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Skewness and Kurtosis on Vertex Colouring of Splitting Graphs

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Abstract: In this paper we found minimum vertex colouring sum based on a minimum proper colouring of a given Splitting graph $S'(G)$ and we compute the statistical measures Mean, Variance, Median, Standard deviation, Skewness and Kurtosis.

Keywords: Graph Colouring; colouring sum of Splitting graphs; colouring mean; colouring variance; colouring median; colouring standard deviation; colouring skewness; colouring kurtosis; χ-chromatic.

I. INTRODUCTION

The splitting graph of a graph $S'(G)$ is obtained by adding to each vertex v a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G. We extend the concepts of mean, median, variance, standard deviation, skewness and kurtosis are some important statistical measures, to the theory of splitting graphs colouring and determine the values of these parameters for a number of standard splitting graphs.

A. Preliminaries

1) Let C = $\{c_1,c_2,....,c_k\}$ be a particular type of proper k-colouring of a given splitting graph $S'(G)$ and $\theta(c_i)$ denotes the number of times a particular color c_i is assigned to the vertex of $S'(G)$. Then, the vertex colouring sum of a colouring C of a given splitting graph $S'(G)$ denoted by $\omega_C(S'(G))$ is defined to be,

$$\omega_C(S'(G)) = \sum_{i=1}^{k} i\theta(c_i)$$

2) A Graph G is said to be Friendship Graph if a planar undirected graph with 2n + 1 vertices and 3n edges. The friendship graph can be constructed by joining n copies of the cycle graph with a common vertex.
3) The Helm graph H_n is the graph obtained from an n-wheel graph by adjoining a pendant edge at each node of the cycle.
4) The n-sunlet graph is the graph on 2n vertices obtained by attaching n pendant edges to a cycle graph C_n.
5) The hairy cycle C_m is obtained by applying that construction to the cycle C_m and the graph K_1 consisting of a single vertex. Therefore C_m is the corona graph $C'_m \circ K_1$.
6) A wheel graph is obtained from a cycle graph $C_n - 1$ by adding a new vertex adjacent to all the vertices of the cycle

B. Colouring Statistical Parameters of Splitting Graphs

We can identify the colouring of the vertices of a given splitting graph $S'(G)$ with a random experiment. Let C = {c1,c2,c3,...,ck} be a proper k-colouring of $S'(G)$ and let X be the random variable (r.v) which denotes the number of vertices in $S'(G)$ having a particular color. Since the sum of all weights of colors of $S'(G)$ is the order of $S'(G)$, the real valued function f(i) defined by,

$$f(i) = \begin{cases} \dfrac{\theta(c_i)}{V(S'(G))}; i = 1,2,....k \\ 0 \end{cases}$$

is the probability mass function (p.m.f) of the r.v X. If the context is clear, we can also say that f(i) is the p.m.f of the splitting graph $S'(G)$ with respect to the given colouring C. Hence, analogous to the definitions of the mean,median,variance,standard deviation,skewness and kurtosis of random variables, those statistical parameters of a splitting graph $S'(G)$, with respect to a general colouring of $S'(G)$ can be defined as follows.

1) *Definition .1:* Let C = {c_1, c_2, c_3,...,c_k} be a certain type of proper k-colouring of a given splitting graph $S'(G)$ and $\theta(c_i)$ denotes the number of times a particular color c_i is assigned to vertices of $S'(G)$. Then, the colouring mean of a colouring C of a given splitting graph G, denoted by $\mu_C S'(G)$ is given by, $\mu_C S'(G) = \dfrac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}$
2) *Definition .2:* The colouring median of a colouring C of a given splitting graph $S'(G)$, denoted by $M_C S'(G)$ and is defined to be $M_C S'(G) = \dfrac{\sum_{i=1}^{k}\theta(c_i)}{2}$
3) *Definition .3:* The colouring variance of a colouring C of a given splitting graph $S'(G)$, denoted by $\sigma^2{}_C S'(G)$ is given by,

$$\sigma^2{}_C S'(G) = \frac{\sum_{i=1}^{k} i^2\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)} - \left[\frac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}\right]^2$$

4) *Definition .4:* The colouring standard deviation of a colouring C of a given splitting graph $S'(G)$, denoted by by $\sigma_C S'(G)$ is given by, $\sigma_C S'(G) = \sqrt{\dfrac{\sum_{i=1}^{k} i^2\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)} - \left[\dfrac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}\right]^2}$
5) *Definition.5:* The colouring skewness of a colouring C of a given splitting graph $S'(G)$, denoted by $\gamma_C S'(G)$ and is defined to be $\gamma_C S'(G) = 3\dfrac{Mean - Median}{Standard\ \ Deviation}$
6) *Definition 6:* For a positive integer r, the r-th moment of the colouring C is denoted by $\mu_{C^r} S'(G)$ is given by,

$$\mu_{C^r} S'(G) = \frac{\sum_{i=1}^{k} i^r\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}$$

