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# Simultaneous Solutions of Three Dimensional MHD Rotating Flow of Non-Newtonian Fluid over an Elongated Surface

R. Vijayaragavan<sup>1</sup>, M. Angeline Kavitha<sup>2</sup>

<sup>1</sup>Department of Mathematics, Thiruvalluvar University, Vellore-632115, India.

<sup>2</sup>Department of Mathematics, Voorhees College, Vellore-632001, Tamil Nadu, India.

**Abstract:** *Magnetohydrodynamic 3D flow of non-Newtonian fluid past a rotating extending surface in with thermophoresis and Brownian moment is investigated theoretically. The governing partial differential equations are transformed as ordinary differential equations with the help of suitable similarities and solved numerically using the bvp4c Matlab package. Simultaneous solutions are presented for Casson and regular fluid cases. From the solution, it is perceived that the flow is affected by sundry physical quantities like Brownian moment and thermophoresis. The influence of various parameters on the flow, temperature and concentration fields are studied with the help of graphical and tabular results. From the results we infer that the rotation decreases the flow field and encourages the thermal field.*

**Keywords:** *MHD, Heat transfer, Mass transfer, Thermophoresis and Brownian moment, Rotation.*

## I. INTRODUCTION

The study of MHD is the relation between electric and magnetic properties. It has extensive applications in the manufacturing fields like bearing, MHD generators, pumps, flow meter, etc. MHD boundary layer is noticed by many technical systems retaining plasma flow and liquid metal opposite to the magnetic fields. This type of force is called resistive force. Now, many researchers illustrated the study of MHD flows past a stretching surface [1-5]. The study of non-Newtonian fluids have multiple applications in industry and engineering. In particular separation of crude oil from petroleum products. Casson fluid is also considered as a non-Newtonian fluid. In 1859 Casson introduced the Casson fluid. It reveals the yield stress. When shear stress is small it acts like a solid otherwise it is a liquid. Tomato sauce, Jelly, Paint, vigorous fruit and soup are Casson fluid examples. The heat transfer flow over a stretching sheet with radiation was illustrated by Pramanik [6] and observed that rising values of Casson parameter enhances the surface stress. The MHD heat transfer Casson fluid past a stretching sheet in the presence of magnetic field analysed by Hayat et al. [7] and Nadeem et al. [8]. Sulochana and Sandeep [9] discussed the convective MHD flow towards a stretching sheet with the slip effects. They concluded that rising values of magnetic field parameter depreciate Nusselt number and skin friction. The researchers [10-12] analysed the convective heat transfer in MHD flows. Abdul Hakeem et al. [13] presented the influence of inclined Lorentz force on Casson fluid towards a stretching surface. The numerical solution of entropy generation towards the radiation of MHD Casson fluid flow over a stretching surface was explained by Bhatti et al. [14]. Sathish Kumar et al. [15] explained the MHD flow on heat and mass transfer over a stretching surface with a suction/injection effect. The study of MHD heat and mass transfer on Casson fluid towards a stretching surface was analyzed by Nadeem et al. [16]. Gireesha et al. [17] presented the numerical solution for boundary layer MHD heat and mass transfer past a stretching sheet with chemical reaction effect. Sandeep [18] studied the effect of aligned magnetic field on thin film flow of a nano fluid. Khan et al. [19] discussed the magnetic field effect on MHD flow of Casson fluid with convective boundary condition. The effect of Brownian motion and thermophoresis on heat and mass transfer of viscous fluid over a cylinder was discussed by Hayat et al. [20]. The effect of Cattaneo-Christov heat flux model on heat transfer of a Jeffrey fluid past a stretching sheet was analysed by Hayat et al. [21,22] and found that for larger value of the Schmidt number depreciates the concentration field. The numerical study of heat transfer of nanofluid flow through a channel with velocity slip condition was investigated by Khan et al. [23]. The 3D MHD squeezing flow of nanofluid towards a stretching wall and plate with magnetic fields effects was investigated by Khan et al. [24-30]. In this paper, we analysed the effect of thermophoresis and Brownian moment on the MHD rotation flow of Casson fluid towards a stretching sheet with magnetic field effect. We discussed the dual solutions for the Newtonian fluid and non-Newtonian fluid cases. The resulting non-linear PDE were changed into the set of ODE and solved numerically using the bvp4c Matlab package. The effect of various parameters on flow, thermal and concentration fields, along with skin friction and Nusselt number are investigated through tables and graphs.

