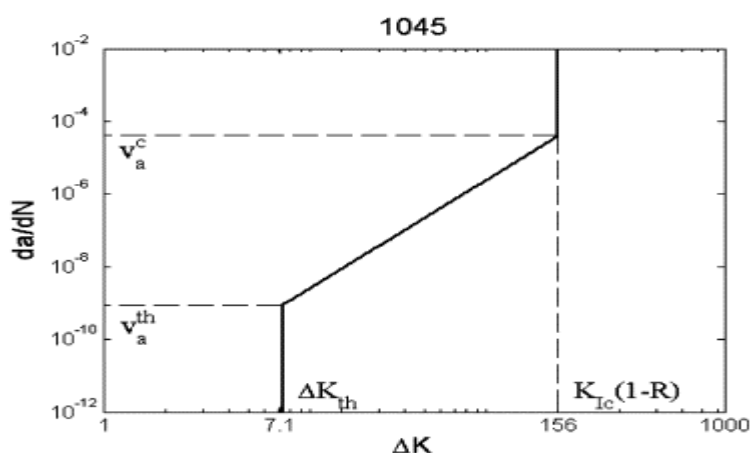

Figure 2- Paris Law for 1045 Steel

As observed from the graph in the figure given above, the law seems to be mostly valid in the range of $10^{-8} - 10^{-6}$ mm/cycle approximately. ΔK_{th} is at $v_a{}^{th}=10^{-9}$ mm/cycle where $v_a{}^{th}$ is conventional velocity at the threshold. K_{tc} is at $v_a{}^c =10^{-5}$ mm/cycle where $v_a{}^c$ is defined as the velocity at the critical conditions. This means that the constant C is not arbitrary as previously thought of. Observing the linearity of the graph between certain conditions, Fleck et al.[16] devised a formula to calculate m:

$$mLog\, F_k = log\frac{v_a^c}{v_a^{th}}$$

III. EXTENDED KALMAN FILTER

The extended Kalman filter is regarded as technique that uses Kalman filter as a base for non-linear function. The filter allows state transition and observation models to be differentiable functions.

$$x_k = f(x_{k-1}, u_k) + w_k$$
$$z_k = h(x_k) + v_k$$

Where w_k and v_k are the process and observation noises with covariance as W_k and V_k respectively and u_k is the control vector.

To explain the equations in words, the function f predicts the state from given estimate and the function h predicts the measurement from the former. Then the Jacobian function is computed.

The papers published by Kalman[17] and Kalman & Bucy[18] unified the above equations as:

$$\hat{x}_k = \hat{x}_{k-1} + k_k(z_k - \hat{x}_{k-1})$$

Where the carat '^' signifies that the variable is estimated. The variable k is known as the Kalman gain.

A. Modification Of Paris Law With Example Of A Griffith Crack

Consider an infinite elastic plate with symmetric crack of length 2a. Here the stress intensity factor can be represented as:

$$K = \sigma\sqrt{\pi a}$$

By integrating, we get:

$$K^* = \sigma\sqrt{\pi\left(a + \frac{\Delta a}{2}\right)}$$

Although the crack propagation is a continuous process, a discrete process can measure the size of the crack after every flight cycle. The reason for a non-continuous algorithm is that perhaps the EKF algorithm might not be computationally correct in integrating the system dynamics for a continuous system.

Although the crack propagation is a continuous process, a discrete process can measure the size of the crack after every flight cycle. The reason for a non-continuous algorithm is that perhaps the EKF algorithm might not be computationally correct in integrating the system dynamics for a continuous system.

Thus the Paris's law equation for each flight cycle k in a recursive format can be written as:

$$a_k = a_{k-1} + C\left(\frac{p_{k-1}{}^r}{t}\sqrt{\pi a_{k-1}}\right)^m$$
$$= g(a_{k-1}, p_{k-1}) \qquad \ldots\ldots (1)$$

We will use this equation during the parameter estimation and designing of the algorithm.

Since m and C are the parameters that depend on the material properties and remain constant, we only have p varying at every flight cycle. The pressure p_k can be expressed as:

$$p_k = \bar{p} + \Delta p_k$$

The Δp_k is regarded as a disturbance and is modelled as a centred normal distribution with variance σ^2. We will model this using a perturbation term.

