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# Identification of Influential Node in a Complex Network using WSM (Weighted Sum Model)

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**Abstract:** Identification of influential nodes is a significant issue in the structural analysis of complex network. To address this issue, different centrality measures have been proposed such as betweenness centrality, closeness centrality, degree centrality, but all of them suffered from some drawbacks. This research proposes a new method to identify influential nodes based on Weighted Sum Method (WSM). Weighted Sum Method is one of the widely used and simplest multi-criteria decision making method. In this paper, different centrality measures are considered as the multi-attribute of complex network, so as to take advantage of each centrality measure. The multi-attribute of complex network are aggregated using WSM to compute the importance of each node in the network. The efficacy and feasibility of the proposed method is established by conducting experiments on four real-world networks and an informative network.

**Keywords:** WSM, BC, CC, DC, PGP, MADM

## I. INTRODUCTION

Complex network theory has received significant attention from researchers due to their widespread application, which ranges from the viral marketing of products to the outbreak of disease and from the diffusion of influence to the spread of ideas [1,13, 21]. There are various functions of complex networks such as transfer of ideas, information, diseases, influences and trust. Determination of influential nodes leads to maximum information spread in the underlying network. Hence, identification of influential spreaders is one of the major concern in complex network that can be applied in diverse fields such as in creation of new marketing strategy, conduct study on public opinion, rumour dynamics and vaccination strategies [12, 15, 16, 22].

In recent years, many approaches based on node centrality such as betweenness centrality (BC) [26], closeness centrality (CC) [27], page rank (PR), leader rank and degree centrality (DC) have been proposed, each having its own limitation and advantage. Although, DC was computationally less complex, but took into account local information of the topology. This problem was overcome by BC and CC that considered global information of the topology to identify important nodes, but due to their high computational complexity they were not applicable to large-scale networks. Another drawback of CC was their inefficiency to be applied to disconnected components. The performance of PR was good for scale-free networks, but failed for random networks. Later, spectral centrality measures were proposed such as, Leader rank (LR), Eigen vector centrality (EC) and semi-local centrality (SLC). EC had advantage over DC, CC and BC since it was applicable for signed and valued graphs while the latter was applicable for simple graphs, but it failed for asymmetric graphs. LR and PR were effective for directed networks but were not applicable for undirected networks. In other words, there is an urgent need to develop an efficient and reasonable ranking algorithm that can identify influential nodes in complex network.

Utilizing multi-attributes of node's data to compute nodes importance can resolve the problem incurred by individual centrality measures. Recently, MADM technology is being widely used to compute node influence in a promising way. Evidence theory, a mathematical tool that can model uncertain information efficiently, is extensively used to compute node importance in complex network. Another mathematical tool such as fuzzy logic is also being used in this domain. The proposed research aims to employ MADM methods to identify influential nodes in the complex network. In the literature there exists many optimization methods of which weighted sum method (WSM), proposed by Churchman and Ackoff (1954) is one of the simplest and widely used method has been applied to various application domains such as recommending DBMS, query optimization etc. The method combines different objectives and their corresponding weights to compute score for every objective and according to that score, objectives are ranked in order of preference.

## II. LITERATURE SURVEY

Influence maximization (IM) problem was proved as NP-Hard problem by Kemp et al. [20] presented greedy approximate algorithms that considered IC (Independent Cascade), Weighted Independent Cascade and LT (Linear Threshold) as influence models and the optimal solution obtained had  $(1-1/e)$  computational complexity. Later a lot of research was conducted by the research community and they presented different algorithms to identify influential nodes with maximum influence spread.

A greedy algorithm named “Cost Efficient Lazy Forward Scheme” (CELf) was proposed by Leskovec et al. [24] and it showed an improved speed up by 700 times over the greedy algorithm.

Later presented CELf++ algorithm which was an improvement over CELf. In CELf++ algorithm, the performance of CELf was optimized by exploiting sub modularity and it was 35-50% fast in terms of execution time than CELf. Another work was presented by Chen et al. [8], where they proposed Mixed Greedy and Greedy algorithms that were improvement over the greedy algorithm from a different perspective.