## II. FORMULATION OF THE PROBLEM

3D, steady, laminar, incompressible flow of magnetohydrodynamic Casson fluid past a stretching sheet is considered. Transverse magnetic field is applied as shown in Fig 1. The flow of the rotating fluid is considered in the positive z-plane. It is considered that the sheet extends in its individual plane with the velocity commensurate to  $e^{(x/L)}$ . It is also considered that the fluid is rotating about the z-axis with steady rate  $\Omega$ . Here  $T_w$  is the fluid temperature near the wall.

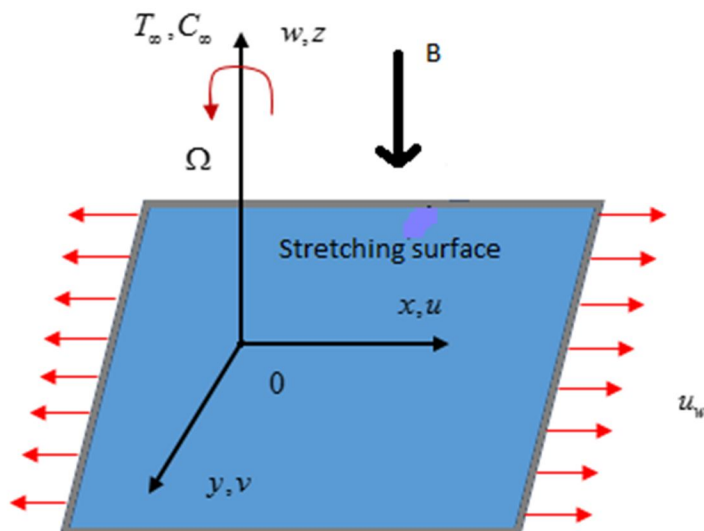


Fig.1 Flow configuration

As per the above assumptions, the governing equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = 2\Omega v + v \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho} u, \tag{2}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -2\Omega u + v \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho} v, \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} + \tau \left( D_B \frac{\partial T}{\partial z} \frac{\partial C}{\partial z} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial z} \right)^2 \right), \tag{4}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D_B \frac{\partial^2 C}{\partial z^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial z^2}, \tag{5}$$

With the boundary conditions

$$u = u_w(x) = u_0 e^{(x/L)}, v = 0, -k \frac{\partial T}{\partial z} = h(T_w - T), D_B \frac{\partial C}{\partial z} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial z} = 0 \text{ at } z = 0, \tag{6}$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, \text{ as } z \rightarrow \infty$$

Where  $\Omega$  is rotation parameter,  $B_0$  is the magnetic induction parameter,  $\beta$  is Casson parameter,  $\rho$  is the fluid density,  $\alpha$  is the thermal diffusivity,  $D_m$  is mass diffusivity,  $D_B$  is coefficient of diffusion,  $T_w, T_\infty$  are wall temperature and free stream temperature and  $C_\infty$  is the free stream concentration.

The following similarities are used to convert the governing equations as dimensionless

$$u = u_0 e^{(x/L)} f'(\eta), w = -\sqrt{\frac{\nu u_0}{2L}} (f(\eta) + \eta f'(\eta)), \tag{7}$$

$$v = v_0 e^{(x/L)} g(\eta), \eta = \sqrt{\frac{u_0}{2\nu L}} e^{(x/L)z}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_\infty},$$

with the help of Eq. (7), Eqs. (1)-(6) can be reduced as

$$\left(1 + \frac{1}{\beta}\right) f''' + ff'' - 2f'^2 + 4\Omega g - Mf' = 0, \tag{8}$$

$$\left(1 + \frac{1}{\beta}\right) g'' + fg' - 2f'g - 4\Omega f' - Mg = 0, \tag{9}$$

$$\frac{1}{Pr} \theta'' + f\theta' + Nt\theta'^2 + Nb\theta'\phi' = 0, \tag{10}$$

$$\phi'' + \left(\frac{Nt}{Nb}\right) \theta'' + Scf\phi' = 0, \tag{11}$$

with the transformed boundary conditions

$$f'(0) = 1, f(0) = 0, g'(0) = \lambda, \phi'(0) + \left(\frac{Nt}{Nb}\right) \theta'(0) = 0, \tag{12}$$

$$\theta'(0) = -\gamma(1 - \theta), f'(\infty) = g'(\infty) = \theta(\infty) = \phi(\infty) = 0,$$