Modifying equation (1),

$$a_k = g(a_{k-1}, \bar{p} + \Delta p_{k-1})$$

The indices produced by each mechanically performed function should be the same irrespective of method of evaluation. Wang[19] have used the Mean Value First Order Second Moment (MVFOSM) approach but, in some cases the approach does not support the above condition. Hence this paper will consider using the second order Taylor expression

$$a_k = g(a_{k-1}, \bar{p}) + \frac{\partial g(a_{k-1}, \bar{p})}{\partial p}\Delta p_{k-1} + \frac{1}{2}\frac{\partial^2 g(a_{k-1}, \bar{p})}{\partial p^2}\Delta p_{k-1}$$

Where $\frac{\partial g(a_{k-1}, \bar{p})}{\partial p}\Delta p_{k-1}$ and $\frac{\partial^2 g(a_{k-1}, \bar{p})}{\partial p^2}\Delta p_{k-1}$ are first order and second order partial derivatives of g with respect to variable p at the point $(a_{k-1}, \bar{p})$.

$$\frac{\partial g(a_{k-1}, \bar{p})}{\partial p} = Cm\left(\frac{r}{t}\sqrt{\tau a_k}\right)^m(\bar{p})^{m-1}$$

$$\frac{\partial^2 g(a_{k-1}, \bar{p})}{\partial p^2} = Cm(m-1)(^r/_t \sqrt{\tau a_k})^m (\bar{p})^{m-2}$$

Taking the above two derivatives as addictive noise w_{k-1} and considering $\bar{p}$ as a given constant, the equation can be written as:

$$a_k = f(a_{k-1}) + w_{k-1}$$

The additive process noise w_k is assumed to be zero mean additive white Gaussian noise i.e.

1) Interpreted as the sum of two components: a noise-free component and the noise component (additive).
2) The noise component is random but it is assumed that it is drawn at each sample time from a fixed Gaussian distribution.
3) White signifies the noise signal containing same power samples at equal frequencies.

This noise is now defined as Q_k:

$$Q_k = \left[\frac{\partial g(a_{k-1}, \bar{p})}{\partial p} + \frac{\partial^2 g(a_{k-1}, \bar{p})}{\partial p^2} \right]^2$$

$$Q_k = Cm(^r/_t \sqrt{\pi a_k})^m (\bar{p})^{m-1} \left[1 + \frac{m-1}{\bar{p}} \right]$$

The measurement data is stated since crack size measured by sensors will always contain noise from measurement environment and instrument inaccuracies, how small the latter might be.

$$z_k = h(a_k) + v_k$$

Where h is measurement function and v_k is the noise. Thus the system and measurement equations are defined.

IV. CONCLUSIONS

Various attempts have been made to study crack-propagation due to cyclic fatigue. The fatigue process that caused the mid-air disintegration of two de Havilland Comets in 1954 has been worked upon to develop a proper quantification theory. Engineers initially used empirical approaches like the S-N curve and the Miner's Equation. Various approaches modified by material properties lead Paris to define a unique equation that later came to be known as Paris Law. Modifications by other prominent researchers lead to some minor modifications and classification of short cracks thus clarifying the concept of 'short cracks' better. However, some properties were defined empirically and this paper has tried to address the resulting problem by defining a new approach to estimate the Paris Law constants and formalize crack length evolution as a nonlinear filtering problem. Extended Kalman Filter (EKF) technique has been used here to determine the Paris Law constants. The paper has proposed using the second order Taylor expression instead of the pre-existing Mean Value First Order Second Moment (MVFOSM) approach. Future work involves implementation of the aforementioned approach and determining its computational accuracy.