A bound linear approach was proposed by Liu et al. [19] to solve influence maximization problem and influence computation. Different meta-heuristic algorithms such as cuckoo search algorithm, particle swarm optimization algorithm, simulated annealing algorithm, and genetic algorithm have been employed to cope with the problem of IM.

Another area that is being explored in the IM domain is to identify influential nodes w.r.t more than one criteria. This research area of IM was first explored by in 2013, where they proposed a novel approach that used TOPSIS, a multi-criteria approach to select salient nodes in the underlying network.

The task of identifying influential nodes was given due importance by Zhang et al. [25]. Eight criteria were taken into consideration by them to find the key node in the network.

The eight criteria were hierarchy, constraint, efficiency, effective size, closeness centrality, betweenness centrality, eigenvector centrality, and degree centrality. The weight of the criteria was computed using AHP method and TOPSIS method was applied to obtain the rank list of nodes based on the importance of nodes. An improved TOPSIS method was proposed by D et al. [18] that considered multiple criteria for the identification of influential nodes in the network. The shortcoming of the approach was the way of computation of weights for criteria. This limitation was overcome by where he proposed a dynamic way of calculating the weight of the criteria.

## III. PROPOSED FRAMEWORK

The proposed W-WSM consists of the following steps and is presented in Figure 1:

- A. Compute the values for 3 different criteria, that are considered here as 3 different centrality measures (cm). The measures here are Betweenness Centrality (BC), Closeness Centrality (CC), and Degree Centrality (DC)[5]. Construct a decision matrix  $D(O_{xy})$  or  $D(O_{x3})$  with x rows representing alternatives and y (here  $y=3$ ) columns representing criteria.

$$D(O_{xy}) = \begin{bmatrix} DC_1 & CC_1 & BC_1 \\ DC_2 & CC_2 & BC_2 \\ \vdots & \vdots & \vdots \\ DC_x & CC_x & BC_x \end{bmatrix}_{x \times y}$$

- B. Matrix  $D(O_{xy})$  is normalized and matrix  $R(r_{pq})$  is constructed such that

$$r_{pq} = \frac{O_{pq}}{\sum_{p=1}^x O_{pq}} ; p = 1, 2..x; q = 1, 2, 3$$

- C. The weight of every attribute is determined according to their importance i.e. Betweenness Centrality is given weight 3, since it takes into account global information of the topology to compute important nodes, Closeness centrality comes next to betweenness centrality, as it determines the bridge node of the network, and is given weight 2. At last comes Degree centrality, since it computes important nodes based on local topology of the network. Hence, it is given weight 1.

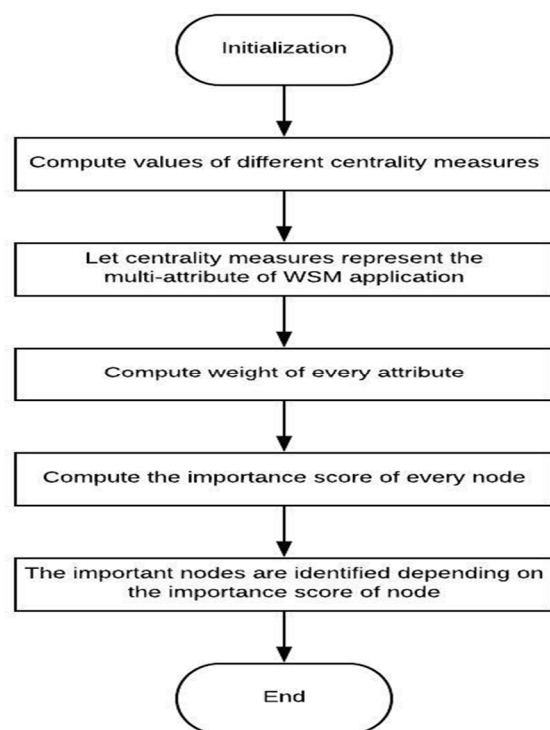


Figure 1: Flowchart for WSM

D. The total importance of the p-th node is computed using eq. 2:

$$M_p^{WSM} = \sum_{q=1}^l w_q c_{pq}, \quad p = 1, 2, 3, \dots, x; \quad q = 1, 2, 3 \quad (2)$$