Where the nondimensional parameters are defined as Ref [10].

$$Sc = \frac{\nu}{D_B}, M = \frac{\sigma B_0^2}{\rho u_0}, \lambda = \frac{v_0}{u_0}, Pr = \frac{\nu}{\alpha}, \gamma = \frac{h}{k} \sqrt{\left(\frac{2\nu L}{u_0}\right)}, Nt = \frac{\tau D_T (T_w - T_\infty)}{T_\infty \nu}, Nb = \frac{\tau D_B C_\infty}{\nu} \tag{13}$$

Wall friction and local Nusselt number can be defined as

$$C_{fx} = \left(1 + \frac{1}{\beta}\right) \frac{\tau_{zx}|_{z=0}}{\rho u_w^2}, C_{fy} = \left(1 + \frac{1}{\beta}\right) \frac{\tau_{zy}|_{z=0}}{\rho u_w^2},$$

$$Re_x^{1/2} C_{fx} = \left(1 + \frac{1}{\beta}\right) f''(0), Re_y^{1/2} C_{fy} = \left(1 + \frac{1}{\beta}\right) g'(0), \tag{14}$$

Where,  $Re_x = \frac{u_w L}{\nu}$  - reduced Reynolds number.

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)}$$

$$\frac{L}{x} \sqrt{\frac{2}{Re_x}} Nu_x = -\theta'(0) \tag{15}$$

### III. RESULTS AND DISCUSSION

The system of ordinary differential equations (8)-(11) through the boundary conditions in equation (12) are numerically solved using R-K based shooting method. For numerical computations we considered the non-dimensional parameter values as Sc=0.3,  $\beta=0.5$ , Pr=6, Nb=0.5, Nt=0.5, m=0.1, M=1. These values are invariant unless otherwise specified in the tables and graphs. The influence of the of the different relevant parameters on the flow, temperature and concentration distribution along with the reduced Nusselt number and wall friction is presented and discussed through tables and graphs.

Figs. 2 and 3 depict the Effect of magnetic field parameter  $M$  on velocity and temperature profiles for both Newtonian and non-Newtonian cases. We observed that an increasing values of  $M$  increases the temperature profile and depreciate the velocity profile for both cases. Generally, the transverse magnetic field produce the body force, to be exact the Lorentz force, which opposes the motion of the fluid and encourage the thermal boundary layer thickness.