1st Moment:

$$\mu_{C^1} S'(G) = \frac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}$$

2nd Moment:

$$\mu_{C^2} S'(G) = \frac{\sum_{i=1}^{k} i^2\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)} - \left[\frac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}\right]^2$$

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3rd Moment

$$\mu_{C^3} S'(G) = \frac{\sum_{i=1}^{k} i^3 \theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} - 3 \frac{\sum_{i=1}^{k} i^2 \theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} \left[\frac{\sum_{i=1}^{k} i \theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} \right] + 2 \left[\frac{\sum_{i=1}^{k} i^3 \theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} \right]^3$$

4^{th} Moment

$$\mu_{C^4} S'(G) = \frac{\sum_{i=1}^{k} i^4 \theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} - 4 \frac{\sum_{i=1}^{k} i^3 \theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} \left[\frac{\sum_{i=1}^{k} i \theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} \right] + 6 \left[\frac{\sum_{i=1}^{k} i^3 \theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} \right] \left[\frac{\sum_{i=1}^{k} i \theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} \right] - 3 \left[\frac{\sum_{i=1}^{k} i \theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} \right]^4$$

7) *Definition .7:* The colouring kurtosis of a colouring C of a given splitting graph $S'(G)$, denoted by $\beta_2 S'(G)$ and defined by, $\beta_2 S'(G) = \frac{\mu_{C^4} S'(G)}{(\mu_{C^2} S'(G))^2}$

χ - Chromatic Statistical Parameters of Splitting Graphs:

Colouring mean, median, variance, standard deviation, skewness and kurtosis corresponding to a particular type of minimal proper colouring of the vertices of $S'(G)$ are defined as follows.

8) *Definition .8:* A colouring mean of a splitting graph $S'(G)$, with respect to a proper vertex colouring C is said to be a χ - chromatic mean of $S'(G)$, if C is the minimum proper colouring of $S'(G)$ and the colouring sum $\omega_C S'(G)$ is also minimum. The χ -chromatic mean of a splitting graph $S'(G)$ is denoted by $\mu_\chi S'(G)$.
9) *Definition .9:* A colouring median of a graph $S'(G)$, with respect to a proper vertex colouring C is said to be a χ - chromatic median of $S'(G)$. The χ -chromatic median of a splitting graph $S'(G)$ is denoted by $M_\chi S'(G)$.
10) *Definition.10:* The χ chromatic variance of $S'(G)$, denoted by $\sigma_\chi^2 S'(G)$, is a colouring variance of $S'(G)$ with respect to a minimal proper vertex colouring of $S'(G)$ which yields the minimum colouring sum.
11) *Definition .11:* The χ chromatic standard deviation of $S'(G)$, denoted by $\sigma_\chi S'(G)$, is a colouring standard deviation of $S'(G)$ with respect to a minimal proper vertex colouring of $S'(G)$ which yields the minimum colouring sum.
12) *Definition.12:* The χ chromatic skewness of $S'(G)$, denoted by $\gamma_\chi S'(G)$, is a colouring variance of $S'(G)$ with respect to a minimal proper vertex colouring of $S'(G)$ which yields the minimum colouring sum.
13) *Definition .13:* The χ chromatic kurtosis of $S'(G)$, denoted by $\beta_{2\chi} S'(G)$, is a colouring kurtosis of $S'(G)$ with respect to a minimal proper vertex colouring of $S'(G)$ which yields the minimum colouring sum. If β_2 = 3, then it is known as MESOKURTIC Curve, if β_2 < 3, then it is known as PLATYKURTIC Curve and if β_2 > 3, then it is known as LEPTOKURTIC Curve

Let us now determine the χ - chromatic mean, median, variance, standard deviation, skewness and kurtosis of certain standard splitting graph classes. The following result discusses on Friendship splitting graph $S'(F_n)$.

a) *Proposition 1.* The χ -Chromatic mean of a friendship splitting graph $S'(F_n)$ is $\frac{5n+1}{2n+1}$, χ-chromatic variance is $\frac{n(n+5)}{(2n+1)^2}$, χ-chromatic median is 2n + 1, χ- chromatic standard deviation, $\sqrt{\frac{n(n+5)}{(2n+1)^2}}$, χ- chromatic skewness $3\left(\frac{-4n^2+n}{\sqrt{n(n+5)}}\right)$ and χ -chromatic kurtosis $\frac{n^4+42n^3-6n^2+17n}{n^4+10n^3+25n^2}$.

