iJRASET
International Journal For Research in Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 7 **Issue:** IV **Month of publication:** April 2019

DOI: https://doi.org/10.22214/ijraset.2019.4288

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 7 Issue IV, Apr 2019- Available at www.ijraset.com

On the Negative Pell Equation $y^2 = 48x^2 - 23$

S. Mallika[1], G. Ramya[2]
[1]*Assistant Professor,* [2]*M. Phil Scholar, Department of Mathematics, SIGC, Trichy-620002, Tamilnadu, India.*

Abstract: ***The binary quadratic equation represents by negative Pellian*** $y^2 = 48x^2 - 23$ ***is analyzed for its distinct integer solutions. A few interesting relations among the solution are given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.***
Keywords: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.
AMS Mathematics subject Classification (2010):11D09

I. INTRODUCTION

The binary quadratic Diophantine equations (both homogeneous and non-homogeneous) are rich in variety. In [1-17] the binary quadratic non-homogeneous equations representing hyperbolas respectively are studied for their non-zero integral solutions. This communication concerns with yet another binary quadratic equation given by $y^2 = 48x^2 - 23$ .The recurrence relations satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

II. METHOD OF ANALYSIS

The negative Pell equation representing hyperbola under consideration is

$$y^2 = 48x^2 - 23 \tag{1}$$

whose smallest positive integer solution is

$$x_0 = 1,\ y_0 = 5$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 48x^2 + 1$$

whose smallest positive integer solution is

$$\tilde{x}_0 = 1,\ \tilde{y}_0 = 7$$

whose general solution is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{48}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(7 + \sqrt{48}\right)^{n+1} + \left(7 - \sqrt{48}\right)^{n+1}$$

$$g_n = \left(7 + \sqrt{48}\right)^{n+1} - \left(7 - \sqrt{48}\right)^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = \frac{1}{2} f_n + \frac{5}{8\sqrt{3}} g_n$$

$$y_{n+1} = \frac{5}{2} f_n + \frac{6}{\sqrt{3}} g_n$$

The recurrence relations satisfied by x and y are given by

$$x_{n+3} - 14x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 14y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x and y satisfying (1) are given in the Table: 1 below:

©IJRASET: All Rights are Reserved

Table: 1 Numerical Examples

n	x_{n+1}	y_{n+1}
-1	1	5
0	12	83
1	167	1157
2	2326	16115
3	32397	224453
4	451232	3126227

From the above table, we observe some interesting relations among the solutions which are presented below:

