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# Efficiency Enhancement in a Pre-Bunched Cerenkov Free Electron Laser

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*Abstract-- An electron beam passing through a slow wave structure is unstable to an electromagnetic perturbation whose phase velocity equals the velocity of the beam. This phenomenon of cerenkov emission is the basis of all slow wave devices. The Cerenkov free electron laser (CFEL) model under consideration consists of a pre-bunched relativistic electron beam and a dielectric loaded waveguide. The increase in the growth rate and gain by using a pre-bunched electron beam in a Cerenkov free electron laser (CFEL) has been studied. The growth rate and gain is calculated at experimentally known CFEL and FEL parameters and it is seen that beam pre-bunching on Cerenkov free electron laser (CFEL) offers considerable enhancement in gain and efficiency when the phase of the pre-bunching electron beam is $-\pi/2$, i.e., when the pre-bunched beam is in the retarding zone. Moreover, the growth rate and gain of the CFEL is found to increase with the modulation index and reaches maximum when the modulation index approaching unity in addition to the frequency and wave number of the pre-bunched beam are comparable to that of the radiation wave, i.e., when the pre-bunched beam velocity is comparable to the phase velocity of the radiation wave.*

*Keywords--Pre-bunched electron beam, Cerenkov free electron laser, Cerenkov emission, efficiency, Growth rate, modulation index, slow wave device.*

## I.    INTRODUCTION

The Cerenkov free electron laser (CFEL) [1] is an attractive source of high power microwave generation at short wavelengths, which employs a slow wave medium to slow down the phase velocity of transverse electric (TE) or transverse magnetic (TM) modes to less than c, so that they can be excited by a moderately relativistic electron beam by the process of cerenkov emission. In this device, an electron beam passing through wave structure resonantly interacts with wave whose phase velocity equals the drift velocity of electrons and the wave grows at the expense of energy of the beam. A slow wave structure is needed to slow down the phase velocity of electromagnetic modes,  A Cerenkov free electron laser generally employs a dielectric whose dielectric constant is $|\varepsilon| > 1$ as a slow wave medium to reduce the phase velocity of the radiation below c (velocity of light). A moderately relativistic electron beam can excite the electromagnetic radiation by cerenkov emission. Recently, there has been strong motivation for generating sub-millimeter waves via Cerenkov emission in a dielectric loaded waveguide. A CFEL consisting of two dielectrically lined parallel plates driven by relativistic electron beam has been studied. Power levels of 10 KW at 400 micrometers and 200 KW at 1mm were measured. As the electron beam density was high and the resonant interaction region was long enough for the beam plasma oscillations to occur, it is expected that CFEL was operating in the collective regime [2-4]. Tripathi and Lui [5] have proposed the operation of a free electron laser in a dielectric loaded waveguide and have found that the main effect of loading dielectric

in free electron lasers is shortening of operating wavelengths by using the same beam energy as in vacuum free electron lasers. More recently, a lot of research work has been carried out in studying the free electron laser [6-15] by pre-bunched electron beams. A high power microwave free electron laser experiment has been performed using pre-bunched electron beam of 35Mev [11]. Here when the electron beam is pre-bunched at a frequency close to an eigen frequency of the cavity, the oscillation build process is speed up and the radiation build time is shortened significantly. Free electron maser experiment with a pre-bunched electron beam is also under way at Tel Aviv University [7]. In this case, they utilize a 1.0A current pre-bunched electron beam obtained from a microwave tube. The electron beam is bunched at 4.87GHz frequency and is subsequently accelerated to 70 keV. The bunched beam is injected into a planar wiggler ($B_w$ = 300 gauss, $\lambda_W$ =4.4 cm, where $B_w$ is the wiggler field and $\lambda_W$ is the wiggler

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wavelength) constructed in a Halbach configuration with 17 periods.