Proof. Consider a friendship splitting graph $S'(F_n)$ has 4n+2 vertices. The vertices of $S'(F_n)$ can be coloured using c_1, c_2, c_3; Exactly two vertices has colour c_1; 2n vertices has c_2 and c_3.Hence, the p.m.f of the corresponding r.v.X is

Therefore, $\mu_x(S'(F_n)) = \frac{5n+1}{5n+1}$ $\sigma^2{}_x(S'(F_n)) = \frac{n(n+5)}{(2n+2)^2}$, $M_x(S'(F_n)) = 2n+1$, $\sigma_x(S'(F_n)) = \sqrt{\frac{n(n+5)}{(2n+2)^2}}$,

$$\gamma_x(S'(F_n)) = 3\left(\frac{-4n^2+n}{\sqrt{n(n+5)}}\right)$$

We can obtain kurtosis by various moments,

$$\mu'_1 = \frac{10n+2}{4n+2},\ \mu'_2 = \frac{26n+2}{4n+2}\ \mu'_3 = \frac{70n+2}{4n+2},\ \mu'_4 = \frac{194n+2}{4n+2}$$

$$\mu_3 = \frac{9n(1-n)}{(2n+1)^3},\ \mu_4 = \frac{n^4+42n^3-6n^2+17n}{(2n+1)^4},\ \beta_{2x}(S'(F_n)) = \frac{n^4+42n^3-6n^2+17n}{n^4+10n^3+25n^2}$$

If $\beta_{2x}(S'(F_n)) < 3$, then it is known as PLATYKURTIC Curve.

b) *Proposition 2:* The χ–chromatic mean of a $s'(C_n \circ K_1)$ is

$$\mu_x(s'(C_n \circ K_1)) = \begin{cases} \frac{3}{2}; even \\ \frac{3n+1}{2n}; odd \end{cases}$$

And the χ – chromatic variance of a $s'(C_n \circ K_1)$ is

$$\sigma^2{}_x(s'(C_n \circ K_1)) = \begin{cases} \frac{1}{4}; even \\ \frac{n^2+4n-1}{4n^2}; odd \end{cases}$$

the χ – chromatic median of $s'(C_n \circ K_1)$=2n,

the χ – chromatic standard deviation of $s'(C_n \circ K_1)$ is $\sigma_x(s'(C_n \circ K_1)) = \begin{cases} \frac{1}{2}; even \\ \sqrt{\frac{n^2+4n-1}{4n^2}}; odd \end{cases}$

the χ – chromatic skewness of $s'(C_n \circ K_1)$ is

$$\gamma_x(s'(C_n \circ K_1) = \begin{cases} 3(3-4n);\ even \\ 3(\frac{-4n^2+3n+1}{\sqrt{n^2+4-1}};\ odd \end{cases}$$

The χ - chromatic kurtosis of is $\beta_2 x(s'(C_n \circ K_1) = \begin{cases} 1;\ even \\ \frac{n^4+40n^3-46n^2 24n-3}{n^4+8n^3+14n^2-8n+1};\ odd \end{cases}$

Proof:. Consider a splitting sunlet cycle $s'(C_n \circ K_1)$ on 4n vertices. Then, we have the following cases.

(1) If n is even and is 2-colourable then $s'(C_n \circ K_1)$ has exactly $\frac{4n}{2}$ vertices having colour c_1 and c2 each. Then, as explained in the first part of previous theorem, we have

the χ -chromatic mean of $s'(C_n \circ K_1)$ is $\mu_x(s'(C_n \circ K_1) = \frac{3}{2}$,

the χ chromatic variance of $s'(C_n \circ K_1)$ is $\sigma^2 x(s'(C_n \circ K_1) = \frac{1}{4}$,

the χ chromatic median of $s'(C_n \circ K_1)$ is $\frac{4n}{2} = 2n$,

the χ -chromatic standard deviation of $s'(C_n \circ K_1)$ is $\sigma_x(s'(C_n \circ K_1) = \frac{1}{2}$,

the χ chromatic skewness of $s'(C_n \circ K_1)$ is $= 3(3-4n)$

we can obtain kurtosis by various moments $\mu'_1 = \frac{3}{2}, \mu'_2 = \frac{5}{2}, \mu'_3 = \frac{9}{2}, \mu'_3 = \frac{17}{2}$, $\mu_3 = 0, \mu_4 = \frac{1}{16}$, $\beta_{2x}(s'(C_n \circ k_1) = 1$,

If $\beta_{2x}(s'(C_n \circ k_1) < 3$ then it is known as PLATYKURTIC Curve.