E. A rank list of nodes is constructed according to the importance score of the node

Explanation using example

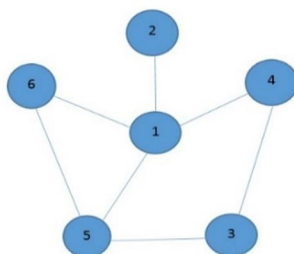


Figure 2: A toy network consisting of 6 nodes and 7 edges

I) Compute the values of three different centrality measures DC, CC, and BC. A decision matrix  $D(O_{xy})$  or  $D(O_{x3})$  is constructed with x rows representing nodes and y columns representing different centrality measures.

$$D(O_{xy}) = \begin{bmatrix} 4 & 0.166 & 0.55 \\ 1 & 0.100 & 0 \\ 2 & 0.111 & 0.05 \\ 2 & 0.125 & 0.10 \\ 3 & 0.142 & 0.2 \\ 2 & 0.125 & 0 \end{bmatrix}$$



2) Matrix D is normalized and matrix R is obtained

$$r_{11} = \frac{4}{4+1+2+2+3+2} = 0.285$$

And matrix  $R(r_{pq})$  is obtained as below:

$$R = \begin{bmatrix} 0.285 & 0.216 & 0.611 \\ 0.071 & 0.129 & 0 \\ 0.142 & 0.144 & 0.055 \\ 0.142 & 0.162 & 0.111 \\ 0.214 & 0.185 & 0.222 \\ 0.142 & 0.162 & 0 \end{bmatrix}$$

3) The weights are determined according to Step 3 where (BC = 3, CC = 2, DC = 1)

4) The importance of the Hep-th (here p = 1 i.e. importance of 1st node) node is computed using Eq.(2)

$$M_1^{WSM} = \sum_{q=1}^3 w_q r_{1q} = 2.55$$

6) A rank list is constructed based on the importance score of the node computed in the previous step:

Node	Score	Rank
1	2.55	1
2	0.329	5
3	0.595	6
4	0.799	4
5	1.25	3
6	1.25	2

Table 1: A rank list of nodes according to their importance score

The rank list is obtained according to node importance score using Eq. (6) and is shown in Table 1. According to the proposed method, node 1 is the most important node, since it is the central node and acts as a connecting node between two sub-networks (2) and (3, 4, 5, 6). Thus, it can be said that W-WSM is able to rank nodes correctly according to their spreading ability.

## IV. RESULTS AND DISCUSSION

### A. Dataset

The performance of the proposed approach is validated using four real networks:

- 1) *Yeast* [9]: The dataset is an un-weighted network representing protein and yeast interactions.
- 2) *PGP* [9]: The dataset represents an encrypted communication network. Pretty Good Privacy algorithms are used to maintain privacy among users; iii) *HEP-th*: This dataset is also known as “High Energy Physics” and represents an un-weighted citation network. The can be downloaded from: [www.casos.cs.cmu.edu/computational\\_tools/datasets/external/hep-th/index11.php](http://www.casos.cs.cmu.edu/computational_tools/datasets/external/hep-th/index11.php).

Network	a	b	$N_d$	$PL_{avg}$	$Deg_{avg}$	$Bet_{max}$
USAir	332	2126	6	2.563	12.807	5286.21
PGP	10680	48632	24	7.485	9.107	14959584.71
Yeast	2361	14364	16	4.647	6.084	36248.02
HEP-th	8361	31502	17	5.61	3.768	25686.02

Table 2. Topological features of dataset (a and b denote the number of nodes and the number of edges,  $N_d$  denote network diameter,  $PL_{avg}$  denote average path length,  $Deg_{avg}$  denote average degree and  $Bet_{max}$  denote maximum betweenness)

### B. Yeast

The list of top-15 nodes computed by BC, CC, DC and the proposed method is presented in table. As can be seen in the given table that BC and the WSM computes the same set of top-13 nodes. Hence, we compute Kendall's tau to compute the correlation between ranking lists.