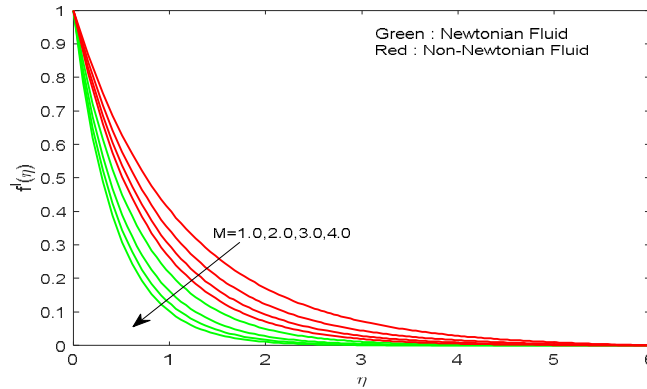


Fig. 2. Effect of  $M$  on  $f'(\eta)$

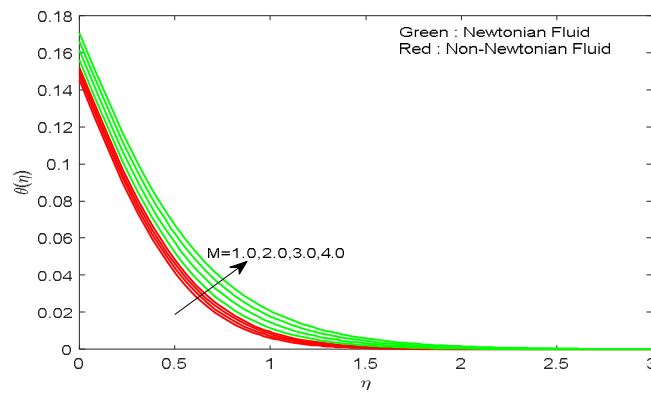


Fig. 3. Effect of  $M$  on  $\theta(\eta)$

Figs. 4 and 5 depict the effect of rotation parameter  $\Omega$  on velocity and temperature profiles. We observed that the increasing values of rotation parameter  $\Omega$  increases the temperature field and depreciate the velocity profile. Physically, this may happen due to the nanoparticle concentration being directly proportional to the rotation parameter. Fig. 6. Shows the nature of the temperature field for increasing values of  $Nt$ . Here, we noticed that rising values of  $Nt$  enhances the temperature causing an increase in the wall temperature.

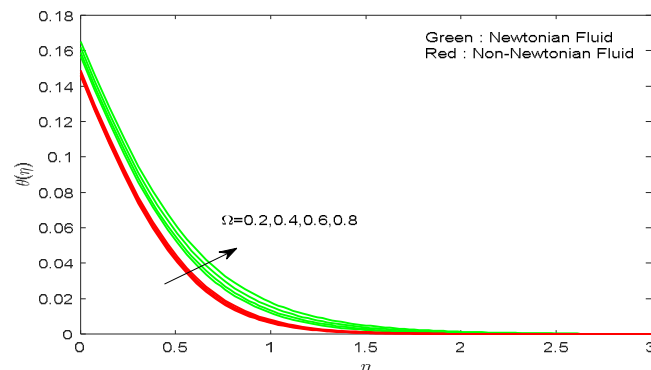


Fig. 4. Effect of  $\Omega$  on  $\theta(\eta)$

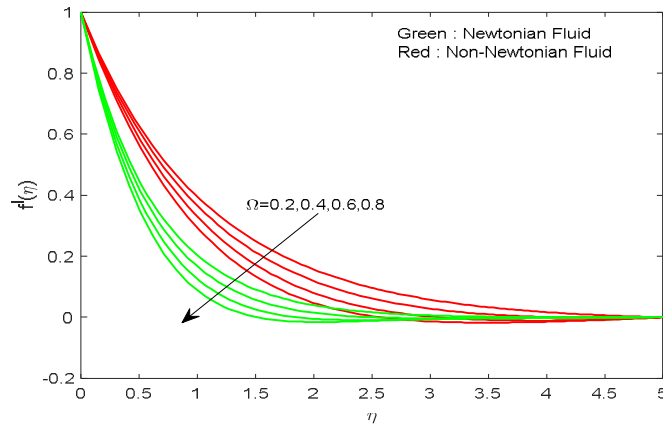


Fig. 5. Effect of  $\Omega$  on  $f'(\eta)$

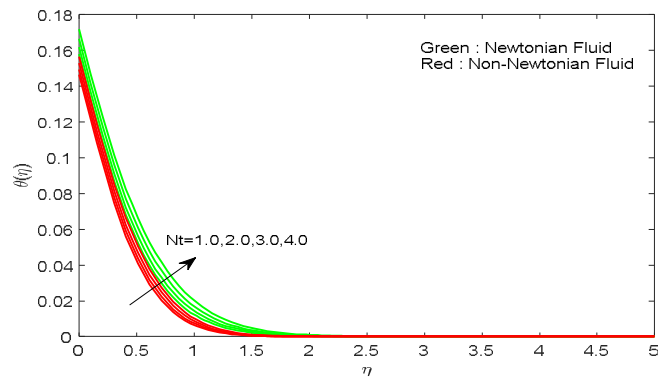


Fig. 6. Effect of  $Nt$  on  $\theta(\eta)$

The effect of  $Sc$  on the temperature field is plotted in Fig. 7. It can be seen that increasing values of  $Sc$  enhances the temperature field. Fig. 8. Shows the influence of  $\Omega$  on  $g^1(\eta)$ . It is observed that the rising values of  $\Omega$  increases  $g^1(\eta)$ . Fig. 9. depict the effect of magnetic field parameter on  $g^1(\eta)$ . Here, we see that an increasing values of  $M$  enhances  $g^1(\eta)$ .

