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# Strongly Nano Generalized Closed Sets in Nano Topological Spaces

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**Abstract:** The basic objective of this paper is to introduce and investigate the properties of Strongly nano generalized closed sets in Nano Topological Spaces which is the extension of Nano generalized closed sets introduced by Lellis Thivagar

**Keywords:** Nano closed set, Nano open set, Generalized closed set, Nano generalized closed set, Strongly Nano generalized closed set.

## I. INTRODUCTION

Levine[2] introduced the class of generalized closed sets, a super class of closed sets in 1970. This concept was introduced as a generalization of closed sets in Topological spaces through which new results in general topology were introduced. Lellis Thivagar [1] introduced Nano topological space with respect to a subset  $X$  of a universe which is defined in terms of lower and upper approximations of  $X$ . The elements of Nanotopological space are called Nano open sets. He has also defined Nano closed sets, Nano-interior and Nano closure of a set. He also introduced the weak forms of Nano open sets namely Nano- $\alpha$  open sets, Nano semi open sets and Nano preopen sets. Nano generalized closed and nano strongly generalized closed was introduced by K. Bhuvaneswari[5,6]. In this paper some properties of strongly nano generalized closed sets in Nano topological spaces are studied.

## II. PRELIMINARIES

- 1) **Definition 2.1:** A subset  $A$  of a topological space  $(X, \tau)$  is called a generalized closed set (briefly  $g$ -closed) if  $Cl(A) \subseteq U$  Whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$
- 2) **Definition 2.2:** Let  $U$  be a non-empty finite set of objects called the universe and  $R$  be an equivalence relation of  $U$  named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with in another. The pair  $(U, R)$  is said to be the approximation space. Let  $X \subseteq U$ 
  - a) The lower approximation of  $X$  with respect to  $R$  is the set of all objects, which can be for certain classified as  $X$  with respect to  $R$  and it is denoted by  $L_R(X)$ . That is  $L_R(X) = U \{R(x) : R(x) \subseteq X\}$  where  $R(x)$  denotes the equivalence class determined by  $X$ .
  - b) The upper approximation of  $X$  with respect to  $R$  is the set of all objects which can be possibly classified as  $X$  with respect to  $R$  and it is denoted by  $U_R(X)$ . That is  $U_R(X) = U \{R(x) : R(x) \cap X \neq \emptyset\}$
  - c) The boundary region of  $X$  with respect to  $R$  is the set of all objects, which can be classified neither  $X$  nor as not  $X$  with respect to  $R$  and it is denoted by  $B_R(X)$ . That is  $B_R(X) = U_R(X) - L_R(X)$ .
- 3) **Property 2.3:** If  $(U, R)$  is an approximation space and  $X, Y \subseteq U$ , then
  - a)  $L_R(X) \subseteq X \subseteq U_R(X)$
  - b)  $L_R(\emptyset) = U_R(\emptyset) = \emptyset$  &  $L_R(U) = U_R(U) = U$
  - c)  $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
  - d)  $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
  - e)  $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
  - f)  $L_R(X) \subseteq L_R(Y)$  and  $U_R(X) \subseteq U_R(Y)$  whenever  $X \subseteq Y$
  - g)  $U_R(X^c) = [L_R(X)]^c$  and  $L_R(X^c) = [U_R(X)]^c$
  - h)  $U_R U_R(X) = L_R U_R(X) = U_R(X)$
  - i)  $L_R L_R(X) = U_R L_R(X) = L_R(X)$
- 4) **Definition 2.4:** Let  $U$  be the universe,  $R$  be an equivalence relation on  $U$  and  $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$  where  $X \subseteq U$ . Then by property 2.3,  $\tau_R(X)$  satisfies the following axioms:
  - a)  $U$  and  $\emptyset$  belongs to  $\tau_R(X)$ .
  - b) The union of the elements of any sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

c) The intersection of the elements of any finite sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

That is  $\tau_R(X)$  is a topology on  $U$  called the Nano topology on  $U$  with respect to  $X$ . We call

$(U, \tau_R(X))$  as the Nano topological space. The elements of  $\tau_R(X)$  are called as Nano open sets. The elements of  $(\tau_R(X))^c$  are called as nano closed sets.

5) **K. Remark 2.5:** If  $\tau_R(X)$  is the Nano topology on  $U$  with respect to  $X$ , then the set  $B = \{U, L_R(X), B_R(X)\}$  is the basis for  $\tau_R(X)$ .

