iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 7 **Issue:** III **Month of publication:** March 2019

DOI: http://doi.org/10.22214/ijraset.2019.3327

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International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 7 Issue III, Mar 2019- Available at www.ijraset.com

A Comparative Study of MAP Models with Multiple Queues and Multiple Servers

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Abstract: In this paper, various scenario of MAP (Markov Arrival Processes) with exponential service rate is presented. The data regarding the arrivals and service times at an Orthopaedic clinic was collected for an evening session. This was analyzed. A single queue with 2 servers and 2 queues with 2 servers were studied and the results were compared with respect to expected total cost.
Keywords: Markov Arrival Processes (MAP), Service rate, Poisson, Exponential, Performance measures.

I. INTRODUCTION

The need for same service by many customers results in the formation of queues**.** The service and the desire to be served should be reasonably balanced for a good queue. This can be summed up in table I as characteristics of a primary queue. Here the service rate and waiting time are vital parameters. A mathematical analysis of the above forms the core of queuing theory.

TABLE I
Reason for formation of queues.

	Service rate s Arrival rate a	
1	s .= a	Ideal
2	s < a	Queue forms
3	s > a	Sometimes server is idle

Kendall notation is followed to represent queues. (A/S/c) : (K/N/D) where A is inter arrival time distribution of customers, S service time distribution, c is the no of servers, N is system capacity and D the queue discipline K is the capacity of the queue. For a Markov arrival system the arrival rate is Poisson (MAP). Here we consider the service rate to be Exponential and hence Markov queues are considered.

A. Objectives Of The Study

Service counters are found with customers awaiting service, at times resulting in congestion, when they have to wait for service. An appropriate queuing model can be established from the data collected from this system. They can be analyzed scientifically based on the available formulae and an alternative model can be suggested to improve the service or reduce the expected total costs involved in the queuing system.

B. Abbreviations Used

1) L_S is the mean number of customers in the system (waiting area plus the service area).
2) Ws is the mean time for a customer to go through the system.
3) Lq is the mean number of customers in the queue (waiting area).
4) Wq is the mean waiting time for a customer in the queue (waiting area).
5) $E(TC)$ is the expected total cost.
6) C_w is the waiting cost of the customer.
7) C_s is the server cost.

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II. THE M|M|C QUEUEING MODEL

A. In an M|M|c queue, there are c parallel servers, each serving customers, (c>1).
B. The arrivals follows Poisson distribution and service process follows exponential distribution, in which case the model is Markovian.
C. All arriving customers when coming into the service system are part of a queue. If all the servers are busy in serving the customers, the primary client within the queue are served by any of the servers that become idle. The service rate during this case will be μc.

Table II gives the performance measures of different Markovian queuing models.

TABLE II
Performance Measures Of M/M/1 , M/M/2 And M/M/C Queues

Measure	M/M/1	M/M/2	M/M/c $P_Q = \frac{(c\rho)^c}{c!(1-\rho)} P_0$
Utilization rate	$\rho = \lambda/\mu$.	$\rho = \lambda/2\mu$.	ρ = λ/cμ.
Mean number of customers in the system L_S	$\rho/1\text{-}\rho$	$\frac{2\rho}{1-\rho^2}$	$\frac{P_Q \rho}{(1-\rho)} + c\rho$
Mean time to go through the system Ws	$\frac{1}{\mu(1-\rho)}$	$\frac{1}{\mu(1-\rho^2)}$	$\frac{P_Q \rho}{\lambda(1-\rho)} + \frac{1}{\mu}$
Mean waiting time in the queue Wq	$\frac{\rho}{\mu(1-\rho)}$	$\frac{\rho^2}{\mu(1-\rho^2)}$	$\frac{P_Q \rho}{\lambda(1-\rho)}$
Mean number of customers in the queue Lq	$\lambda\ Wq$	$\frac{2\rho}{(1-\rho^2)}$	$\frac{P_0 \rho \left(\frac{\lambda}{\mu}\right)^c}{c!(1-\rho)^2}$

The data regarding the number of patients arriving for each hour after the clinic was opened for the evening session was collected from an orthopedic clinic. Similarly the number of patients who completed the consultation with the doctor in each hour was also observed. The same is tabulated and given below.

