iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 7 **Issue:** IV **Month of publication:** April 2019

DOI:

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 7 Issue IV, Apr 2019- Available at www.ijraset.com

Strongly Nano Generalized Closed Sets in Nano Topological Spaces

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Abstract: The basic objective of this paper is to introduce and investigate the properties of Strongly nano generalized closed sets in Nano Topological Spaces which is the extension of Nano generalized closed sets introduced by Lellis Thivagar
Keywords: Nano closed set, Nano open set, Generalized closed set,Nano generalized closed set, Strongly Nano generalized closed set.

I. INTRODUCTION

Levine[2] introduced the class of generalized closed sets , a super class of closed sets in 1970.This concept was introduced as a generalization of closed sets in Topological spaces through which new results in general topology were introduced. LellisThivagar [1] introduced Nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of Nanotopological space are called Nano open sets. He has also defined Nano closed sets, Nano-interior and Nano closure of a set. He also introduced the weak forms of Nano open sets namely Nano – α open sets , Nano semi open sets and Nano preopen sets. Nano generalized closed and nano strongly generalized closed was introduced by K.Bhuvaneswari[5,6]. In this paper some properties of strongly nano generalized closed sets in Nano topological spaces are studied.

II. PRELIMINARIES

1) *Definition 2.1*: A subset Aof a topological space (X , τ) is called a generalized closed set (briefly g -closed) if Cl(A)⊆ U Whenever A ⊆U and U is open in(X , τ)
2) *Definition 2.2:* Let U be a non-empty finite set of objectscalled the universe and R be an equivalence relation of U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with in another. The pair (U, R) is said to be the approximation space. Let X⊆ U
   a) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = U\{R(x) : R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by X.
   b) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = U\{R(x): R(x)\cap X\neq\phi\}$
   c) The boundary region of X with respect to R is theset of all objects, which can be classified neither X nor as not X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) – L_R(X)$.
3) *Property 2.3*: If (U, R) is an approximation space and X, Y⊆U, then
   a) $L_R(X) \subseteq X\subseteq U_R(X)$
   b) $L_R(\phi) = U_R(\phi) =$ & $L_R(U) =U_R(U)=U$
   c) $U_R(XUY) = U_R(X)\ U\ U_R(Y)$
   d) $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$
   e) $L_R(X\cap Y) = L_R(X) \cap L_R(Y)$
   f) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
   g) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$
   h) $U_RU_R (X) = L_R U_R(X) = U_R(X)$
   i) $L_R L_R(X) = U_R L_R(X) = L_R(X)$
4) *Definition 2.4:* Let U be the universe, R be an equivalencerelation on U and$\tau_R(X) = \{U,\phi , L_R(X), U_R(X), B_R(X)\}$ where X⊆ U. Then by property 2.3, $\tau_R(X)$ satisfies the following axioms :
   a) U andϕ belongs to$\tau_R(X)$.
   b) The union of the elements of any sub collection of$\tau_R(X)$ is in $\tau_R(X)$.

c) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call
$(U,\tau_R(X))$ as the Nano topological space. The elements of$\tau_R(X)$ are called as Nano open sets. The elements of $(\tau_R(X))^C$ are called as nano closed sets.

5) K. Remark 2.5: If $\tau_R(X)$ is the Nano topology on U with respectto X, then the set B={U, $L_R(X)$, $B_R(X)$} is the basis for $\tau_R(X)$.
6) *Definition 2.6:* If $(U,\tau_R(X))$ is a Nano topological space with respect to X where X⊆U and if A⊆U, then the nano interior of the set A is defined as the union of all Nano open subsets contained in A and it is denoted by NInt(A). That is NInt(A) is the largest Nano open subset of A. The Nano closure of the set A is defined as the intersection of all Nano closed sets containing A and it is denoted by NCl(A). That is NCl(A) is the smallest Nano closed set containing A.

III. STRONGLY NANO GENERALIZED CLOSED SET

Throughout this paper (U, $\tau_R(X)$) is a Nano topological space with respect to X where X⊆ U , R is an equivalence Relation on U , U/R denotes the family of equivalence classes of U by R.