(2) If n is odd, then is $s'(C_n \circ K_1)$ 3-colourable. Let C = { c_1, c_2, c_3 } be the minimal proper colouring of $s'(C_n \circ K_1)$. Then, the p:m:f of the corresponding r:v:X is given by

$$f(i) = \begin{cases} \frac{2n}{4n};\ i=1 \\ \frac{2n-2}{4n};\ i=2 \\ \frac{2}{4n};\ i=3 \\ 0;\ otherwise \end{cases}$$

the χ chromatic mean of $s'(C_n \circ K_1)$ is $\mu_x(s'(C_n \circ K_1) = \frac{3n+1}{2n}$,

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the χ -chromatic variance of $s^{'}(C_n \circ K_1)$ is $\sigma^2{}_x(s^{'}(C_n \circ K_1) = \frac{n^2+4n-1}{4n^2}$,

the χ -chromatic median of $s^{'}(C_n \circ K_1)$ is $\frac{4n}{2} = 2n$,

the χ -chromatic standard deviation of $s^{'}(C_n \circ K_1)$ is $\sigma_x(s^{'}(C_n \circ K_1) = \sqrt{\frac{n^2+4n-1}{4n^2}}$,

the χ chromatic skewness of $s^{'}(C_n \circ K_1)$ i $= 3(\frac{-4n^2+3n+1}{\sqrt{n^2-4n-1}}$,

we can obtain kurtosis by various moments,

$\mu^{'}_1 = \frac{3n+1}{2n}, \mu^{'}_2 = \frac{5n+5}{2n}, \mu^{'}_3 = \frac{9n+19}{2n}, \mu^{'}_3 = \frac{17n+65}{2n}$, $\mu_3 = \frac{5n^2-6n+1}{4n^3}, \mu_4 = \frac{n^4+40n^3-46n^2+24n-3}{16n^4}$,

$\beta_{2x}(S^{'}(F_n)) = \frac{n^4+40n^3-46n^2+24n-3}{n^4+8n^3+14n^2-8n+1}$, If $\beta_{2x}(S^{'}(F_n)) < 3$, then it is known as PLATYKURTIC Curve.

c) *Proposition 3:* The χ –chromatic mean $\mu_x(s^{'}(C_n \circ mK_1) = \begin{cases} \frac{3}{2}; even \\ \frac{3nm}{nm+n}; odd \end{cases}$

χ – chromatic variance of a $s^{'}(C_n \circ mK_1)$ is $\sigma^2{}_x(s^{'}(C_n \circ mK_1) = \begin{cases} \frac{1}{4}; even \\ \frac{-2n^2m^2+7mn^2}{(nm+n)^2}; odd \end{cases}$,

the χ – chromatic median of $s^{'}(C_n \circ mK_1)$=nm+n,

the χ – chromatic standard deviation of a $s^{'}(C_n \circ mK_1)$ is $\sigma_x(s^{'}(C_n \circ mK_1) = \begin{cases} \frac{1}{2}; even \\ \sqrt{\frac{-2m^2n^2+7mn^2}{(nm+n)^2}}; odd \end{cases}$,

the χ – chromatic skewness $s^{'}(C_n \circ mK_1)$ is

$$\gamma_x(s^{'}(C_n \circ mK_1) = \begin{cases} 3(3-2nm+2n); even \\ 3(\frac{-m^2n^2-2mn^2+3mn-n^2}{\sqrt{-2m^2n^2+7mn^2}}); odd \end{cases},$$

the χ – chromatic kurtosis of $s^{'}(C_n \circ K_1)$ is $\beta_2 x(s^{'}(C_n \circ mK_1) = \begin{cases} 1; even \\ \frac{-32m^4n^4+93m^3n^4-69m^3n^4+49mn^4}{49m^2n^4-28m^3n^4+m^4n^4}; odd \end{cases}$

Proof: Consider a splitting sunlet cycle $s^{'}(C_n \circ mK_1)$ on (nm+n) vertices. Then, we have the following cases.

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i) If n is even and is 2-colourable then $s^{'}(C_n \circ mK_1)$ has exactly $\frac{nm+n}{2(nm+n)}$ vertices having colour c_1 and c_2 each. Then, as explained in the first part of previous theorem,we have

1. the χ chromatic mean of $s^{'}(C_n \circ mK_1)$ is

$$\mu_x(s^{'}(C_n \circ mK_1) = \frac{3}{2}$$

2. the χ chromatic variance of $s^{'}(C_n \circ mK_1)$ is $\sigma^2{}_x(s^{'}(C_n \circ mK_1) = \frac{1}{4}$

3. the χ chromatic median of $s^{'}(C_n \circ mK_1)$ is $nm + n$

4. the χ chromatic standard deviation of $s^{'}(C_n \circ mK_1)$ is

$$\sigma_x(s^{'}(C_n \circ mK_1) = \frac{1}{2}$$

5. the χ chromatic skewness of $s^{'}(C_n \circ mK_1)$ is

$$3\left(\frac{mean - median}{s\tan drad \quad deviation}\right) = 3(3 - 2nm + 2n)$$