A. x_{n+1} values are alternatively odd and even according as n values & y_{n+1} values are alternatively odd.

B. Relations Among The Solutions

1) $x_{n+3} - 14x_{n+2} + x_{n+1} = 0$

2) $7x_{n+1} - x_{n+2} + y_{n+1} = 0$

3) $x_{n+1} - 7x_{n+2} + y_{n+2} = 0$

4) $7x_{n+1} - 97x_{n+2} + y_{n+3} = 0$

5) $97x_{n+1} - x_{n+3} + 14y_{n+1} = 0$

6) $x_{n+1} - x_{n+3} + 2y_{n+2} = 0$

7) $x_{n+1} - 97x_{n+3} + 14y_{n+3} = 0$

8) $y_{n+2} - 7y_{n+1} - 48x_{n+1} = 0$

9) $y_{n+3} - 97y_{n+1} - 672x_{n+1} = 0$

10) $7y_{n+3} - 97y_{n+2} - 48x_{n+1} = 0$

11) $97x_{n+2} - 7x_{n+3} + y_{n+1} = 0$

12) $7x_{n+2} - x_{n+3} + y_{n+2} = 0$

13) $x_{n+2} - 7x_{n+3} + y_{n+3} = 0$

14) $7y_{n+3} - 7y_{n+1} - 672x_{n+2} = 0$

15) $48x_{n+2} + y_{n+1} - 7y_{n+2} = 0$

16) $y_{n+3} - 7y_{n+2} - 48x_{n+2} = 0$

17) $97y_{n+2} - 7y_{n+1} - 48x_{n+3} = 0$

18) $97y_{n+3} - y_{n+1} - 672x_{n+3} = 0$

19) $7y_{n+3} - y_{n+2} - 48x_{n+3} = 0$

20) $y_{n+1} - 14y_{n+2} + y_{n+3} = 0$

C. Each Of The Following Expressions Represents A Nasty Number

1) $\frac{1}{23}\left(996x_{2n+2} - 60x_{2n+3} + 276\right)$

©IJRASET: All Rights are Reserved

2) $\frac{1}{161}(6942x_{2n+2} - 30x_{2n+4} + 1932)$

3) $\frac{1}{23}(576x_{2n+2} - 60y_{2n+2} + 276)$

4) $\frac{1}{161}(6912x_{2n+2} - 60y_{2n+3} + 1932)$

5) $\frac{1}{2231}(96192x_{2n+2} - 60y_{2n+4} + 26772)$

6) $\frac{1}{23}(13884x_{2n+3} - 996x_{2n+4} + 276)$

7) $\frac{1}{161}(576x_{2n+3} - 996y_{2n+2} + 1932)$

8) $\frac{1}{23}(6912x_{2n+3} - 996y_{2n+3} + 276)$

9) $\frac{1}{161}(96192x_{2n+3} - 996y_{2n+4} + 1932)$

10) $\frac{1}{2231}(576x_{2n+4} - 13884y_{2n+2} + 26772)$

11) $\frac{1}{161}(6912x_{2n+4} - 13884y_{2n+3} + 1932)$

12) $\frac{1}{23}(96192x_{2n+4} - 13884y_{2n+4} + 276)$

13) $\frac{1}{138}(72y_{2n+3} - 864y_{2n+2} + 1656)$

14) $\frac{1}{161}(6y_{2n+4} - 1002y_{2n+2} + 1932)$

15) $\frac{1}{69}(432y_{2n+4} - 6012y_{2n+3} + 828)$

D. Each Of The Following Expressions Represents A Cubical Integer

1) $\frac{1}{23}[166x_{3n+3} - 10x_{3n+4} + 498x_{n+1} - 30x_{n+2}]$

2) $\frac{1}{161}[1157x_{3n+3} - 5x_{3n+5} + 3471x_{n+1} - 15x_{n+3}]$

3) $\frac{1}{23}[96x_{3n+3} - 10y_{3n+3} + 228x_{n+1} - 30y_{n+1}]$

4) $\frac{1}{161}[1152x_{3n+3} - 10y_{3n+4} + 3456x_{n+1} - 30y_{n+2}]$

5) $\frac{1}{2231}[16032x_{3n+3} - 10y_{3n+5} + 48096x_{n+1} - 30y_{n+3}]$

©IJRASET: All Rights are Reserved

6) $\frac{1}{23}\left[2314x_{3n+4} - 166x_{3n+5} + 6942x_{n+2} - 498x_{n+3}\right]$

7) $\frac{1}{161}\left[96x_{3n+4} - 116y_{3n+3} + 288x_{n+2} - 498y_{n+1}\right]$

8) $\frac{1}{23}\left[1152x_{3n+4} - 166y_{3n+4} + 3456x_{n+2} - 498y_{n+2}\right]$

9) $\frac{1}{161}\left[16032x_{3n+4} - 166y_{3n+5} + 48096x_{n+2} - 498y_{n+3}\right]$

10) $\frac{1}{2231}\left[96x_{3n+5} - 2314y_{3n+3} + 288x_{n+3} - 6942y_{n+1}\right]$

11) $\frac{1}{161}\left[1152x_{3n+5} - 2314y_{3n+4} + 3456x_{n+3} - 6942y_{n+2}\right]$

12) $\frac{1}{23}\left[16032x_{3n+5} - 2314y_{3n+5} + 48096x_{n+3} - 6942y_{n+3}\right]$

13) $\frac{1}{138}\left[12y_{3n+4} - 144y_{3n+3} + 36y_{n+2} - 432y_{n+1}\right]$

14) $\frac{1}{161}\left[y_{3n+5} - 167y_{3n+3} + 3y_{n+3} - 501y_{n+1}\right]$

15) $\frac{1}{69}\left[72y_{3n+5} - 1002y_{3n+4} + 216y_{n+3} - 3006y_{n+2}\right]$

E. Each Of The Following Expressions Represents A Bi-Quadratic Integer

1) $\frac{1}{23}\left[166x_{4n+4} - 10x_{4n+5} + 664x_{2n+2} - 40x_{2n+3} + 138\right]$

2) $\frac{1}{161}\left[1157x_{4n+4} - 5x_{4n+6} + 4628x_{2n+2} - 20x_{2n+4} + 966\right]$

3) $\frac{1}{23}\left[96x_{4n+4} - 10y_{4n+4} + 384x_{2n+2} - 40y_{2n+2} + 138\right]$