Ranking	WSM	BC	CC	DC
1	1441	156	842	465
2	886	1441	672	289
3	656	886	1131	199
4	156	656	1332	1441
5	199	199	2081	886
6	648	638	98	156
7	289	289	612	478
8	497	497	832	109
9	2008	2008	902	541
10	109	109	899	141
11	478	181	1121	497
12	239	478	1212	2008
13	181	239	1261	498
14	280	280	1298	54
15	54	191	1314	413

Table 3. Rank lists of top-15 nodes generated by proposed WSM, BC, CC, and DC in Yeast network

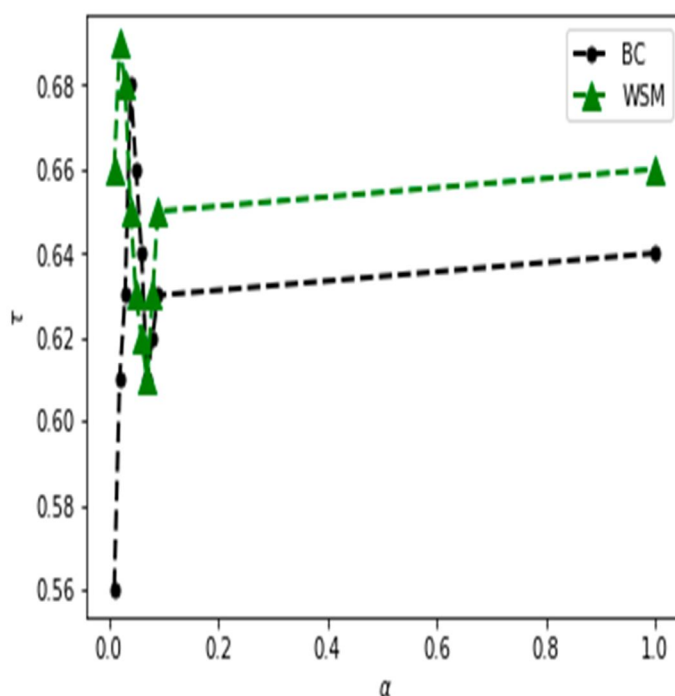


Fig.3 Kendall Tau coefficient of WSM and BC in Yeast network

### C. PGP

Table presents the top-15 nodes rank lists computed by WSM, CC, BC and DC. As can be seen from the table that WSM and BC compute the same set of top-15 nodes.

Ranking	WSM	BC	CC	DC
1	62	62	62	23
2	127	128	125	124
3	489	499	488	162
4	239	239	233	58
5	28	192	193	347
6	192	28	29	1389
7	698	698	699	1390
8	289	289	288	1391
9	23	117	118	192
10	191	191	190	57
11	117	141	140	621
12	141	23	231	1392
13	36	36	361	1393
14	219	219	27	1394
15	27	27	291	1395

Table 5. Rank lists of top-15 nodes generated by proposed WSM, BC,CC, and DC in PGP network

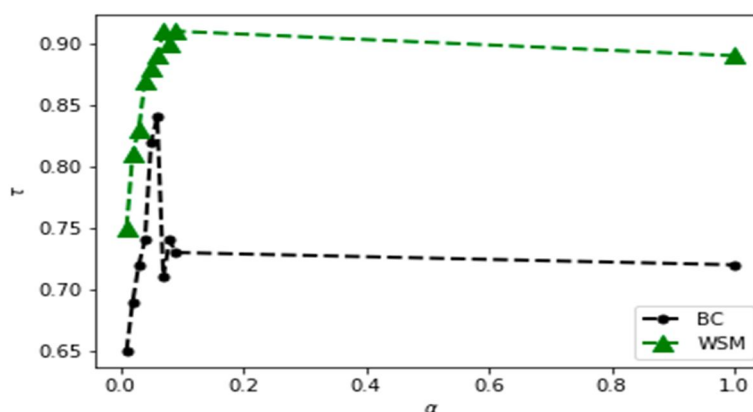


Fig.4 Kendall Tau coefficient of WSM and BC in PGP network

### D. Correlation Analysis

A node is considered to be important if it has large spreading ability. One of the widely used method for computing rank correlation is Kendall's Tau coefficient that measures the correlation between two variables.