(vi) we can obtain kurtosis by various moments,

$$\mu'_1 = \frac{3}{2}, \mu'_2 = \frac{5}{2}, \mu'_3 = \frac{9}{2}, \mu'_3 = \frac{17}{2},$$

$$\mu_3 = o, \mu_4 = \frac{1}{16},$$

$$\beta_{2x}(s'(C_n \circ mk_1) = 1$$

If $\beta_{2x}(s'(C_n \circ mk_1) < 3$ then it is known as PLATYKURTIC Curve.

ii) If n is odd, then is $s^{'}(C_n \circ mK_1)$ 3-colourable.Let C={ c_1, c_2, c_3 }be the minimal proper colouring of $s^{'}(C_n \circ mK_1)$. Then, the p.m.f of the corresponding r.v.X is given by

$$f(i) = \begin{cases} \frac{nm}{2(nm+n)}; i = 1,2,3 \\ 0; elsewere \end{cases}$$

1. the χ chromatic mean of $s'(C_n \circ mK_1)$ is $\mu_x(s'(C_n \circ mK_1)) = \frac{3nm}{nm+n}$
2. the χ chromatic variance of $s'(C_n \circ mK_1)$ is $\sigma^2{}_x(s'(C_n \circ mK_1)) = \frac{-2m^2n^2+7mn^2}{(nm+n)^2}$
3. the χ chromatic median of $s'(C_n \circ mK_1)$ is $nm+1$
4. the χ chromatic standard deviation of $s'(C_n \circ mK_1)$ is

$$\sigma_x(s'(C_n \circ mK_1)) = \sqrt{\frac{-2m^2n^2+7mn^2}{(nm+n)^2}}$$

5. the χ chromatic skewness of $s'(C_n \circ mK_1)$ is

$$3\left(\frac{mean - median}{s\tan dareddevia\ tion}\right) = 3\left(\frac{-m^2n^2-2mn^2-3mn-n^2}{\sqrt{-2m^2n^2+7mn^2}}\right)$$

6. we can obtain kurtosis by various moments,

$$\mu'_1 = \frac{3nm}{nm+n}, \mu'_2 = \frac{7nm}{nm+n}, \mu'_3 = \frac{18nm}{nm+n}, \mu'_4 = \frac{49nm}{nm+n},$$

$$\mu_3 = \frac{9m^3n^3-27m^2n^3+18mn^3}{(nm+n)^3}, \mu_4 = \frac{-32m^4n^4+93m^3n^4-69m^2n^4+49mn^4}{(nm+n)^2},$$

$$\beta_{2x}(s'(C_n \circ mk_1)) = \frac{-32m^4n^4+93m^3n^4-69m^2n^4+49mn^4}{49m^2n^4-28m^3n^4+4m^4n^4}$$

If $\beta_{2x}(s'(C_n \circ mk_1) < 3$ then it is known as PLATYKURTIC Curve.

d) *Proposition 4:* The X chromatic mean of a splitting wheel graph $s'(W_n)$

$$\mu_x(s'(W_n)) = \begin{cases} \frac{3n+8}{2n}; even \\ \frac{3n+35}{2n}; odd \end{cases}$$

And the x – chromatic variance of $s'(W_n)$ is

$$\sigma^2{}_x(s'(W_n)=\begin{cases}\dfrac{n^2+32n-64}{4n^2}; even\\ \dfrac{n^2+8n-9}{4n^2}; odd\end{cases}$$

the x – chromatic median of $(s'(W_n))$= n

the x – chromatic standard deviation of $(s'(W_n))$is

$$\sigma_x(s'(W_n)=\begin{cases}\sqrt{\dfrac{n^2+32n-64}{4n^2}}; even\\ \sqrt{\dfrac{n^2+8n-9}{4n^2}}; odd\end{cases}$$

the x – chromatic skewness of $(s'(W_n))$is

$$\gamma_x(s'(W_n)=\begin{cases}3(\dfrac{-2n^2+3n+8}{\sqrt{n^2+32n-64}}); even\\ 3(\dfrac{2n^2-3n+3}{\sqrt{n^2+8n-9}}); odd\end{cases}$$

the x – chromatic kurtosis of $(s'(W_n))$is

$$\beta_2 x(s'(W_n)\begin{cases}\dfrac{n^4+704n^3-4480n^2+12288n+122888}{n^4-64n^2+1024}; even\\ \dfrac{n^4+1456n^3-1026n^2+432n-243}{n^4-n^3+46n^2+144n+81}; odd\end{cases}$$