4) $\frac{1}{161}\left[1152x_{4n+4} - 10y_{4n+5} + 4608x_{2n+2} - 40y_{2n+3} + 966\right]$

5) $\frac{1}{2231}\left[16032x_{4n+4} - 10y_{4n+6} + 64128x_{2n+2} - 40y_{2n+4} + 13386\right]$

6) $\frac{1}{23}\left[2314x_{4n+5} - 166x_{4n+6} + 9256x_{2n+3} - 664x_{2n+4} + 138\right]$

7) $\frac{1}{161}\left[96x_{4n+5} - 166y_{4n+4} + 384x_{2n+3} - 664y_{2n+2} + 966\right]$

8) $\frac{1}{23}\left[1152x_{4n+5} - 166y_{4n+5} + 4608x_{2n+3} - 664y_{2n+3} + 138\right]$

9) $\frac{1}{161}\left[16032x_{4n+5} - 166y_{4n+6} + 64128x_{2n+3} - 664y_{2n+4} + 966\right]$

©IJRASET: All Rights are Reserved

10) $\frac{1}{2231}\left[96x_{4n+6}-2314y_{4n+4}+384x_{2n+4}-9256y_{2n+2}+13386\right]$

11) $\frac{1}{161}\left[1152x_{4n+6}-2314y_{4n+5}+4608x_{3n+5}-9256y_{3n+4}+966\right]$

12) $\frac{1}{23}\left[16032x_{4n+6}-2314y_{4n+6}+64128x_{2n+4}-9256y_{2n+4}+138\right]$

13) $\frac{1}{138}\left[12y_{4n+5}-144y_{4n+4}+48y_{2n+3}-576y_{2n+2}+828\right]$

14) $\frac{1}{161}\left[y_{4n+6}-167y_{4n+4}+4y_{2n+4}-668y_{2n+2}+966\right]$

15) $\frac{1}{69}\left[72y_{4n+6}-1002y_{4n+5}+288y_{2n+4}-4008y_{2n+3}+414\right]$

F. Each Of The Following Expressions Represents A Quintic Integer

1) $\frac{1}{23}\left[166x_{5n+5}-10x_{5n+6}+830x_{3n+3}-50x_{3n+4}+1660x_{n+1}-100x_{n+2}\right]$

2) $\frac{1}{161}\left[1157x_{5n+5}-5x_{5n+7}+5785x_{3n+3}-25x_{3n+5}+11570x_{n+1}-50x_{n+3}\right]$

3) $\frac{1}{23}\left[96x_{5n+5}-10y_{5n+5}+480x_{3n+3}-50y_{3n+3}+960x_{n+1}-100y_{n+1}\right]$

4) $\frac{1}{161}\left[1152x_{5n+5}-10y_{5n+6}+5760x_{3n+3}-50y_{3n+4}+11520x_{n+1}-100y_{n+2}\right]$

5) $\frac{1}{2231}\left[16032x_{5n+5}-10y_{5n+7}+80160x_{3n+3}-50y_{3n+5}+160320x_{n+1}-100y_{n+3}\right]$

6) $\frac{1}{23}\left[2314x_{5n+6}-166x_{5n+7}+11570x_{3n+4}-830x_{3n+5}+23140x_{n+2}-1660x_{n+3}\right]$

7) $\frac{1}{161}\left[96x_{5n+6}-166y_{5n+5}+480x_{3n+4}-830y_{3n+3}+960x_{n+2}-1660y_{n+1}\right]$

8) $\frac{1}{23}\left[1152x_{5n+6}-166y_{5n+6}+5760x_{4n+5}-830y_{4n+5}+11520x_{n+2}-1660y_{n+2}\right]$

9) $\frac{1}{161}\left[16032x_{5n+6}-166y_{5n+7}+80160x_{3n+4}-830y_{3n+5}+160320x_{n+2}-1660y_{n+3}\right]$

10) $\frac{1}{2231}\left[96x_{5n+7}-2314y_{5n+5}+480x_{3n+5}-11570y_{3n+3}+960x_{n+3}-23140y_{n+1}\right]$

11) $\frac{1}{161}\left[1152x_{5n+7}-2314y_{5n+6}+5760x_{3n+5}-11570y_{3n+4}+11520x_{n+3}-23140y_{n+2}\right]$

12) $\frac{1}{23}\left[16032x_{5n+7}-2314y_{5n+7}+80160x_{3n+5}-11570y_{3n+5}+160320x_{n+3}-23140y_{n+3}\right]$