1) *Definition 3.1:* Let (U, $\tau_R(X)$)) be a Nano topological space. A subset A of $(U,\tau_R(X))$ is called Strongly Nanogeneralized closed set (briefly SNg- closed) if NCl(NInt(A))⊆ V where A⊆V and V is Nano open.
2) *Example 3.2:* Let U = { w,x,y,z } with U/R ={ {w} ,{x} {y,z},{z,y} } and X= {w,y}.Then the Nanotopology $\tau_R(X)$) = { U, ø , {w},{w,y,z},{y,z}}. Nano closed sets are {ø , U ,{x,y,z},{x},{w,x}}. Let V= {x,y} and
A= {x}. Then NCl(A) = {x}⊆ V . That is A is said to be SNg- closed in (U,$\tau_R(X)$).
3) *Theorem 3.3*: A subset A of (U,$\tau_R(X)$) is SNg –closed ifNCl(NInt(A)) – A contains no nonempty SNg- closed set.
   a) *Proof:* Suppose if A is SNg- closed. Then NCl(NInt(A))⊆ V where A⊆ V and V is Nano open. Let Y be a Nanoclosed subset of NCl(NInt(A))–A . Then A⊆Y^C andY^C is Nano open. Since A is SNg – closed,NCl(NInt(A))⊆Y^C or Y ⊆$[NCl(NInt(A))]^C$. That is Y ⊆NCl(NInt(A)) andY ⊆$[NCl(NInt(A))]^C$ implies that Y⊆ ø . So Y is empty.
4) *Theorem 3.4:* If A and B are SNg- closed , then A U B is SNg- closed.
   a) *Proof:* Let A and B are SNg- closed set. Then NCl(NInt(A))⊆ V where A ⊆ V and V is Nano open and NCl(NInt(B**))**⊆ V where B ⊆ V and V is Nano open . Since A and B are subsets of V , (A U B) is a subset of V and V is Nano open . Then NCl(NInt(A U B)) = NCl(NInt(A)) U NCl(NInt(B**))**⊆V which implies that AUB is SNg- closed.
5) *Remark 3.5*: The Intersection of two SNg- closed sets is again an SNg-closed set which is shown in the following example.
6) *Example 3.6:* Let U={a,b,c,d}, X={a,b}, U/R = {{a},{c},{b,d}},$\tau_R(X)$).= {U,ø,{a},{a,b,d},{b,d}}.Let A= {a,b,c} , B= {a,c,d} and A∩ B = {a,c}.Here NCl(NInt(A∩B))⊆ V when (A∩B) ⊆V and V is Nano open.
7) *Theorem 3.7:* If A is SNg – closed andA⊆B⊆NCl(NInt(A)) , then B is SNg-closed.
   a) Proof: Let B⊆ V where V is Nano open in$\tau_R(X)$.Then A⊆ B implies A⊆ V. Since A in SNg-closed ,NCl(NInt(A) ⊆ V. Also B ⊆NCl(NInt(A)) implies NCl(NInt(B))⊆NCl(NInt(A)). Thus NCl(NInt(B))⊆ V and so B is SNg-closed.
8) *Theorem 3.8:* Every Nano closed set is Nano generalized closed set.
   a) *Proof:* Let A⊆V and V is Nano open in $\tau_R(X)$. Since A is Nano closed ,NCl(NInt(A))⊆A. That is NCl(NInt(A))⊆ A⊆V. Hence A is Strongly Nano generalized closed set.The converse of the above theorem need not be true as seen from the following example.
9) *Example 3.9:* Let U = {a,b,c,d} with X= {a,c} with U/R = {{a},{b},{c,d}}. $\tau_R(X)$= {U, ø, {a},{a,c,d},{c,d}}. Nano closed sets are U,ø, {b,c,d},{b},{a,b}.Here {b,c} is strongly nano generalized closed set but it is not nano closed set.
10) *Theorem 3.10:* A SNg- closed set A is Nano closed if and only if NCl(NInt(A))– A is Nano closed.
   a) *Proof:* **(**Necessity) Let A is Nano closed . Then NCl(NInt(A)) = A and so NCl(NInt(A)) – A = ø which is Nano closed.

(Sufficiency) Suppose NCl(NInt(A)) – A is Nano closed. Then NCl(NInt(A)) – A = ø since A is Nano closed. That is NCl(NInt(A)) = A or A is Nano closed.

11) *Theorem 3.11*: Suppose that B⊆A⊆U,B is an SNg-closed set relative to A and that A is an SNg-closed subset of U. Then B is SNg-closed relative to U.
   a) *Proof:* Let B⊆V and suppose that V is Nanoopen in U.Then B⊆A∩V. Therefore NCl(NInt(B))⊆A∩V. It follows then that A∩NCl(NInt(B))⊆A∩V and A ⊆ V U NCl(NInt(B)). Since A is SNg- closed in U , we have NCl(NInt(A))⊆ V U NCl(NInt(B)).Therefore NCl(NInt(B))⊆NCl(NInt(A))⊆ V U NCl(NInt(B) and so NCl(NInt(B))⊆ V. Then B is SNg-closed relative to V.