Proof: Consider a splitting wheel $s'(W_n)$ on 4-colourable, when n is even and 3-colourable when n is odd. Then, we have the following cases.First assume that n is an even integer. Then, the outer cycle C_{n-1} of $s'(W_n)$)is an odd cycle. Hence, $\frac{2n-2}{2}$ vertices of C_{n-1} have colour C_1C_2 , two verticesof C_{n-1} have colour C_3 and the central vertex of W_n has colour C_4. Hence thecorresponding p:m:f of the corresponding W_n is given by

$$f(i)=\begin{cases}\frac{2n-4}{4n}; i=1,2\\ \frac{2}{2n}; i=3,4\\ 0; elsewhere\end{cases}$$

i) the χ chromatic mean of $s'(W_n)$ is $\mu_x(s'(W_n)=\frac{3n+8}{2n}$

ii) the χ chromatic variance of $s'(W_n)$ is $\sigma^2{}_x(s'(W_n)=\frac{n^2+32n-64}{4n^2}$

iii) the χ chromatic median of $s'(W_n)$ is n

iv) the χ chromatic standard deviation of $s'(W_n)$ is $\sigma_x(s'(W)=\sqrt{\frac{n^2+32n+64}{4n^2}}$

v) the χ chromatic skewness of $s'(W_n)$ is $3\left(\frac{mean-median}{s\tan dareddeviation}\right)=3(\sqrt{\frac{-2n^2+3n+8}{n^2+32n-64}})$

vi) we can obtain kurtosis by various moments,

$$\mu'_1=\frac{3n+8}{2n}, \mu'_2=\frac{5n+40}{2n}, \mu'_3=\frac{9n+164}{2n}, \mu'_3=\frac{17n+640}{2n},$$

$$\mu_3=0, \mu_4=\frac{n^4+704n^3-4480n^2+12288n+12288}{(2n)^4},$$

$$\beta_{2x}(s'(W_n)=1$$

If $\beta_{2x}(s'(W_n)<3$ then it is known as PLATYKURTIC Curve.

Next assume that n is an odd integer. Then, the outer cycle C_{n-1} of $s'(W_n)$ is an even cycle. Hence, $\frac{2n-2}{2}$ vertices of the outer cycle C_{n-1} have colour c1, $\frac{2n-2}{2}$ vertices of C_{n-1} have colour c2 and the central vertex of C_{n-1} has colourc_3. Hence, the p: m: f for $s'(W_n)$ is given by $f(i)=\begin{cases}\frac{2n-4}{4n}; i=1,2\\ \frac{2}{2n}; i=3,\\ 0; elsewhere\end{cases}$

1. the χ chromatic mean of $s'(W_n)$ is $\mu_x(s'(W_n)) = \frac{3n+3}{2n}$
2. the χ chromatic variance of $s'(W_n)$ is $\sigma^2{}_x(s'(W_n)) = \frac{n^2+8n-9}{4n^2}$
3. the χ chromatic median of $s'(W_n)$ is n
4. the χ chromatic standard deviation of $s'(W_n)$ is $\sigma_x(s'(W)) = \sqrt{\frac{n^2+8n-9}{4n^2}}$
5. the χ chromatic skewness of $s'(W_n)$ is

$$3\left(\frac{mean - median}{s\tan dareddeviation}\right) = 3(\sqrt{\frac{-2n^2+3n+3}{n^2+8n}})$$

6. we can obtain kurtosis by various moments,

$$\mu'_1 = \frac{3n+3}{2n}, \mu'_2 = \frac{5n+13}{2n}, \mu'_3 = \frac{9n+18}{2n}, \mu'_3 = \frac{17n+155}{2n},$$

$$\beta_{2x}(S'(W_n)) = \frac{n^4 + 1456n^3 - 1026n^2 + 432n + 243}{n_4 + 16n^3 + 46n^2 - 144n + 81},$$

If $\beta_{2x}(s'(W_n) < 3$ then it is known as PLATYKURTIC Curve.

e) *Proposition 5:* The χ - chromatic mean of a splitting helm graph S'(Hn) is

$$\mu_x(s'(H_n)) = \begin{cases} \frac{3n+2}{2n-1}; even \\ \frac{3n}{2n-1}; odd \end{cases}$$

The χ - chromatic variance of a splitting helm graph S'(Hn) is

$$\sigma^2{}_\chi(s'(H_n)) = \begin{cases} \frac{n^2+15n-20}{(2n-1)^2}; even \\ \frac{n^2+3n-4}{(2n-1)^2}; odd \end{cases}$$

The χ - chromatic median of a splitting helm graph S'(Hn) is 2n-1

The χ - chromatic standard deviation of a splitting helm graph S'(Hn) is

$$\sigma_{\chi}(s'(H_n)) = \begin{cases} \sqrt{\dfrac{n^2+15n-20}{(2n-1)^2}}; \ even \\ \sqrt{\dfrac{n^2+3n-4}{(2n-1)^2}}; \ odd \end{cases}$$

The χ - chromatic skewness of a splitting helm graph S'(Hn) is

$$\gamma_x(s'(H_n)) = \begin{cases} 3\left(\dfrac{-4n^2+7n+1}{\sqrt{n^2+15n-20}}\right); \ even \\ 3\left(\dfrac{-4n^2+7n-1}{\sqrt{n^2+3n-4}}\right); \ odd \end{cases}$$

