iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 7 **Issue:** IV **Month of publication:** April 2019

DOI: https://doi.org/10.22214/ijraset.2019.4184

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 7 Issue IV, Apr 2019- Available at www.ijraset.com

[J, K]-Set Domination of Path Graphs

N. Murugesan[1], P. Elangovan[2]
[1, 2]*PG and Research Department of Mathematics, Government Arts College, Coimbatore-641018*

Abstract: Domination is an important graph theoretic concept in graph theory. Various types of dominations have been studied in the literature. In this paper, the [j, k]-dominations have been considered for path graphs. By [j, k]-domination we mean, every vertex of the complement of the dominating set has at least j adjacent vertices and atmost k adjacent vertices in the dominating set. In particular the [j, k]- domination number of a graph is the cardinality of the smallest such set . In this paper, the [j, k]-domination number for path graphs have been studied.Mathematics subject classification: 05C69
Keywords: Dominating set, Domination number, [j, k]- dominating set, [j, k]- domination snumber.

I. INTRODUCTION

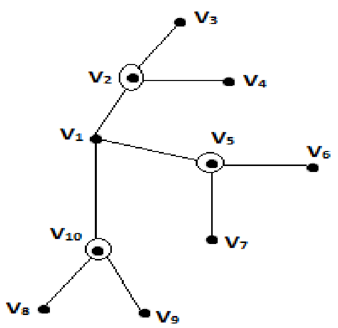
Let G= (V, E) be a simple graph. A subset D of V is a dominating set of G if every vertex v ∈ V-D is adjacent to a vertex of D. The domination number of G denoted by γ(G) is the minimum cardinality of a dominating set G. A dominating set is a total dominating set if every vertex in G (including the vertices in D) have a neighbour in D.

II. [J,K] – SET DOMINATION

A. Definition

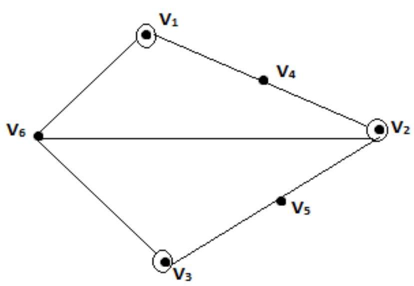
A set D ⊆ V is called [J,K] – set dominaton if for any vertex v ∈ V-D , j ≤ |N(v)∩D| ≤ k, i.e. there are atleast j vertices adjacent to v, but not more than k vertices in D. The smallest cardinality of [j,k] – set is called [j,k] – dominating set. The [j,k] – domination number is denoted by γj,k(G)

B. Example



In the above graph { v2, v5 ,v10} is a dominating set but it is not [1,2] – dominating set because the vertex v1 is adjacent to 3 vertices, Also this is not a total dominating set.

C. Example



The set D= { v1, v2,v3} is a dominating set,
V-D= { v4, v5,v6}
N(v4) ∩ D= { v1 , v2} ∩ D = { v1 , v2} ; Therefore │N(v4) ∩ D│= 2
N(v5) ∩ D= { v2, v3} ∩ D = { v2, v3} ; Therefore │N(v5) ∩ D│= 2
N(v6) ∩ D= { v1, v2, v3 } ∩ D = { v1, v2, v3 } ; Therefore │N(v6) ∩D│= 3
Thus D is [2,3]- dominating set.

D. Note

In general, every dominating set need not be a [j,k]- dominating set, but the converse is always true. The trivial example for the converse part is that the dominating numbers of paths and cycles. As a generalization we have the following lemma.

E. Lemma

Let G be a graph with $\Delta(G)=2$. Then $\gamma(G)=\gamma_{[j,k]}(G)$.

1) Proof: *Let G be graph with $\Delta(G)=2$. Then every vertex v in V has atmost 2 neighbours. Therefore j=1 and k=2, the least possible values of j and k. Hence the Lemma.*

III. [J,K]-DOMINATION IN PATHS

A. Theorem[1]

The domination number of path P is $\gamma(P_n) = \lfloor \frac{n+2}{3} \rfloor$

The domination number of cycle C is $\gamma(C_n) = \lfloor \frac{n+2}{3} \rfloor$

B. Theorem[2]

For n>2, P_n has a [1,2]-dominating set except n=3k, k=1,2,3….

1) *Proof:* To prove this theorem, we prove that there is atleast one v ∈ V-D, such that N(v) has two vertices in D, when n is not a multiple of 3, and when n=3k, for every v ∈ V-D, N(v) has exactly one vertex in D.

As an example, consider P_8, P_9

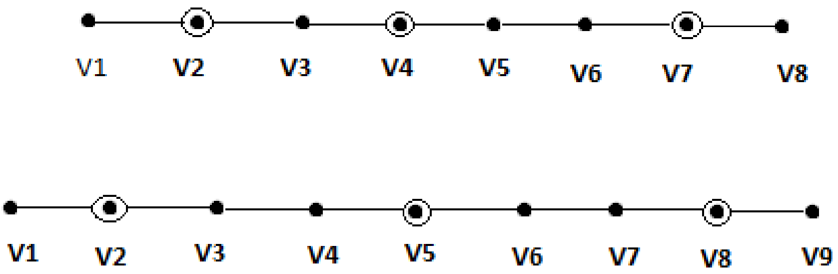


Fig 3.2 paths P_8 and P_9

Where {v_2, v_4, v_7} and {v_2, v_5, v_8} are dominating sets of P_8 and P_9 respectively and {v_2, v_5, v_8} is the only dominating set of P_9, therefore P_9 has no [1, 2]- dominating sets.