Table shows the effect of various pertinent parameter on the skin friction and Nusselt number. It is evident that rising values of magnetic field parameter and rotation parameter declines the heat transfer rate and friction factor for both cases.

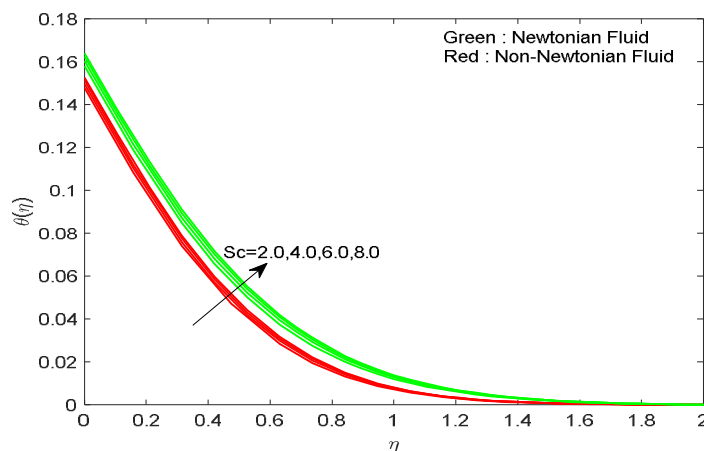


Fig. 7. Effect of  $Sc$  on  $\theta(\eta)$

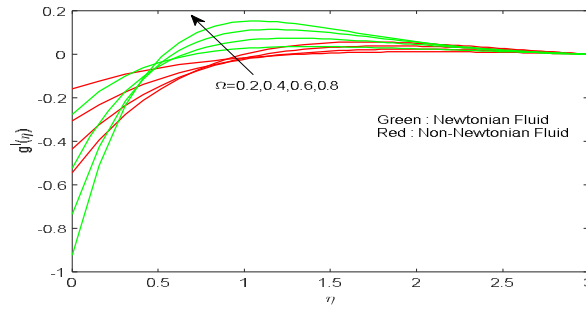


Fig. 8. Effect of  $\Omega$  on  $g^1(\eta)$

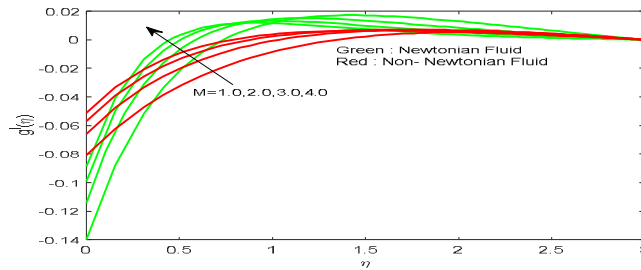


Fig. 9. Effect of  $M$  on  $g^1(\eta)$

TABLE-1: Physical parameter values for various values of the pertinent parameters.

$M$	$\Omega$	$Nt$	$Sc$	$Nb$	$\left(1 + \frac{1}{\beta}\right) f''(0)$	$\left(1 + \frac{1}{\beta}\right) g'(0)$	$-\theta'(0)$
1.0					-2.842809	-0.243288	0.256453
2.0					-3.321997	-0.197764	0.255697
3.0					-3.744197	-0.171458	0.255004
4.0					-4.124553	-0.153566	0.254359
	0.2				-2.899544	-0.480048	0.256320
	0.4				-3.005328	-0.921701	0.256080
	0.6				-3.150937	-1.304072	0.255736
	0.8				-3.309939	-1.629018	0.255343
		1.0			-2.870458	-0.242660	0.255960
		2.0			-2.870458	-0.242660	0.255048
		3.0			-2.870458	-0.242660	0.254039
		4.0			-2.870458	-0.242660	0.252913
			2.0		-3.003609	-0.230114	0.255688
			4.0		-3.003609	-0.230114	0.255087
			6.0		-3.003609	-0.230114	0.254604
			8.0		-3.003609	-0.230114	0.254203
				1.0	-2.842809	-0.243288	0.256453
				2.0	-2.842809	-0.243288	0.256453
				3.0	-2.842809	-0.243288	0.256453
				4.0	-2.842809	-0.243288	0.256453

#### IV. CONCLUSIONS

Brownian motion with a combination of Cross diffusion and thermophoresis has several real time industrial applications. With this influence, this paper reports on the velocity, heat and mass transfer attributes of the 3D motion of a MHD rotating flow of non-Newtonian fluid past an elongated surface. The diffusion and energy equations are constructed by the joint influence of thermophoresis, Brownian motion and Cross diffusion.

