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Graph Colouring Parameters - A Survey

J. Suresh Kumar
Assistant Professor and Research Supervisor, PG and Research Department of Mathematics, N.S.S. Hindu College, Changanacherry, Kottayam District, Kerala, India-686102

Abstract: Graph Coloring is one of the most popular topics in Graph Theory and Discrete Mathematics. In this paper, we review the various types of graph coloring concepts and the respective graph parameters defined and studied so far and discuss the significant results.

I. INTRODUCTION

The chromatic number, χ(G) of a graph G. is one of the most popular graphical invariants. Dirac [23, 24] introduced the concept of k-critical graphs as the graphs, which are k-chromatic and are inclusion minimal with respect to this property.

It is well known that if G is k-critical, minimum degree of G is at least k-1. Motivated by this observation, Gallai [42, 43], introduced the concept of high and low vertices. A vertex of a k-critical graph is said to be a low or high vertex, according as its degree is k-1 or not. The subgraphs induced by the set of high vertices and by the set of low vertices have been studied extensively by Gallai [42, 43], and Sachs and Stiebitz [82, 83]. A survey of the results about k-critical graphs having low vertices is given by Sachs and Stiebitz [82]. These concepts play a significant role in characterizing and constructing edge-critical graphs.

The Pseudo achromatic number of a graph G is defined as the maximum number of colors that can be assigned to the vertices of G such that for any two distinct colors, there must exist an edge whose end vertices have those pair of colors and is denoted by ψ_s(G).

A graph G is called edge critical, if ψ_s(G-e) < ψ_s(G),for every edge, e, of G. Further, if then G is called k-edge critical, It is well known that if G is k-edge critical graph, then its maximum degree is at most k-1. Motivated from this result, Suresh Kumar [61] defined a vertex of a k-edge critical graph as a high vertex, if its degree is k-1 and as a low vertex, otherwise.

In this paper, we review the various types of graph colorings available in the literature and the respective graph parameters defined and studied so far and discuss the significant results.

II. RESULTS AND DISCUSSION

Let G be a graph with vertex set V(G) and let C be a set of colors. A coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have distinct colors. The set of vertices with any one color is called a color class of G. Each color class forms an independent set of vertices. A k-coloring of G is a coloring of G using k colors. The minimum cardinal k for which G has a k-coloring is called the chromatic number of the graph G and is denoted by X(G). If $\chi(G) = k$, then G is called a k-chromatic graph. A graph G is called a k-colorable graph, if G has a coloring using at most k colors. An edge coloring of a graph G is an assignment of colors to the edges of a graph .G such that adjacent edges have distinct colors. A k-edge coloring of G is an edge coloring of G using k colors. The minimum cardinal k for which G has a kedge coloring is called the edge chromatic number of the graph G and is denoted by Vizing [96, 97] obtained the best bound for the edge chromatic number of a graph as follows: $\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1$. The inequalities of these types are known as Vizing-type results.

A. Complete and Pseudo- Complete Colorings

The achromatic number of a graph G was introduced by Harary [53, 54]. Gupta [50] introduced the concept of pseudo-achromatic number of G.

A coloring is also called as a *proper coloring.* A *Pseudo-complete coloring* is an assignment of colors with the property that for any two distinct colors, there exist adjacent' vertices whose end vertices receive that pair of colors. A pseudo-complete coloring, which is also a proper coloring, is called a complete coloring of G and the maximum number of color classes in any complete coloring of G is called the *achromatic number* of G and is denoted by i(G). The maximum number of colors in any pseudo-complete coloring of G is called the *pseudo-achromatic number of G* and is denoted by "(G). The set of vertices with any one color is called a *color class* of G.

A pseudo-complete coloring using k colors is called a k- pseudo-complete coloring and a graph having a pseudo-complete coloring using at most k colors is called a k-pseudo-complete colorable graph.

Let G be a k-pseudo-complete colorable graph and consider any k-pseudo-complete coloring of G. Let {S1,S2,...,S,1}be the color classes of G. Then S 1 ,S2 , ... ,S,,} is a partition of V(G). Since, there exist edges with any pair of colors, the general graph obtained by sequentially identifying the sets {S1,S2.....S} is a spanning super graph of K,. Thus G is homomorphic to is a spanning supergraph of K. Thus we have

1) *Theorem 1.4.* An n-pseudo-complete colorable graph is homomorphic to a spanning supergraph of K_n
2) *Theorem 1.5.* An n-pseudo-complete colorable cycle is homomorphic to an eulerian spanning supergraph of K_n.
3) *Theorem 1.6.* A cycle in an n-pseudo-complete colourable graph is homomorphic to a closed trail of K_n
4) *Theorem 1.7.* A path in an n-pseudo-complete colourable graph is homomorphic to a trail of same length in K_n

B. A Survey Of Various Coloring Parameters

The Chromatic number is the first of all coloring parameters to be studied by graph theorists and history dates back to the famous *Four Color Problem* and the early works of Kempe [65] in 1879 and Heawood [56] in 1890. Galvin and Komjath [44] in 1991 proved that the Axiom of choice is equivalent to the statement that "Any finite or infinite graph has a chromatic number".