C. Theorem [3,4,5]

The number of [1, 1]- dominating sets in the path graph P_n , n≥3

$$n[\gamma_{[1,1]}(P_n)] = \begin{cases} 1 & \text{if } n=3k,\ k=1,2,3,\dots \\ 2 & \text{if } n=4 \text{ and } n=3k+2,\ k=1,2,3,\dots \\ 3 & \text{if } n=3k+1,\ k=2,3,4,\dots \end{cases}$$

Proof: Let $v_1, v_2, \dots\dots, v_n$ are the vertices of the path P_n, such that v_i is adjacent to v_i+1, i=1, 2, 3 ,….,n-1.

In path graph degree of each internal vertex is 2 and the degree of end vertices is 1. Hence every vertex dominates at most 2 vertices. Now we claim that in path graphs: P_n, when n=3k, k=1,2,3… there is a minimum dominating set D in which every vertex dominates exactly 2 vertices and every vertex in V-D is dominated by exactly one vertex. This proves that there is exactly only one [1,1] - dominating set. For this let us decompose the vertex set V in P_{3k} into k number of sets each contains 3 vertices in the form v_i,v_{i+1},v_{i+2}. Here V_{i+1} dominates v_i and v_{i+2}. Hence every set contains a vertex of D and the remaining two vertices in that set are dominated by only this vertex. Thus there are k such vertices namely $v_2,v_5,\ldots\ldots v_{n-1}$.
Which can be generalized as given below.

$$D_{[1,1]}(P_n)=\{v_{n+2-3k1};\ k_1=1, 2, 3, 4,\ldots.,k\}.$$

$$\text{Hence, } \gamma_{[1,\,1]}(P_n) = k$$

Suppose if n=4, the [1, 1]-dominating set are $\{v_1,v_4\}$ and $\{v_2, v_3\}$.

$$\text{Hence, } \gamma_{[1,1]}(P_4) = 2$$

In path graph P_n when n=3k+2, k=1,2,3……. there are two dominating sets and there is a minimum dominating set D in which end vertex v_1 dominates v_2 in the first dominating set and the end vertex v_n dominates v_{n-1} in the second dominating set. The remaining vertices in two dominating sets dominate exactly 2 vertices and every vertex in V-D is dominated by exactly one vertex. This proves that there are two [1,1]-dominating sets.
Which can be generalized as given below.
Therefore if n=3k+2, k=1, 2, 3,…. the [1, 1]- dominating set is

$$D_{[1,1]}(P_n) = \begin{cases} V_{n+2-3k1}, & \text{when } k1=1, 2,\ldots..,(k+1) \\ V_{n+3-3k1}, & \text{when } k1=1, 2, \ldots\ldots,(k+1) \end{cases}$$

$$\text{Hence, } \gamma_{[1,1]}(P_n)=k+1$$

In path graph Pn when n=3k+1, k=2,3…..there are 3 dominating sets and there is a minimum dominating set D in which end vertices v_1 dominates v_2 and vn dominates v_{n-1} in the first dominating set. In the second dominating set v_{n-1} dominates v_n and v_{n-2} dominates v_{n-3}. In the third dominating set v_2 dominates v_1 and v_3 dominates v_4. The other vertices in three dominating sets dominate exactly two vertices. Every vertex in V-D is dominated by exactly one vertex. This proves that there is exactly three [1,1]-dominating sets. Decompose the vertex set in V of P_{3k+1} except the end vertices in the first dominating set, the vertices vn-1, vn-2 in the second dominating set and v_2,v_3 in the third dominating set in the form v_i,v_{i+1}, v_{i+2}. Here v_{i+1} dominates v_i and v_{i+2}. Hence every set contains a vertex of D and the remaining two vertices in that set are dominated by only this vertex. Such vertices are $v_4,v_7,v_{10},\ldots.$ in the first dominating set, $v_2,v_5,v_8\ldots..$in the second dominating set and $v_6,v_9,v_{12}\ldots$ in the third dominating set.
Which can be generalized as given below.
Therefore if $n=3_{k+1}$, k=2, 3, 4,….. [1, 1]- dominating set is

$$D_{[1,1]}(P_n) = \begin{cases} V_{n+3-3k1}, & \text{when } k1=1, 2,\ldots., k+2 \\ V_{n+1-3k1},\ V_{n+2-3k1}, & \text{when } k1=1, 2,\ldots.,k+2 \\ V_{n+1-3k1},\ V_{n+2-3k1}, & \text{when } k1=1, 2,\ldots.,k+2 \end{cases}$$

$$\text{Hence, } \gamma_{[1,1]}(P_n)=k+2$$

IV. CONCLUSION

In this paper we have generalized the [j, k]-dominating number of path graphs. Similarly we can study [j, k]-dominating number of some other special graphs like cycles, helm and etc.

REFERENCES

[1] Haynes, T.W.; Hedetniemi, S.T.; and Slater, P.J. Fundamentals of Domination in Graphs. Newyork:Dekker, 1998.
[2] N.Murugesan and Deepa.S.Nair, (1, 2)-Domination In Graphs, J.Math. Comput. Sci. 2 (2012), No. 4, 774-783, Issn: 1927-5307.
[3] Chengye zhao2 and chao Zhang, [1,2]-Domination in Generalized Petersen Graphs1, Applied Mathematical Science, Vol. 9, 2015, no. 64, 3187 – 3191.
[4] K.Ameenal Bibi1, A.Lakshmi2 and R.Jothilakshmi3, Applications of Distance – 2 Dominating Sets of Graph in Networks, Advances in Computational Sciences and Technology, ISSN 0973 – 6107 Volume 10, Number 9 (2017) pp. 2801 – 2810.
[5] Mustapha Chellalia, Teresa W. Haynesb,*, Stephen T. Hedetniemic, [1, 2]- sets in graphs, Discrete Applied Mathematics 161 (2013) 2885 – 2893.