The arrival and service rates are calculated and given below. The waiting cost of the patients is taken as Rs200 per hour which is the approximate wage earned by a middle class family which frequents this clinic. The doctor charges Rs 200 as consultation fee (CF). Hence the total service cost is CF * number of patients divided by total time in hrs.

1) The expected total cost in the case of one queue (single queue – multi server) Expected total cost E(TC)=E(SC)+ E(WC)=s C_s+ L_s *C_w
2) The expected total cost in the case of two queues (multi queue – multi server) Expected total cost E(TC)=S(E(SC)+ E(WC))=S(C_s+ L_s C_w)

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TABLE IIII

Data representing the arrivals ans service on an hourly basis.

Arrivals | Service

S.No	TIME-INTERVAL	NO OF PATIENTS		S.No	TIME-INTERVAL	NO OF PATIENTS
1	4.00 -5.00	9		1	4.30 -5.30	7
2	5.00 - 6.00	7		2	5.45 - 6.45	6
3	6.00 - 7.00	4		3	6.45 -7.45	8
4	7.00 -8.00	3		4	8.00 – 8.30	4
5	8.00 – 8.30	2				

Arrival rate = λ= 5.5 per hr

Service rate = μ = 7.14 per hr

Service cost = 200*25/4.5 = Rs 1111.11/hr

The above data was analysed using Easy Fit software which gives the Arrivals as Poisson. This is given in Fig.1 below.

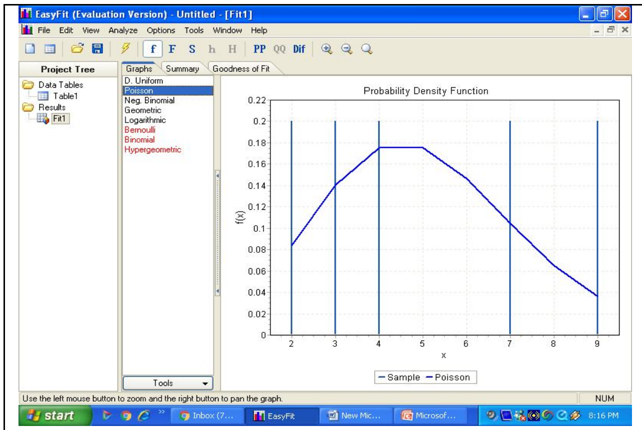

Fig.1. Probability density function for arrivals is Poisson.

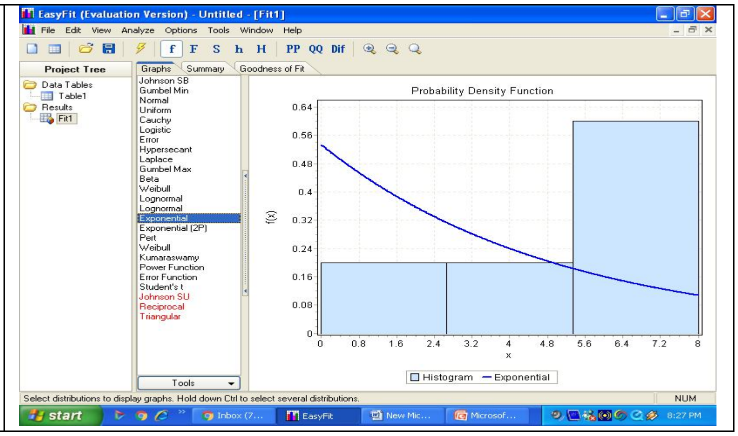

Fig.2. Probability density function for service rate is exponential.