The achromatic number was the first studied as a graph parameter by Harary, Hedetniemi and Prins [53] in 1967 and later by Harary and Hedetniemi [54] in 1970. The definition of the pseudo-achromatic number of a graph appeared in a paper by Gupta [50] in 1969. In 1972, Sampathkumar and Bhave [84] has studied these parameters using the concept of the partition graphs. The most important reference to the achromatic number was the paper by Farber, Hahn, Hell and Miller [38].

We now give some of the other vertex coloring parameters. Subchromatic number and subachromatic number were first introduced by Mynhardt and Broere [77] as a special case of a more general concept and was studied by Albertson et.al. [6] in 1989, the Ochromatic number and the Ordered colorings were studied by Simmons [85, 86, 87] and later by Erdos et.al.[36] in 1987. The Partite chromatic number and partite achromatic number were studied by Domke et.al. [27] in 1986. The Harmonious chromatic number was introduced by Frank, Harary and Plantholt [40] and independently by Hoperoft and Krishnamoorthy [59]. These parameters were studied in detail by Lee and Mitchem [67] in 1987, by Mitchem [75] in 1989, by Miller and Pritikin [74] in 199l and by Kundrik [67] in1992.

The *T-colorings and T-chromatic number* were introduced by Hale [52], when studying the practical problem of assigning a transmitting frequency to each of a number of radio and television stations, with the condition that any pair of stations geographically enough to interfere with each other should transmit signals that differ significantly in frequency. This practical problem of assignment of frequencies to each station reduces to the T-coloring of a graph G, whose vertices are the radio stations with an edge between any pair of stations if they are mutually close. When T={O}, the T-coloring reduces to usual coloring. Further results on T-colorings were available in Roberts [81], Liu [69] and Tesman [91, 92]

The Game chromatic number was introduced by Bodlaender [10] in 1991 and was studied by Faigle, Kern, Kierstead and Trotter [37]. The Cochromatic Number was first introduced by Lesniak and Straight [68] in 1997 and was also studied by Straight [88, 89], Gimbel [46, 47], Broere and Burger [13, 14], Erdos, Gimbel and Kratsch [35] and Erdos and Gimbel [33]. The Star Chromatic Number was introduced and studied by Bondy and Hell [11], Zhu [100], Guichard [49] and Abbot and Zhou [1]. The List colorings and List chromatic number were introduced by Vizing [98] in 1976 and also studied by Borodin [12] and Fleischner and Stiebitz [40].

The other coloring problems include the problem of coloring the real line, so that the distance between the like colored numbers does not lie in some specified set which was studied by Eggleton et.al. [28, 29, 30, 31, 32], the Online-coloring studied by Lovasz, Saks and Trotter [71] and Viswanathan [95]. The Total coloring was introduced by Vizing [96, 97] and independently by Behzad [9] and was studied also by Vijayaditya [94]. The Acyclic coloring was introduced by Grunbaum [48] in 1973 and was studied by Albertson and Berman [5], and Borodin [12]. The Equitable colorings were studied by Hajnal and Szemeredi [51] and Meyer [73]. The strong chromatic number was introduced and studied by Alon [8] and independently by Fellows [39]. The computational aspects of graph coloring parameters is, in general, difficult as it is clear from the following results on computational complexity:

1) *Problem.1:* Complexity of chromatic number.

Instance: A Graph G and a positive integer, k.

Question: Is G a k-colorable graph?

Karp [63] showed in 1972 that the above problem is NP-complete. Garey and Johnson [45] justified the solvability of the above problem in polynomial time for k = 2 and provided their own proof of the NP-completeness for k>2 in their work.

2) *Problem.2:* Complexity of achromatic number.

Instance: A Graph G and a positive integer, k.

Question: Does G have achromatic number k or more?

This above problem was shown to be NP-complete by Yannakakis and Gavril [99] in 1980.