12) *Corralary 3.12:* Let A be a SNg-closed set and suppose that F is a Nanoclosedset . Then A∩F is anSNg-closed set which is given in the following example.
13) *Example 3.13*: Let U = {a,b,c,d} with X= {a,b} with U/R = {{a},{c},{b,d}}. $\tau_R(X)$ = {U, ø, {a},{a,b,d},{b,d}}. Nano closed sets are U,ø, {b,c,d},{c},{a,c}. Let A= {a,b,c} and F= {b,c,d}. Then A∩F= {b,c} is an SNg closed set.
14) *Theorem 3.14:* For each a∊ U , either {a} is Nano closed (or) $\{a\}^c$is Strongly Nano generalized closed in $\tau_R(X)$.
a) *Proof:* Suppose {a} is not Nano closed in U. Then$\{a\}^c$is not nano open and the only nano open set containing$\{a\}^c$ is V ⊆ U. That is$\{a\}^c$⊆ U. Therefore NCl(NInt($\{a\}^c$)⊆ U which implies $\{a\}^c$is Strongly Nano generalized closed set in $\tau_R(X)$

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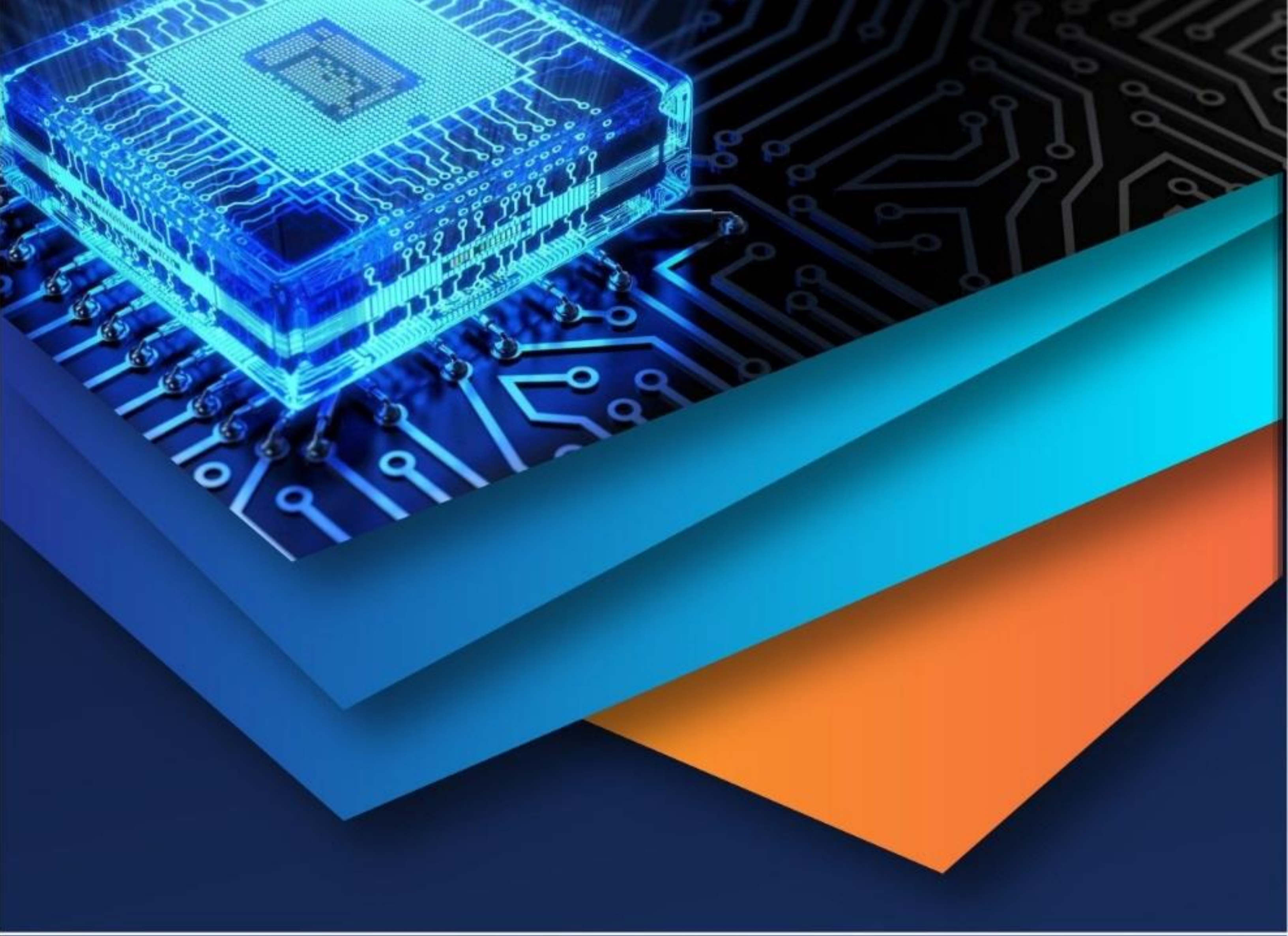

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