The service data was also identified to follow exponential distribution .The same is given in Fig.2.

The data collected was approximated as λ= 5 and μ = 7 and used to find the performance measures of various scenarios.

TABLE IV Performance Measures Of Various Scenarios

Performance measure	1 Doctor	2 Doctors	3 Doctors	4 Doctors	5 Doctors
λ=Arrival rate	5	5	5	5	5
μ = Service rate	7	7	7	7	7
ρ = System utilization= $\lambda/n\mu$	71%	35.713%	23.809%	17.857%	14.28%
W_q= Waiting time in queue	.3571	.0209	.0024	.0003	0
W_s = Waiting time in system	.5	.1637	.1453	.1431	.1429
L_s = Length of system	2.5	.8187	.7264	.7157	.7144
L_q =Length of queue	1.785	.1044	.0122	.0014	.0001
P_0= Probability of no customer in queue	.285	.473	.488	.4893	.4895
Total system cost /hr in Rs.	1321.42	985.17	990.71	994.56	994.42

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There is no significant difference in W_s = 0.14 and costs = Rs.990 approx. for more than 3 doctors Also the system utilization changes from 71% to 23% which is very significantly low. Optimal assignment is 2 doctors for evening session.

The total cost of a single queue with 2 doctors from the above table is Rs 985.17.

If the same is considered with multi queues (2) with arrival rate λ= 5/2 =2.5 per hr and service rate μ = 7 per hr, the total expected cost is Rs.436.50 for a single queue. Since there are two queues the total expected cost is 436.50 * 2 = Rs 873.00.

In this case it is optimal to divide the queue into two in front of the two server than have a single queue with two servers if cost consideration is essential.

a) *Numerical Illustration 1:* Here a multi queue multi server system is studied using the same values of the data above for which Arrival rate = λ= 5.5 per hr, Service rate = μ = 7.14 per hr. In the first case the queue is divided into two queues, each with an arrival rate of 5.5/2= 2.5 /hr and the service rate remains same at 7 patients per hour approx. This can be considered as a single queue for the application of performance formulae. The total expected cost is found to be Rs 387.55.Since there are two queues the total expected cost is 387.55 *2 = Rs 775.10.

b) *Numerical Illustration 2:* Here a multi queue multi server system is studied using the same values of the data above for which Arrival rate = λ= 5.5 per hr, Service rate = μ = 7.14 per hr. In this case the queue is divided into two queues, each with an arrival rate of 5.5/2= 2.5 /hr and the service rate is also share by the two servers at 7.14/2 =3.57 patients per hour. This can be considered as a single queue for the application of performance formulae. The total expected cost is found to be Rs945.64.Since there are two queues the total expected cost is 945.64*2 = Rs 1891.28. From the above numerical illustrations it is observed that a multi queue (2) with arrival rate λ/2 and service rate μ has a lower expected total cost than a multi queue (2) with arrival rate λ/2 and service rate μ/2.

c) *Future Scope For Work*

i) The study can be extended for Non Markovian queues where arrivals are non Poisson.

ii) The service distribution and arrival rate can be varied, and the performance measures can be compared.

III.CONCLUSIONS

In this paper, various scenario of MAP (Markov Arrival Processes) with exponential service rates were compared. The data regarding arrivals and service times at an Orthopaedic clinic was collected for an evening session. This was analyzed. A single queue with 2 servers and 2 queues with 2 servers were studied and the results were compared. It was found that the number of servers may be increased to two for better efficiency of the system. Also a single queue with multi servers (2) and Multi queue (2) with multi servers (2) was studied. It was found that that a multi queue (2) with arrival rate λ/2 and service rate μ has a lower expected total cost than a multi queue (2) with arrival rate λ/2 and service rate μ/2. Also it is optimal to divide the queue into two in front of the two servers than have a single queue with two servers if cost consideration is essential.