The χ - chromatic skewness of a splitting helm graph S'(Hn) is

$$\beta_{2\chi}(s'(H_n)) \begin{cases} \dfrac{n^4+350n^3-1420n^2+2387n-1568}{n^4+30n^3+185n^2-600n+400}; even \\ \dfrac{n^4+38n^3-126n^2+151n-64}{n^4+6n^3+n^2-24n+16}; odd \end{cases}$$

Proof: Consider a splitting helm graph $s'(H_n)$ on 4-colourable, when n is even and 3-colourable when n is odd. Then, we have the following cases.First assume that n is an even integer. Then, the outer cycle C_{n-1} of $s'(H_n)$)is an odd cycle. Hence, $\frac{2n-2}{4n-2}$ vertices of C_{n-1} have colour C_1, Hence, $\frac{2n-4}{4n-2}$ vertices of C_{n-1} have colour C_2, two vertices of C_{n-1} have colour C_3 and C_4. Hence the corresponding p:m:f of the corresponding H_n is given by

$$f(i) = \begin{cases} \dfrac{2n-2}{4n-2}; i = 1, \\ \dfrac{2n-4}{4n-2}; i = 2 \\ \dfrac{2}{2n-1}; i = 3,4 \end{cases}$$

i) the χ chromatic mean of $s'(H_n)$ is $\mu_x(s'(H_n)) = \dfrac{3n+2}{2n-1}$

ii) the χ chromatic variance of $s'(H_n)$ is $\sigma^2{}_x(s'(H_n)) = \dfrac{n^2+15n-20}{(2n-1)^2}$

iii) the χ chromatic median of $s'(H_n)$ is 2 n-1

iv) the χ chromatic standard deviation of $s'(H_n)$ is $\sigma_x(s'(H_n)) = \sqrt{\dfrac{n^2+15n-20}{(2n-1)^2}}$

v) the χ chromatic skewness of $s'(H_n)$ is $3\left(\dfrac{mean-median}{s\tan dareddeviation}\right) = 3(\sqrt{\dfrac{-4n^2+7n+1}{n^2+15n-20}})$

vi) we can obtain kurtosis by various moments,

$$\mu'_1 = \frac{3n+2}{2n-1}, \mu'_2 = \frac{5n+16}{2n-1}, \mu'_3 = \frac{9n+74}{2n-1}, \mu'_3 = \frac{17n+304}{2n-1},$$

$$\mu_3 = \frac{65n^2-233n+186}{(2n-1)^3}, \mu_4 = \frac{n^4+350n^3-1420n^2+2387n-1568}{(2n-1)^4},$$

$$\beta_{2x}(s'(H_n)) = \frac{n^4+350n^3-1420n^2+2387n-1568}{n^4+30n^3+185n^2-600n+400}$$

If $\beta_{2x}(s'(H_n)) < 3$ then it is known as PLATYKURTIC Curve.

(2) Next assume that n is an odd integer. Then, the outer cycle C_{n-1} of $s'(H_n)$ is an even cycle. Hence, $\frac{2n-2}{4n-2}$ vertices of the outer cycle C_{n-1} have colour c_1 and c_2, and the central vertex of C_{n-1} has colour c_3. Hence, the p:m:f for $s'(H_n)$ is given by

$$f(i) = \begin{cases} \dfrac{2n-2}{4n-2}; i = 1,2 \\ \dfrac{2}{4n-2}; i = 3, \\ 0; elsewhere \end{cases}$$

1. the χ chromatic mean of $s'(H_n)$ is $\mu_x(s'(H_n)) = \dfrac{3n}{2n-1}$
2. the χ chromatic variance of $s'(H_n)$ is $\sigma^2{}_x(s'(H_n)) = \dfrac{n^2+3n-4}{(2n-1)^2}$
3. the χ chromatic median of $s'(H_n)$ is 2 n-1

4. the χ chromatic standard deviation of $s'(H_n)$ is $\sigma_x(s'(H_n)) = \sqrt{\frac{n^2+3n-4}{(2n-1)^2}}$

5. the χ chromatic skewness of $s'(H_n)$ is $3\left(\frac{mean-median}{standard deviation}\right) = 3(\sqrt{\frac{-4n^2+7n+1}{n^2+3n-4}})$

6. we can obtain kurtosis by various moments,

$$\mu'_1 = \frac{3n}{2n-1}, \mu'_2 = \frac{5n+4}{2n-1}, \mu'_3 = \frac{9n+18}{2n-1}, \mu'_3 = \frac{17n+64}{2n-1},$$

$$\mu_3 = \frac{9n^2 - 27n + 18}{(2n-1)^3}, \mu_4 = \frac{n^4 + 38n^3 - 126n^2 + 151n - 64}{(2n-1)^4},$$

$$\beta_{2x}(s'(H_n)) = \frac{n^4 + 38n^3 - 126n^2 + 151n - 64}{n^4 + 6n^3 + n^2 - 24n + 16}$$

If $\beta_{2x}(s'(H_n)) < 3$ then it is known as PLATYKURTIC Curve.