13) $\frac{1}{138}\left[12y_{5n+6}-144y_{5n+5}+60y_{3n+4}-720y_{3n+3}+120y_{n+2}-1440y_{n+1}\right]$

©IJRASET: All Rights are Reserved

14) $\frac{1}{161}\left[y_{5n+7}-167y_{5n+5}+5y_{3n+5}-835y_{3n+3}+10y_{n+3}-1670y_{n+1}\right]$

15) $\frac{1}{69}\left[72y_{5n+7}-1002y_{5n+6}+360y_{3n+5}-5010y_{3n+4}+720y_{n+3}-10020y_{n+2}\right]$

III. REMARKABLE OBSERVATIONS

A. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in the Table: 2 below:

Table: 2 Hyperbolas

S. No	Hyperbola	(X,Y)
1	$X^2-48Y^2=529$	$(83x_{n+1}-5x_{n+2},\ x_{n+2}-12x_{n+1})$
2	$X^2-48Y^2=103684$	$(1157x_{n+1}-5x_{n+3},x_{n+3}-167x_{n+1})$
3	$X^2-48Y^2=529$	$(48x_{n+1}-5y_{n+1},y_{n+1}-5x_{n+1})$
4	$X^2-48Y^2=25921$	$(576x_{n+1}-5y_{n+2},y_{n+2}-83x_{n+1})$
5	$X^2-48Y^2=4977361$	$(8016x_{n+1}-5y_{n+3},y_{n+3}-1157x_{n+1})$
6	$X^2-48Y^2=529$	$(1157x_{n+2}-83x_{n+3},12x_{n+3}-167x_{n+2})$
7	$X^2-48Y^2=25921$	$(48x_{n+2}-83y_{n+1},12y_{n+1}-5x_{n+2})$
8	$X^2-48Y^2=529$	$(576x_{n+2}-83y_{n+2},12y_{n+2}-83x_{n+2})$
9	$X^2-48Y^2=25921$	$(8016x_{n+2}-83y_{n+3},12y_{n+3}-1157x_{n+2})$
10	$X^2-48Y^2=4977361$	$(48x_{n+3}-1157y_{n+1},167y_{n+1}-5x_{n+3})$
11	$X^2-48Y^2=25921$	$(576x_{n+3}-1157y_{n+2},167y_{n+2}-83x_{n+3})$
12	$X^2-48Y^2=529$	$(8016x_{n+3}-1157y_{n+3},167y_{n+3}-1157x_{n+3})$
13	$16X^2-3Y^2=76176$	$(3y_{n+2}-36y_{n+1},83y_{n+1}-5y_{n+2})$
14	$48X^2-Y^2=4976832$	$(y_{n+3}-167y_{n+1},1157y_{n+1}-5y_{n+3})$
15	$16X^2-3Y^2=76176$	$(36y_{n+3}-501y_{n+2},1157y_{n+2}-83y_{n+3})$

©IJRASET: All Rights are Reserved

B. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabola which are presented in the Table: 3 below:

Table: 3 Parabolas

S. No	Parabola	(X,Y)
1	$23X-96Y^2=529$	$(83x_{2n+2}-5x_{2n+3},\ x_{n+2}-12x_{n+1})$
2	$322X-96Y^2=103684$	$(1157x_{2n+2}-5x_{2n+4},x_{n+3}-167x_{n+1})$
3	$23X-96Y^2=529$	$(48x_{2n+2}-5y_{2n+2},y_{n+1}-5x_{n+1})$
4	$161X-96Y^2=25921$	$(576x_{2n+2}-5y_{2n+3},y_{n+2}-83x_{n+1})$
5	$2231X-96Y^2=4977361$	$(8016x_{2n+2}-5y_{2n+4},y_{n+3}-1157x_{n+1})$
6	$23X-96Y^2=529$	$(1157x_{2n+3}-83x_{2n+4},12x_{n+3}-167x_{n+2})$
7	$161X-96Y^2=25921$	$(48x_{2n+3}-83y_{2n+2},12y_{n+1}-5x_{n+2})$
8	$23X-96Y^2=529$	$(576x_{2n+3}-83y_{2n+3},12y_{n+2}-83x_{n+2})$
9	$161X-96Y^2=25921$	$(8016x_{2n+3}-83y_{2n+4},12y_{n+3}-1157x_{n+2})$
10	$2231X-96Y^2=4977361$	$(48x_{2n+4}-1157y_{2n+2},167y_{n+1}-5x_{n+3})$
11	$161X-96Y^2=25921$	$(576x_{2n+4}-1157y_{2n+3},167y_{n+2}-83x_{n+3})$
12	$23X-96Y^2=529$	$(8016x_{2n+4}-1157y_{2n+4},167y_{n+3}-1157x_{n+3})$
13	$552X-3Y^2=38088$	$(3y_{2n+3}-36y_{2n+2},83y_{n+1}-5y_{n+2})$
14	$23184X-3Y^2=7465248$	$(y_{2n+4}-167y_{2n+2},1157y_{n+1}-5y_{n+3})$
15	$552X-3Y^2=38088$	$(36y_{2n+4}-501y_{2n+3},1157y_{n+2}-83y_{n+3})$