- A. Increasing values of  $M$  enhances the temperature field and decreases the velocity field.
- B. Rising values of rotation parameter decreases the velocity field and increases the temperature field.
- C. An increase in the Schmidt number increases the temperature profile.
- D. The Brownian motion and thermophoresis parameters have an affinity to decrease the Nusselt number.
- E. Rotation causes regulation of the flow, thermal and concentration fields.

### REFERENCES

- [1] M.Sajid, I. Ahmad, T. Hayat, M. Ayub, Unsteady flow and heat transfer of a second grade fluid over stretching sheet. *Commun. Nonlin. Sci. Numer. Simul.* 13(2008) 2193-2202.
- [2] A. Ishak, MHD boundary layer flow due to an exponentially stretching sheet with radiation effect, *Sains Malays.* 40 (2011) 391-395.
- [3] T. Hayat, A. Shafiq, A. Alsaedi, S. Asghar, Effects of inclined magnetic field in flow of third grade fluid with variable thermal conductivity. *AIP Adv.* 5 (2015) 087108.
- [4] S. Abelman, E. Momoniat, T. Hayat, Couette flow of a third grade fluid with rotating frame and slip condition, *Nonlin. Anal.: Real World Appl.* 10 (2009) 3329-3334.
- [5] M. Sheikholeslami, M.G. Bandpy, Free convection of ferrofluid in a cavity heated from below in the presence of an external magnetic field, *Powder Technol.* 256 (2014) 490-498.
- [6] S. Pramanik, Casson fluid flow and heat transfer past an exponentially porous stretching surface in the presence of thermal radiation, *Ain Shams Eng. J.* 5 (2014) 205-212.
- [7] T. Hayat, S.A. Shehzad, A. Alsaedi, Soret and Dufour effects on MHD flow of Casson fluid, *Appl. Math. Mech.* 33 (10) (2012) 1301-1312.
- [8] S. Nadeem, R.L. Haq, C. Lee, MHD flow of a Casson fluid over an exponentially shrinking sheet, *Sci. Iran. B* 19 (60) (2012) 1550-1553.
- [9] C. Sulochana, N. Sandeep, Dual solution for radiative MHD forced convective flow of a nanofluid over a slandering stretching sheet in porous medium, *J. Naval Arch. Marine Eng.* <http://dx.doi.org/10.3329/jname.v12i2.23638>.
- [10] E. Magyari, B. Keller, Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface, *Appl. Phys.* 32 (1999) 577-585.
- [11] M. Jayachandra Babu, N. Sandeep, CUM flow across a melting surface in the presence of double stratification and cross-diffusion effects, *J. Mol. Liq.* 232 (2017) 27-35.
- [12] G. Kumaran, N. Sandeep, Thermophoresis and Brownian moment effects on parabolic flow of MHD Casson and Williamson fluids with cross diffusion, *J. Mol. Liq.* 233 (2017) 262-269.
- [13] A. K. Abdul Hakeem, P. Renuka, N. Vishnu Ganesh, R. Kalaivanan, B. Ganga, Influence of inclined Lorence forces on boundary layer flow of Casson fluid over a impermeable stretching sheet with heat transfer, *J. Magn. Mag. Mater.* 401 (2016) 354-361.
- [14] M.M. Bhatti, T. Abbas, M.M. Rashid, M. El-Sayed Ali, Numerical solution of entropy generation with thermal radiation on MHD Carreau nanofluid towards a shrinking sheet, *Entropy*, 18 (6) (2016), Doi: 10.3390/e18060200.
- [15] M. Sathish Kumar, N. Sandeep, B. Rushi Kumar, Dual solution for heat and mass transfer in MHD bio-convective flow over a stretching/shrinking surface with suction/injection, *Int. J. Eng. Res. Africa* 21 (2016) 84-101.