6) **Definition 2.6:** If  $(U, \tau_R(X))$  is a Nano topological space with respect to  $X$  where  $X \subseteq U$  and if  $A \subseteq U$ , then the nano interior of the set  $A$  is defined as the union of all Nano open subsets contained in  $A$  and it is denoted by  $NInt(A)$ . That is  $NInt(A)$  is the largest Nano open subset of  $A$ . The Nano closure of the set  $A$  is defined as the intersection of all Nano closed sets containing  $A$  and it is denoted by  $NCl(A)$ . That is  $NCl(A)$  is the smallest Nano closed set containing  $A$ .

### III. STRONGLY NANO GENERALIZED CLOSED SET

Throughout this paper  $(U, \tau_R(X))$  is a Nano topological space with respect to  $X$  where  $X \subseteq U$ ,  $R$  is an equivalence Relation on  $U$ ,  $U/R$  denotes the family of equivalence classes of  $U$  by  $R$ .

1) **Definition 3.1:** Let  $(U, \tau_R(X))$  be a Nano topological space. A subset  $A$  of  $(U, \tau_R(X))$  is called Strongly Nanogeneralized closed set (briefly SNG- closed) if  $NCl(NInt(A)) \subseteq V$  where  $A \subseteq V$  and  $V$  is Nano open.

2) **Example 3.2:** Let  $U = \{w, x, y, z\}$  with  $U/R = \{\{w\}, \{x\}, \{y, z\}, \{z, y\}\}$  and  $X = \{w, y\}$ . Then the Nanotopology  $\tau_R(X) = \{U, \emptyset, \{w\}, \{w, y, z\}, \{y, z\}\}$ . Nano closed sets are  $\{\emptyset, U, \{x, y, z\}, \{x\}, \{w, x\}\}$ . Let  $V = \{x, y\}$  and  $A = \{x\}$ . Then  $NCl(A) = \{x\} \subseteq V$ . That is  $A$  is said to be SNG- closed in  $(U, \tau_R(X))$ .

3) **Theorem 3.3:** A subset  $A$  of  $(U, \tau_R(X))$  is SNG- closed if  $NCl(NInt(A)) - A$  contains no nonempty SNG- closed set.

a) **Proof:** Suppose if  $A$  is SNG- closed. Then  $NCl(NInt(A)) \subseteq V$  where  $A \subseteq V$  and  $V$  is Nano open. Let  $Y$  be a Nanoclosed subset of  $NCl(NInt(A)) - A$ . Then  $A \subseteq Y^c$  and  $Y^c$  is Nano open. Since  $A$  is SNG- closed,  $NCl(NInt(A)) \subseteq Y^c$  or  $Y \subseteq [NCl(NInt(A))]^c$ . That is  $Y \subseteq NCl(NInt(A))$  and  $Y \subseteq [NCl(NInt(A))]^c$  implies that  $Y \subseteq \emptyset$ . So  $Y$  is empty.

4) **Theorem 3.4:** If  $A$  and  $B$  are SNG- closed, then  $A \cup B$  is SNG- closed.

a) **Proof:** Let  $A$  and  $B$  are SNG- closed set. Then  $NCl(NInt(A)) \subseteq V$  where  $A \subseteq V$  and  $V$  is Nano open and  $NCl(NInt(B)) \subseteq V$  where  $B \subseteq V$  and  $V$  is Nano open. Since  $A$  and  $B$  are subsets of  $V$ ,  $(A \cup B)$  is a subset of  $V$  and  $V$  is Nano open. Then  $NCl(NInt(A \cup B)) = NCl(NInt(A) \cup NCl(NInt(B))) \subseteq V$  which implies that  $A \cup B$  is SNG- closed.

5) **Remark 3.5:** The Intersection of two SNG- closed sets is again an SNG- closed set which is shown in the following example.

6) **Example 3.6:** Let  $U = \{a, b, c, d\}$ ,  $X = \{a, b\}$ ,  $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ ,  $\tau_R(X) = \{U, \emptyset, \{a\}, \{a, b, d\}, \{b, d\}\}$ . Let  $A = \{a, b, c\}$ ,  $B = \{a, c, d\}$  and  $A \cap B = \{a, c\}$ . Here  $NCl(NInt(A \cap B)) \subseteq V$  when  $(A \cap B) \subseteq V$  and  $V$  is Nano open.