IV. CONCLUSION

In this paper, we have presented infinitely many integer solutions for al hyperbola represented by the negative Pell Equation $y^2=48x^2-23$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of negative Pell Equations and determine their integer solutions along with suitable properties.

©IJRASET: All Rights are Reserved

REFERENCES

[1] R.A.Mollin and Anithasrinivasan, A note on the negative pell equation, International Journal of Algebra, 4(19) (2010) 919-922.

[2] E.E.Whitford, some solutions of the Pellian Equation $x^2 - Ay^2 = \pm 4$ JSTOR: Annals of Mathematics, Second series, 1 (1913-1914) 157-160.

[3] S.AhmetTekcan, B.W.Gezer and O.Bizim. On the integer solutions of the pell Equation $x^2 - dy^2 = 2^t$, World Academy of Science, Engineering and Technology, 1(2007) 522-526.

[4] S.Ahmet Tekcan, The Pell Equation $x^2 - (k^2 - k)y^2 = 2^t$, World Academy of Science, Engineering and Technology, 19(2008) 697-701.

[5] V.Sangeetha, M.A.Gopalan and N\M.Somanath. On the Integral Solutions of the Pell equation $x^2 = 13y^2 - 3^t$, International Journal of Applied Mathematical Research, 3(1)(2014) 58-61.

[6] M.A.Gopalan, G.Sumathi and S.Vidhyalakshmi, Observations on the hyperbola $x^2 = 19y^2 - 3^t$, Journal of the Engineering and Technology, 2(2A)(2014) 152-155.

[7] M.A.Gopalan , S.Vidhyalakshmi and A.Kavitha ,On the Integral soluations of the binary quadratic equation $x^2 = 15y^2 - 11^t$, Scholars Journal of the Engineering and Technology, 2(2A)(2014) 156-158.

[8] S.Vidhyalakshmi, V.Krithika and K.Agalya, On the negative pell equation $y^2 = 80x^2 - 31$, Proceeding of the National Conference on MATAM,PSNA college of Engineering and Technology, Dindugal,(2015)4-10.

[9] K.Meena, M.A.Gopalan and R.Karthika, On the Negative pell equation $y^2 = 10x^2 - 6$, International Journal of Multidisciplinary Research and Development,2(12)(2015)390-392.

[10] M.A.Gopalan, S.Vidhyalakshmi, V.Pandichelvi, P.Sivakamasundari and C.Priyadharsini, On the Negative pell equation $y^2 = 45x^2 - 11$ International Journal of Pure Mathematical Science, 16 (2016)30-36.

[11] K.Meena, S.Vidhyalakshmi and A.Rukmani. On the negative pell equation $y^2 = 31x^2 - 6$ Universe of Engineering Technologies and Science, 11(XII) (2015) 1-4.

[12] K.Meena, M.A.Gopalan and E.Bhuvaneswari, On the Negative pell equation $y^2 = 60x^2 - 15$, Scholars Bulletin , 1(11)(2015)310-316.

[13] M.A.Gopalan, S.Vidhyalakshmi, R.Presenna and M.Vanitha, Observation on the negative pell equation $y^2 = 180x^2 - 11$, Universal Journal of Mathematics,2(1)(2016)41-45.

