



IJRASET

International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 7 Issue: IV Month of publication: April 2019

DOI: <https://doi.org/10.22214/ijraset.2019.4184>

www.ijraset.com

Call:  08813907089

E-mail ID: ijraset@gmail.com

[J, K]-Set Domination of Path Graphs

N. Murugesan¹, P. Elangovan²

^{1,2}PG and Research Department of Mathematics, Government Arts College, Coimbatore-641018

Abstract: Domination is an important graph theoretic concept in graph theory. Various types of dominations have been studied in the literature. In this paper, the [j, k]-dominations have been considered for path graphs. By [j, k]-domination we mean, every vertex of the complement of the dominating set has at least j adjacent vertices and at most k adjacent vertices in the dominating set. In particular the [j, k]- domination number of a graph is the cardinality of the smallest such set. In this paper, the [j, k]- domination number for path graphs have been studied. *Mathematics subject classification: 05C69*

Keywords: Dominating set, Domination number, [j, k]- dominating set, [j, k]- domination number.

I. INTRODUCTION

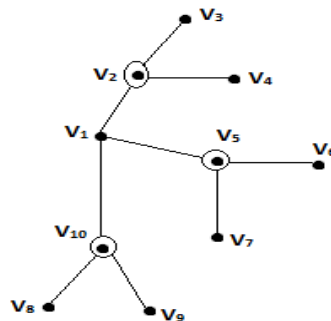
Let $G=(V, E)$ be a simple graph. A subset D of V is a dominating set of G if every vertex $v \in V-D$ is adjacent to a vertex of D . The domination number of G denoted by $\gamma(G)$ is the minimum cardinality of a dominating set G . A dominating set is a total dominating set if every vertex in G (including the vertices in D) have a neighbour in D .

II. [J,K] – SET DOMINATION

A. Definition

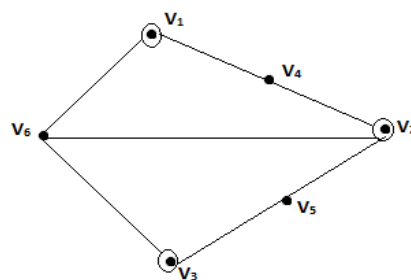
A set $D \subseteq V$ is called [J,K] – set dominaton if for any vertex $v \in V-D$, $j \leq |N(v) \cap D| \leq k$, i.e. there are atleast j vertices adjacent to v, but not more than k vertices in D . The smallest cardinality of [j,k] – set is called [j,k] – dominating set. The [j,k] – domination number is denoted by $\gamma_{j,k}(G)$

B. Example



In the above graph $\{v_2, v_5, v_{10}\}$ is a dominating set but it is not [1,2] – dominating set because the vertex v_1 is adjacent to 3 vertices, Also this is not a total dominating set.

C. Example



The set $D = \{v_1, v_2, v_3\}$ is a dominating set,

$V - D = \{v_4, v_5, v_6\}$

$N(v_4) \cap D = \{v_1, v_2\} \cap D = \{v_1, v_2\}$; Therefore $|N(v_4) \cap D| = 2$

$N(v_5) \cap D = \{v_2, v_3\} \cap D = \{v_2, v_3\}$; Therefore $|N(v_5) \cap D| = 2$

$N(v_6) \cap D = \{v_1, v_2, v_3\} \cap D = \{v_1, v_2, v_3\}$; Therefore $|N(v_6) \cap D| = 3$

Thus D is $[2,3]$ - dominating set.

D. Note

In general, every dominating set need not be a $[j,k]$ - dominating set, but the converse is always true. The trivial example for the converse part is that the dominating numbers of paths and cycles. As a generalization we have the following lemma.

E. Lemma

Let G be a graph with $\Delta(G)=2$. Then $\gamma(G) = \gamma_{[j,k]}(G)$.

1) *Proof:* Let G be graph with $\Delta(G)=2$. Then every vertex v in V has at most 2 neighbours. Therefore $j=1$ and $k=2$, the least possible values of j and k . Hence the Lemma.