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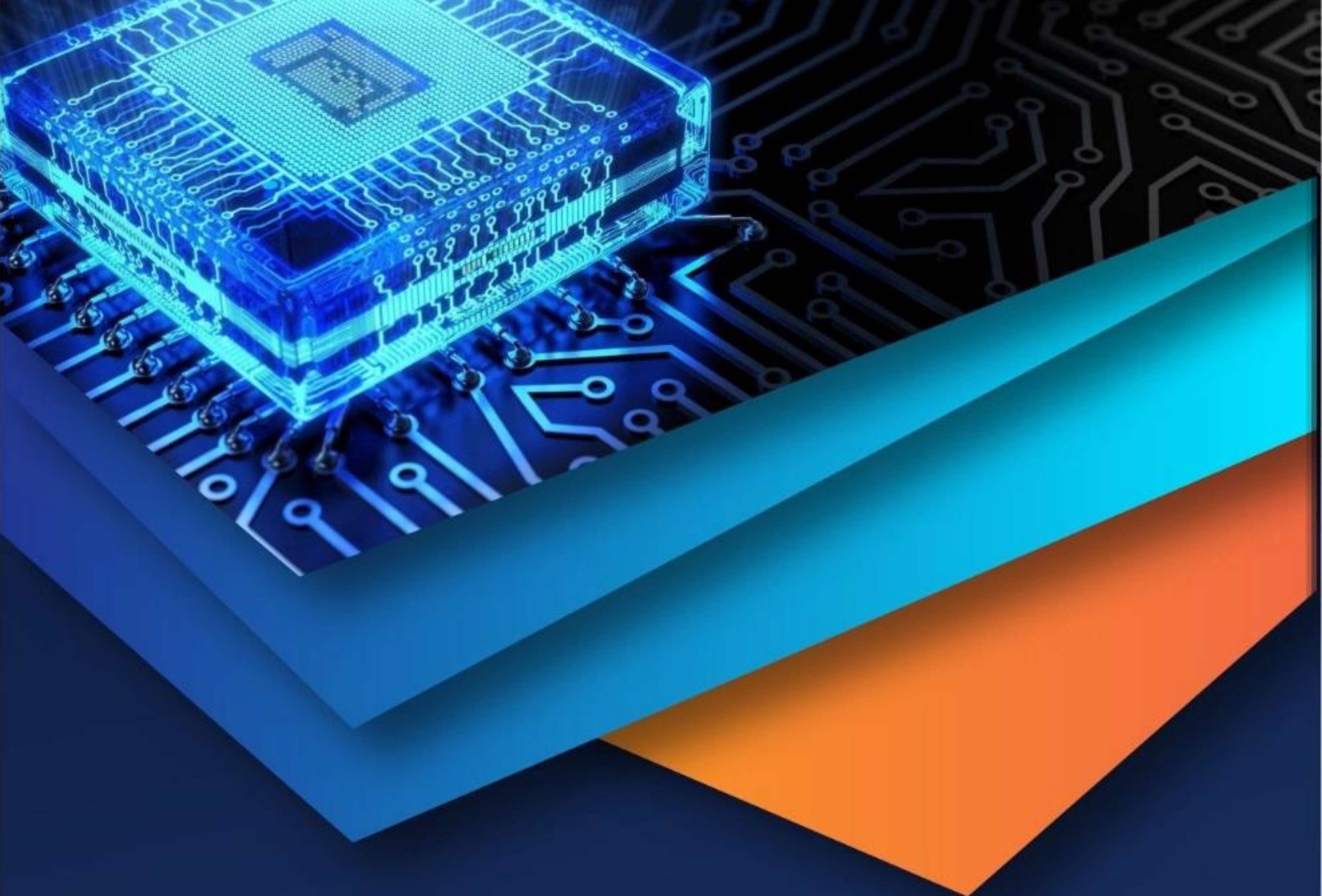

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