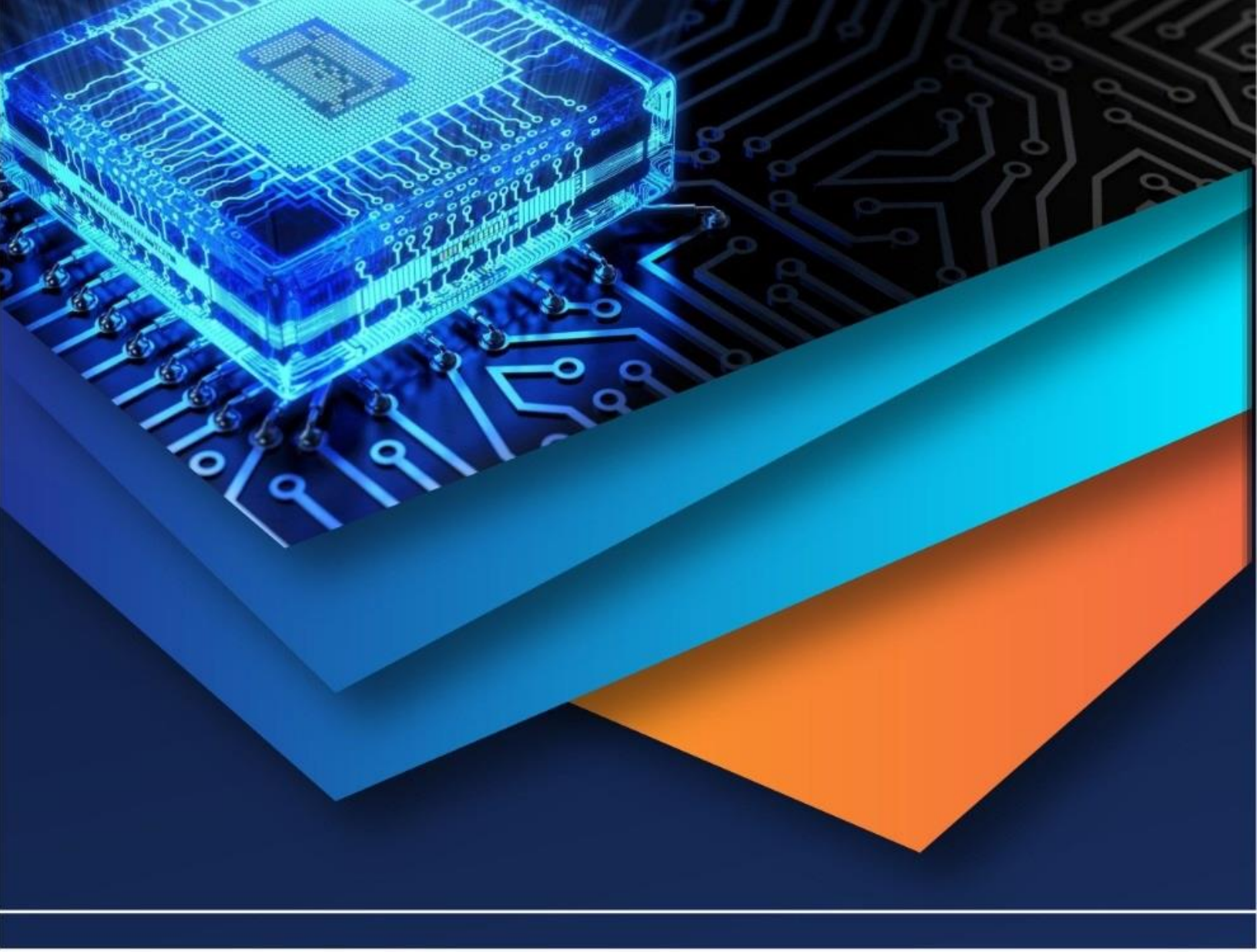
[14] S.Vidhyalakshmi, M.A.Gopalan, E.Premalatha and S.SofiaChristinal , on the Negative Pell Equation $y^2 = 72x^2 - 8$,International Journal of Engineering Research,4(2)(2016)25-28.

[15] M.A.Gopalan, S.Vidhyalakshmi and N.Thiruniraiselvi. A study on the Hyperbola $y^2 = 8x^2 - 31$. International Journal of latest Research in Science and Technology, 2(1)(2013)454-456.

[16] M.A.Gopalan, S.Vidhyalakshmi, T.R.Usha Rani, S. Mallika. Observation on $y^2 = 12x^2 - 3$.Bessel Journal of math, 2(3) (2012)153-158.

[17] S.Mallika, P.Abinaya, On Negative Pellian Equation $y^2 = 40x^2 - 15$, Journal of Mathematics and Informatics, Vol.11, 2017,95-102.

©IJRASET: All Rights are Reserved



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089 (24*7 Support on Whatsapp)