- [16] S. Nadeem, Rizwan UIHaq, c. Lee, MHD flow of a Casson fluid over an exponentially shrinking sheet, *Scientia Iranica* 19(6) (2012) 1550-1553.
- [17] B.J. Gireesha, B. Mahanthesh, M.M. Rashidi, MHD boundary layer heat and mass transfer of a chemically reacting Casson fluid over a permeable stretching surface with non-uniform heat source/sink, *Int. J. Indust. Math.* 7(3) (2015), ISSN2008-5621.
- [18] N. Sandeep, Effect of Aligned Magnetic field on liquid thin film flow of magnetic –nanofluid embedded with grapheme nanoparticles, *Adv. Powder Technol.* 28(2017) 865-875.
- [19] M. Khan, Hashim, Ali Saleh Alshomrani, MHD stagnation- point flow of a Carreau fluid and heat transfer in the presence of convective boundary conditions, *PLoS ONE* 11 (6) (2016) <http://dx.doi.org/10.1371/journal.pone.0157180>.
- [20] T. Hayat, M.I. Khan, M. Waqas, A. Alsaedi, Newtonian heating effect in nano fluid flow by a permeable cylinder, *Result Phys.* 7 (2017) 256-262.
- [21] T. Hayat, M.I. Khan, A. Alsaedi, M. Waqas, T. Yasmeen, Impact of Cattaneo-Christov heat flux model in flow of variable thermal conductivity fluid over a variable thicked surface, *Int. J. Heat Mass Transf.* 99(2016) 702-710.
- [22] T. Hayat, M.I. Khan, M. Farooq, T. Yasmeen, A. Alsaedi, Stagnation point flow with Cattaneo-Christov heat flux and homogeneous-heterogeneous reactions, *J. Molecul. Liquids* 220(2016) 49-55.
- [23] Umar Khan, Naveed Ahmed, Syed Tauseef Mohyud-Din, Heat transfer effects on carbon nanotubes suspended nanofluid flow in a channel with non-parallel walls under the effect of velocity slip boundary condition: a numerical study, *Neural Com. Appl.* 28(1) (2017) 37-46.
- [24] U. Khan, N. Ahmed, Syed Tauseef Mohyud-Din, Numerical investigation for 3D squeezing flow of nanofluid in a rotating channel with lower stretching wall suspended by carbon nanotubes, *Appl. Thermal. Eng.* 113 (2017) 1107-1117.
- [25] U. Khan, N. Ahmed, Bander Bin-Mohsen, Syed Tauseef Mohyud-Din, Nonlinear radiation effects of flow of nanofluid over a porous wedge in the presence of magnetic field, *Int. J. Numer. Meth. Heat Fluid Flow* 27(1) (1017) 1-18.
- [26] U. Khan, N. Ahmed, Syed Tauseef Mohyud-Din, Influence of viscous dissipation and Joule heating on MHD bio-convection flow over a porous wedge in the presence of nanoparticles and gyrotatic microorganisms, *Springerplus* 5 (1) (2016) 2043.
- [27] U. Khan, N. Ahmed, Syed Tauseef Mohyud-Din, Bandar Bin-Mohsin, A bioconvection model for MHD flow and heat transfer over a porous wedge containing both nanoparticles and gyrotatic microorganisms, *J. Biolog. Syst.* 24(4) (2016) 409-429.
- [28] U. Khan, N. Ahmed, Syed Tauseef Mohyud-Din, Analysis of magnetohydrodynamic flow and heat transfer of Cu-water nanofluid between parallel plates for different shapes of nanoparticles, *Neural Com. Appl.* (2016) 1-9.
- [29] A. V. Kuznetsov, D.A. Nield, The Cheng-Minkowycz problem for natural convective boundary layer flow in a porous medium saturated by a nanofluid: a revised model, *Int. J. Heat Mass Transf.* 65 (2013) 682-685.
- [30] U. Khan, N. Ahmed, S.T. Mohyud-Din, W. Sikander, Flow of Carbon nanotubes suspended nanofluid in stretchable non-parallel walls, *Neural Com. Appl.* (2017) 1-13.



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