3) *Problem.3:* Complexity of the Pseudo-achromatic number.

Instance: Graph G and a positive integer, k.

Question: Does G have pseudo-achromatic number *k* or more?

Hell [58] has shown that the above problem is NP-complete by an easy proof of the equality of this parameter with that of the exact achromatic number of a bipartite graph with $n(n-1)/2$ edges, a problem which in turn is NP-complete. Hell and Miller [57] showed that there are only finitely many irreducible graphs with achromatic number less than four. Mate in [72] computed the approximate achromatic number of an irreducible graph with *n* vertices.

Akiyama, Harary and Ostrand [4] characterized the graphs G such that both G and its complement are n-colorable and specified explicitly for the case n=3 all the 171 graphs. They further showed that the 41 of the above graphs have the property that both G and its complement have achromatic number three. Farber et. al. [38] studied the achromatic number from the point of view of computational complexity and showed that the problem of determining whether the achromatic number of G is at least *n* is NP-complete even when the problem is restricted to bipartite graphs.

The achromatic number of a cycle and a path was determined by Hare, Hedetniemi and Laskar [55]. and also by Turner, Rowley, Jamison and Laskar [93] considered the problem of finding the achromatic number of a graph as follows: "Given any simple graph G and an integer *k*,does G have a complete k-coloring?" and investigated the achromatic number of the line graph, L(K), of the complete graph, *K*, for n < 15. Jamison [62] presented certain best available upper and lower bounds for the achromatic number of the line graph, L(G). The problem of obtaining bounds for graph parameters and finding the existence of graphs with prescribed parameters is a fundamental line of investigation in Structural Graph theory. The problem of finding the existence of graphs with prescribed coloring parameters has been studied by many authors. Dirac [18] asked if there exists a graph with no triangles but with arbitrarily high chromatic number. This was answered independently by Descartes and Descartes [17], Mycielski [76] and Zykov [101]. Their result was extended by Kelley and Kelley [64], who have proved that for all *n>1,* there exists an n-chromatic graph, whose girth exceeds five. He also raised the following conjecture: "For any two positive integers *m* and *n*, there exists an n-chromatic graph whose girth exceeds *M"*. This was first settled by Erdos and Rado [34] using a probabilistic argument and later by Lovasz [70] constructively. The graphs which are inclusion minimal with respect to the chromatic number are called the critical graphs and was introduced and studied by Dirac [18-26]. Dirac [22] obtained 6- critical graphs with many edges by completely joining two disjoint odd cycles of the same length. Dirac [24] also investigated the minimum number of vertices in a critical graph.

Kelly and Kelly [64] studied the length of the cycles and the circumference of these graphs. They proved that every large k-critical graph contains a long cycle and obtained some bounds for the circumference of 4-critical graphs. Dirac [18] and Gallai [42] obtained some improved bounds for the circumference of k-critical graphs. Dirac[24], Gallai [42,43] and Sachs and Stiebitz [82, 83] studied the subgraphs of critical graphs and the existence of subgraphs of critical graphs satisfying several properties were investigated. Certain bounds on the sum and product of the chromatic numbers of a graph and its complement were developed by Nordhaus and Gaddum [78].

C. Variations of Graph Parameters using Graphoidal Covers

Acharya and Sampath kumar [2] introduced the concept of a Graphoidal cover of a graph. Suresh Suseela and Arumugam [90] introduced the concept of an Acyclic graphoidal cover of a graph G.A Graphoidal Cover of a graph *G* is a collection P of non-trivial paths (not necessarily open) in G such that every edge of G is in exactly one path in P and every vertex of G is an internal vertex of at most one path in T. Further, if no member of P is a cycle in G, then P is called an Acyclic Graphoidal cover of G. Any path in P is simply called a P-path. Purnima Gupta [80] introduced Graphoidal domination number as the notion of domination number extended to Graphoidal covers of a graph.

In [60], Sureshkumar introduced the concepts of ψ-colorings and ψ-chromatic number of graphs as the notion of the edge-chromatic number of a graph extended to acyclic Graphoidal covers of a graph and initiate a study of these parameters. Some significant results are:

1) *Theorem.* A graph *H* is the intersection graph of an acyclic Graphoidal cover of a tree if and only if each block of *H* is a complete graph.
2) *Theorem.:* A graph *H* is the intersection graph of an acyclic Graphoidal cover P for a unicyclic graph if and only if all except one block of *H* are complete graphs and the exception is a graph consisting of two complete graphs with exactly one edge in commons

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