7) **Theorem 3.7:** If  $A$  is SNG- closed and  $A \subseteq B \subseteq NCl(NInt(A))$ , then  $B$  is SNG- closed.

a) **Proof:** Let  $B \subseteq V$  where  $V$  is Nano open in  $\tau_R(X)$ . Then  $A \subseteq B$  implies  $A \subseteq V$ . Since  $A$  is SNG- closed,  $NCl(NInt(A)) \subseteq V$ . Also  $B \subseteq NCl(NInt(A))$  implies  $NCl(NInt(B)) \subseteq NCl(NInt(A))$ . Thus  $NCl(NInt(B)) \subseteq V$  and so  $B$  is SNG- closed.

8) **Theorem 3.8:** Every Nano closed set is Nano generalized closed set.

a) **Proof:** Let  $A \subseteq V$  and  $V$  is Nano open in  $\tau_R(X)$ . Since  $A$  is Nano closed,  $NCl(NInt(A)) \subseteq A$ . That is  $NCl(NInt(A)) \subseteq A \subseteq V$ . Hence  $A$  is Strongly Nano generalized closed set. The converse of the above theorem need not be true as seen from the following example.

9) **Example 3.9:** Let  $U = \{a, b, c, d\}$  with  $X = \{a, c\}$  with  $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ .  $\tau_R(X) = \{U, \emptyset, \{a\}, \{a, c, d\}, \{c, d\}\}$ . Nano closed sets are  $U, \emptyset, \{b, c, d\}, \{b\}, \{a, b\}$ . Here  $\{b, c\}$  is strongly nano generalized closed set but it is not nano closed set.

10) **Theorem 3.10:** A SNG- closed set  $A$  is Nano closed if and only if  $NCl(NInt(A)) - A$  is Nano closed.

a) **Proof:** (Necessity) Let  $A$  is Nano closed. Then  $NCl(NInt(A)) = A$  and so  $NCl(NInt(A)) - A = \emptyset$  which is Nano closed.

(Sufficiency) Suppose  $NCl(NInt(A)) - A$  is Nano closed. Then  $NCl(NInt(A)) - A = \emptyset$  since  $A$  is Nano closed. That is  $NCl(NInt(A)) = A$  or  $A$  is Nano closed.

11) **Theorem 3.11:** Suppose that  $B \subseteq A \subseteq U$ ,  $B$  is an SNG- closed set relative to  $A$  and that  $A$  is an SNG- closed subset of  $U$ . Then  $B$  is SNG- closed relative to  $U$ .

a) **Proof:** Let  $B \subseteq V$  and suppose that  $V$  is Nano open in  $U$ . Then  $B \subseteq A \cap V$ . Therefore  $NCl(NInt(B)) \subseteq A \cap V$ . It follows then that  $A \cap NCl(NInt(B)) \subseteq A \cap V$  and  $A \subseteq V \cup NCl(NInt(B))$ . Since  $A$  is SNG- closed in  $U$ , we have  $NCl(NInt(A)) \subseteq V \cup NCl(NInt(B))$ . Therefore  $NCl(NInt(B)) \subseteq NCl(NInt(A)) \subseteq V \cup NCl(NInt(B))$  and so  $NCl(NInt(B)) \subseteq V$ . Then  $B$  is SNG- closed relative to  $V$ .

- 12) *Corollary 3.12:* Let A be a SNg-closed set and suppose that F is a Nanoclosedset . Then  $A \cap F$  is anSNg-closed set which is given in the following example.
- 13) *Example 3.13:* Let  $U = \{a,b,c,d\}$  with  $X = \{a,b\}$  with  $U/R = \{\{a\},\{c\},\{b,d\}\}$ .  $\tau_R(X) = \{U, \emptyset, \{a\},\{a,b,d\},\{b,d\}\}$ . Nano closed sets are  $U, \emptyset, \{b,c,d\}, \{c\}, \{a,c\}$ . Let  $A = \{a,b,c\}$  and  $F = \{b,c,d\}$ . Then  $A \cap F = \{b,c\}$  is an SNg closed set.
- 14) *Theorem 3.14:* For each  $a \in U$  , either  $\{a\}$  is Nano closed (or)  $\{a\}^c$  is Strongly Nano generalized closed in  $\tau_R(X)$ .
- a) *Proof:* Suppose  $\{a\}$  is not Nano closed in U. Then  $\{a\}^c$  is not nano open and the only nano open set containing  $\{a\}^c$  is  $V \subseteq U$ . That is  $\{a\}^c \subseteq U$ . Therefore  $NCl(NInt(\{a\}^c)) \subseteq U$  which implies  $\{a\}^c$  is Strongly Nano generalized closed set in  $\tau_R(X)$

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