Let us assume that there exists two ranklists, first is node spreading ability list obtained from SIR model and second is ranking list generated by ranking method. A large value of correlation coefficient between the two ranklists would signify that the performance of ranking method is good.

In the proposed research, Kendall's Tau coefficient is used to analyze the correlation between the proposed D-WSM and WT. Let us consider that there are  $n$  nodes in the network, then Kendall's Tau coefficient  $T_k$  is defined as:-

$$T_k = \frac{(\text{number of concordant pairs}) - (\text{number of discordant pairs})}{n(n-1)/2}$$

Where  $T_k$  is Kendall's Tau coefficient. Let us assume there exists, a sample of  $N(a)$  data points  $(X_a, Y_a)$  where  $V$  variable represents the ranks of  $X_a$  and  $W$  variable represents the ranks of  $Y_a$ . Pairs  $(V_i, W_i)$  and  $(V_j, W_j)$  are said to be concordant if their ranks occur in same order. That is, if

- 1)  $V_i < V_j$  and  $W_i < W_j$ ,
- 2)  $V_i > V_j$  and  $W_i > W_j$

Similarly, pairs  $(V_i, W_i)$  and  $(V_j, W_j)$  are said to be discordant if their ranks occur in reverse order. That is, if

- 1)  $V_i < V_j$  and  $W_i > W_j$
- 2)  $V_i > V_j$  and  $W_i < W_j$

If pairs  $(V_i = V_j)$  or  $(W_i = W_j)$ , then such pairs are neither classified as discordant or concordant and are neglected. A large value of Tau indicates that the ranking method has generated more accurate ranking lists.

If  $T_k = 1$ , this would be an ideal case, where the ranking lists generated by real spreading process and ranking method are exactly same. For large value of infection spreading probability  $\beta$ , the infection (information) would spread to almost all over the network. In the proposed research, the value of  $\beta$  of SIR model is varied from 0.01 to 0.1.

From fig.3, we can see that the performance of WT is better than D-WSM, when the value of  $\beta$  is between 0.03 to 0.05 in Yeast network, but for other cases, the scenario is reverse.

In fig.4, it is observed that D-WSM outperforms WT for every value of  $\beta$  in PGP network. Apart from this, a large value of Kendall's Tau (0.9) is achieved when  $\beta = 0.09$ .

### E. Complexity Analysis

A real complex network comprises of many nodes. Hence, a highly reasonable and efficient ranking method is required to determine node importance. This section deals with the determination of computing complexity of the proposed approach, given that the underlying network consists of  $m$  nodes and  $n$  edges and  $a$  avg represents average degree of the network.

### F. The Proposed Approach Comprises Of Three Phases

In the first phase, a multi-attribute matrix is constructed by computing values of CC, BC, and DC. The computational complexity of CC using Floyd's algorithm is  $O(n^3)$ . DC is computed in time  $O(n)$ . The time complexity for computing BC using Brand's method is  $O(m * n) - O(m^2 a_{avg})$ . Euclidean standardization is used to normalize the matrix and it takes time  $O(n)$ . The second phase concentrates on determining the computational complexity of the weights of attributes using SIR. SIR model is used to compute node spreading ability  $F$ , that takes  $O(m * n) \leq O(m^3)$ . The third phase observes ranking and evaluation of the nodes present in the network using Weighted Sum Method (WSM). The complexity of computing WSM is  $O(m)$ . Finally, according to the above analysis, it is concluded that the computational complexity of the proposed D-WSM is  $O(m^3)$ .

## V. CONCLUSION

Identification of influential nodes is a significant task for solving influence maximization problem. This paper uses Weighted Sum Method, a multi-criteria decision approach to identify a set of influential nodes in the network. The main contribution of the paper is that it utilized three centrality measures as the multi-attributes of nodes to solve IM problem. To analyze the performance of WSM, SIR model is used to simulate the diffusion of influential nodes in the underlying network. Influential nodes identified by WSM had fast influence spreading capability. From the application perspective, it is a good approach to detect influential nodes using multiple centrality measures.

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