II. CONCLUSION

In this paper, we extend the concept of mean,variance, standard deviation, median, skewness and kurtosis,some important statistical parameters to various Splitting graphs based on vertex coloring sum. Based on the vertex coloring sum of splitting graphs, we have investigated thesestatistical inferences for vertex colourable graphs such as Friendship splitting graphs Sunlet cycle splitting graphs, Hairy cycle splitting graphs, Wheel splitting graphs and Helm splitting graphs. This concept can be extended to several other operations on graphs such a
cartesian product, total colouring graphs, Johan colouring , lexicographic product, corona product, sum and product of graphs etc.,

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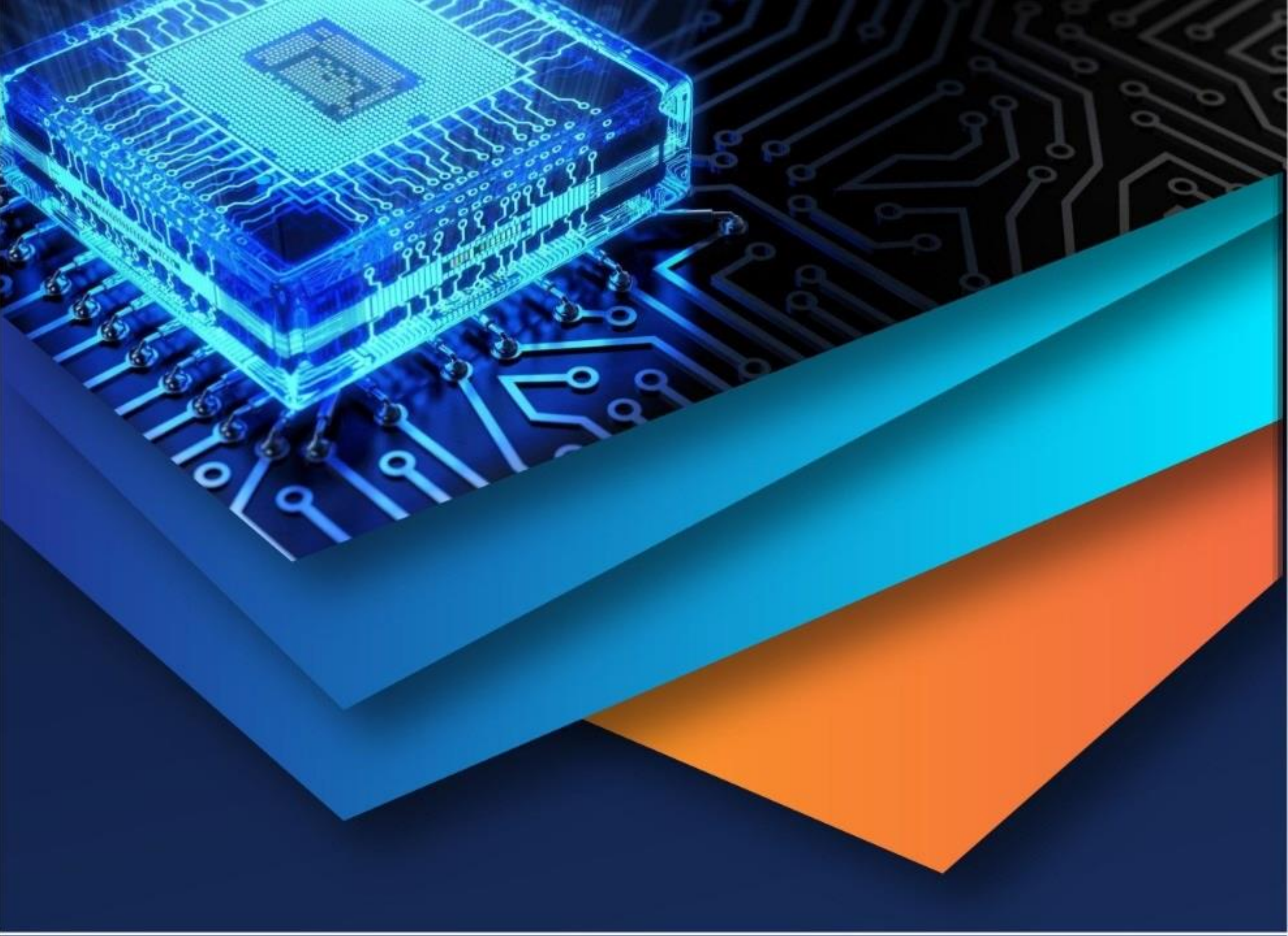

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