REFERENCES

[1] Withey, P. A. (1997). Fatigue failure of the de Havilland comet I. Engineering Failure Analysis, 4(2), 147–154. doi:10.1016/s1350-6307(97)00005-8
[2] Basquin, O.H. (1910). "The exponential law of endurance test". Proceedings of the American Society for Testing and Materials.10:625-630
[3] A. K. Head, "The Growth of Fatigue Cracks," The Philosophical Magazine, vol. 44, series 7, 1953, p. 925.
[4] N. E. Frost and D. S. Dugdale, "The Propagation of Fatigue Cracks in Sheet Specimens," Journal of the Mechanics and Physics of Solids, vol. 6, no. 2, 1958, p. 92.
[5] G. R. Irwin. "Fracture Mode Transition for a Crack Traversing a Plate." Journal of Basic Engineering, Trans ASME Series D, vol. 82. no. 2, June.1960. p. 417.
[6] A. J. McEvily and W. Illg, "The Rate of Crack Propagation in Two Aluminum Alloys," NACA Technical Note 4394, September, 1958.
[7] P. Paris and F. Erdogan, "A Critical Analysis of Crack Propagation Laws", J. Basic Eng 85(4),528-533, December,1963. doi:10.1115/1.3656900
[8] P. Paris, "A rational analytic theory of fatigue", The Trend in Engineering 13, 9, 1961
[9] Elber, W., 1970. Fatigue crack closure under cyclic tension. Eng. Fract. Mech. 2, 37–45.
[10] Ritchie, R.O., Lankford, J. (Eds.), 1986. Small Fatigue Cracks. The Metallurgical Society of the American Institute of Mining, Metallurgical and Petroleum Engineers, Warrendale
[11] Ritchie, R.O., Lankford, J., 1996. Small fatigue cracks: a statement of the problem and potential solutions. Mater.Sci. Eng. 84, 11–16.
[12] El Haddad, M., Topper, T., Smith, K., 1979. Prediction of non-propagating cracks. Eng. Fract. Mech. 11,573–584.
[13] Kitagawa, H., Tanaka, T. (Eds.), 1990. Fatigue 90. Materials and Components Engineering Publications,Birmingham.
[14] Hobson, P.D., Brown, M.W., de los Rios, E.R., 1986. In: Miller, K.J., de los Rios, E.R. (Eds.), EGF (ESIS) Publication No. 1, The Behaviour of Short Fatigue Cracks. Mechanical Engineering Publications, London, pp. 441–459.
[15] Navarro, A., de los Rios, E.R., 1988. A microstructurally short fatigue crack growth equation. Fatigue Fract. Eng. Mater. Struct. 11, 383–396.
[16] Fleck, N.A., Kang, K.J., Ashby, M.F., 1994. Overview 112: the cyclic properties of engineering materials. Acta Metall. Mater. 42, 365–381.
[17] R.E. Kalman 1960. A New Approach to Linear Filtering and Prediction Problems" (PDF). Journal of Basic Engineering. 82: 35–45. doi:10.1115/1.3662552.
[18] R.E. Kalman; R.S. Bucy (1961). "New results in linear filtering and prediction theory". Journal of Basic Engineering. 83: 95–108. doi:10.1115/1.3658902.
[19] Y.Wang,et al., Determination of Paris' law constants and crack length evolution via Extended and Unscented Kalman filter : An application to aircraft fuselage panels, Mech.Syst.SignalProcess.(2016), http://dx.doi.org/ 10.1016/j.ymssp.2016.04.027i

About The Authors

Dhruv Girish Apte is a final year undergraduate student at Department of Mechanical Engineering, G H Patel College of Engineering & Technology, Vallabh Vidhyanagar, Gujarat, India

Mohit Dhoriya is a final year undergraduate student at Department of Mechanical Engineering, G H Patel College of Engineering & Technology, Vallabh Vidhyanagar, Gujarat, India

Prof. Nitinchandra R. Patel is an Assistant Professor in Mechanical Engg. Department of G. H. Patel College of Engg & Technology, V V Nagar, Gujarat, India. He is having Master degree in Machine Design and Bachelor degree in Mechanical Engineering from Sardar Patel University, V V Nagar. He has more than 20 yrs experience including teaching and industries. He has presented 2 technical research papers in International conferences and published 1 technical research paper in National journal and 21 research papers in International journals. He reviewed a book published by Tata McGraw Hill. He is a Member of Institute of Engineers (I) and Life member of ISTE. He is a reviewer / member in Editorial board of various Peer-reviewed journals. He is also recognized as a Chartered Engineer by Institute of Engineers (I) in Mechanical Engineering Division.

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089 (24*7 Support on Whatsapp)