III. [J,K]-DOMINATION IN PATHS

A. Theorem^[1]

The domination number of path P is $\gamma(P_n) = \lfloor \frac{n+2}{3} \rfloor$

The domination number of cycle C is $\gamma(C_n) = \lfloor \frac{n+2}{3} \rfloor$

B. Theorem^[2]

For $n > 2$, P_n has a $[1,2]$ -dominating set except $n=3k, k=1,2,3,\dots$

1) *Proof:* To prove this theorem, we prove that there is atleast one $v \in V - D$, such that $N(v)$ has two vertices in D , when n is not a multiple of 3, and when $n=3k$, for every $v \in V - D$, $N(v)$ has exactly one vertex in D .

As an example, consider P_8, P_9

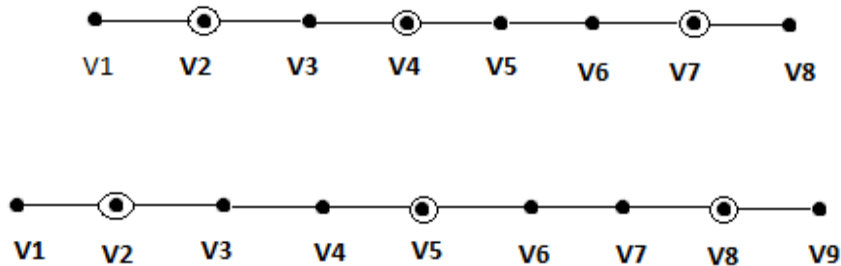


Fig 3.2 paths P_8 and P_9

Where $\{v_2, v_4, v_7\}$ and $\{v_2, v_5, v_8\}$ are dominating sets of P_8 and P_9 respectively and $\{v_2, v_5, v_8\}$ is the only dominating set of P_9 , therefore P_9 has no $[1, 2]$ - dominating sets.

C. Theorem^[3,4,5]

The number of $[1, 1]$ - dominating sets in the path graph $P_n, n \geq 3$

$$n [\gamma_{[1,1]}(P_n)] = \begin{cases} 1 & \text{if } n=3k, k=1,2,3,\dots \\ 2 & \text{if } n=4 \text{ and } n=3k+2, k=1, 2, 3,\dots \\ 3 & \text{if } n=3k+1, k= 2,3,4,\dots \end{cases}$$

Proof: Let v_1, v_2, \dots, v_n are the vertices of the path P_n , such that v_i is adjacent to $v_{i+1}, i=1, 2, 3, \dots, n-1$.

In path graph degree of each internal vertex is 2 and the degree of end vertices is 1. Hence every vertex dominates at most 2 vertices. Now we claim that in path graphs: P_n , when $n=3k, k=1,2,3,\dots$ there is a minimum dominating set D in which every vertex dominates exactly 2 vertices and every vertex in $V-D$ is dominated by exactly one vertex. This proves that there is exactly one $[1,1]$ -dominating set. For this let us decompose the vertex set V in P_{3k} into k number of sets each contains 3 vertices in the form v_i, v_{i+1}, v_{i+2} . Here v_{i+1} dominates v_i and v_{i+2} . Hence every set contains a vertex of D and the remaining two vertices in that set are dominated by only this vertex. Thus there are k such vertices namely v_2, v_5, \dots, v_{n-1} .

Which can be generalized as given below.

$$D_{[1,1]}(P_n) = \{v_{n+2-3k_1}; k_1=1, 2, 3, 4, \dots, k\}.$$

$$\text{Hence, } \gamma_{[1,1]}(P_n) = k$$

Suppose if $n=4$, the $[1, 1]$ -dominating set are $\{v_1, v_4\}$ and $\{v_2, v_3\}$.

$$\text{Hence, } \gamma_{[1,1]}(P_4) = 2$$

In path graph P_n when $n=3k+2, k=1,2,3,\dots$ there are two dominating sets and there is a minimum dominating set D in which end vertex v_1 dominates v_2 in the first dominating set and the end vertex v_n dominates v_{n-1} in the second dominating set. The remaining vertices in two dominating sets dominate exactly 2 vertices and every vertex in $V-D$ is dominated by exactly one vertex. This proves that there are two $[1,1]$ -dominating sets.

Which can be generalized as given below.

Therefore if $n=3k+2, k=1, 2, 3, \dots$ the $[1, 1]$ -dominating set is

$$D_{[1,1]}(P_n) = \begin{cases} V_{n+2-3k_1}, & \text{when } k_1=1, 2, \dots, (k+1) \\ V_{n+3-3k_1}, & \text{when } k_1=1, 2, \dots, (k+1) \end{cases}$$

$$\text{Hence, } \gamma_{[1,1]}(P_n) = k+1$$

In path graph P_n when $n=3k+1, k=2,3,\dots$ there are 3 dominating sets and there is a minimum dominating set D in which end vertices v_1 dominates v_2 and v_n dominates v_{n-1} in the first dominating set. In the second dominating set v_{n-1} dominates v_n and v_{n-2} dominates v_{n-3} . In the third dominating set v_2 dominates v_1 and v_3 dominates v_4 . The other vertices in three dominating sets dominate exactly two vertices. Every vertex in $V-D$ is dominated by exactly one vertex. This proves that there is exactly three $[1,1]$ -dominating sets. Decompose the vertex set in V of P_{3k+1} except the end vertices in the first dominating set, the vertices v_{n-1}, v_{n-2} in the second dominating set and v_2, v_3 in the third dominating set in the form v_i, v_{i+1}, v_{i+2} . Here v_{i+1} dominates v_i and v_{i+2} . Hence every set contains a vertex of D and the remaining two vertices in that set are dominated by only this vertex. Such vertices are v_4, v_7, v_{10}, \dots in the first dominating set, v_2, v_5, v_8, \dots in the second dominating set and v_6, v_9, v_{12}, \dots in the third dominating set.

Which can be generalized as given below.

Therefore if $n=3k+1, k=2, 3, 4, \dots$ $[1, 1]$ -dominating set is

$$D_{[1,1]}(P_n) = \begin{cases} V_{n+3-3k_1}, & \text{when } k_1=1, 2, \dots, k+2 \\ V_{n+1-3k_1}, V_{n+2-3k_1}, & \text{when } k_1=1, 2, \dots, k+2 \\ V_{n+1-3k_1}, V_{n+2-3k_1}, & \text{when } k_1=1, 2, \dots, k+2 \end{cases}$$

$$\text{Hence, } \gamma_{[1,1]}(P_n) = k+2$$

IV. CONCLUSION

In this paper we have generalized the $[j, k]$ -dominating number of path graphs. Similarly we can study $[j, k]$ -dominating number of some other special graphs like cycles, helm and etc.

REFERENCES

- [1] Haynes, T.W.; Hedetniemi, S.T.; and Slater, P.J. Fundamentals of Domination in Graphs. New York: Dekker, 1998.
- [2] N. Murugesan and Deepa.S. Nair, (1, 2)-Domination In Graphs, J. Math. Comput. Sci. 2 (2012), No. 4, 774-783, Issn: 1927-5307.
- [3] Chengye zhao2 and chao Zhang, [1,2]-Domination in Generalized Petersen Graphs1, Applied Mathematical Science, Vol. 9, 2015, no. 64, 3187 – 3191.
- [4] K. Ameenal Bibi1, A. Lakshmi2 and R. Jothilakshmi3, Applications of Distance – 2 Dominating Sets of Graph in Networks, Advances in Computational Sciences and Technology, ISSN 0973 – 6107 Volume 10, Number 9 (2017) pp. 2801 – 2810.
- [5] Mustapha Chellalia, Teresa W. Haynes*, Stephen T. Hedetniemi, [1, 2]- sets in graphs, Discrete Applied Mathematics 161 (2013) 2885 – 2893.



10.22214/IJRASET



45.98



IMPACT FACTOR:
7.129



IMPACT FACTOR:
7.429



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24*7 Support on Whatsapp)