Simulation of enhanced pre-bunching in free electron lasers for the generation of high gain radiation at high frequencies has been demonstrated by Freund *et al.* [13]. It is demonstrated that free electron laser with the pre-bunched beam combines the best characteristics of amplifiers and oscillators. A theoretical model for gain and efficiency enhancement in a FEL using pre-bunched electron beam has been developed and studied by Beniwal *et al.* [14]. It is seen that growth rate increases with the increase in the modulation index. Bhasin and Sharma have studied the gain and efficiency enhancement in a slow wave FEL using pre-bunched electron beam in a dielectric loaded waveguide [15]. They have found that the growth rate and gain of a slow wave FEL increase with the modulation index and is maximum when the pre-bunched beam velocity is comparable to the phase velocity of the radiation wave. It is the purpose of the present work to present a similar theoretical description of CFEL operation using a pre-bunched electron beam for far infrared wavelength generation. We present the analytical analysis for the excitation of electromagnetic waves by pre-bunched beam in a slow wave device. A dielectric loaded waveguide is considered. Boundary effects have been ignored. The organization of the paper is as follows: We employ fluid theory and follow perturbation techniques to obtain the growth rate and efficiency of CFEL. We have compared the increase in growth rate and efficiency with the increase in modulation index calculated at experimentally known CFEL parameters. Finally, we have provided the discussion of the results.

## II. INSTABILITY ANALYSIS

A dielectric loaded waveguide of effective permittivity $\varepsilon_1$ has been considered. A pre-bunched relativistic electron beam of density $n_{b0}$, velocity $v_b \hat{z}$, relativistic gamma factor $\gamma = 1 + \frac{eV_b}{mc^2}(1 + \Delta \sin \omega_0 \tau) \approx \gamma_0 (1 + \Delta \sin \omega_0 \tau)$ [where $\Delta$ is the modulation index (its value lie from 0 to 1), $mc^2$ is the rest mass energy of the electrons, e is the electronic charge, $\omega_0 \left( \approx k_{z0} v_b \right)$ and $k_{z0}$ are the modulation frequency and wave number of the pre-bunched electron beam], respectively propagates through the waveguide (cf. Fig. 1).
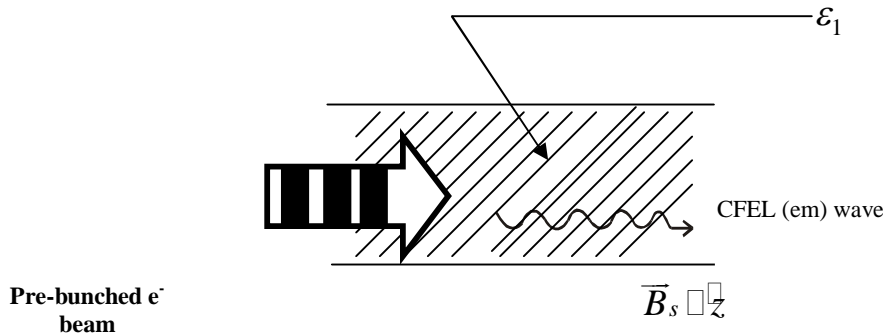


Fig.1. A schematic diagram of the Cerenkov free electron laser

An electromagnetic signal $E_1$ is also present in the interaction region.

$$\vec{E}_1 = \vec{E}_0 e^{-i\left( \omega_1 t - \vec{k}_1 . \vec{x} \right)}, \qquad (1)$$

$$\vec{B}_1 = \frac{c}{\omega_1} \vec{k}_1 \times \vec{E}_1, \qquad (2)$$

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where $\vec{E}_0$ and $\vec{k}_1$ lie in the x-z plane, i.e., $\frac{\partial}{\partial y} = ik_y = 0$. The response of the beam electrons to the signal is governed by the relativistic equation of motion

$$\frac{\partial}{\partial t}(\gamma \vec{v}) + \vec{v}.\nabla(\gamma \vec{v}) = -\frac{e}{m}(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}). \qquad (3)$$

Expanding

$$\vec{v} = v_b \hat{z} + \vec{v}_1, \qquad \gamma' = \gamma + \gamma^3 \frac{\vec{v_b}.\vec{v1}}{c^2}$$

and linearizing equation (3), we get

$$\vec{v_1} + \gamma^3 \frac{v_b^2}{c^2} v_{z1}\hat{z} = \frac{e}{im(\omega_1 - k_z v_b)}\left[\vec{E_1}\left(1 - \frac{k_z v_b}{\omega_1}\right) + \vec{k}\frac{v_b E_z}{\omega_1}\right]. \qquad (4)$$

Velocity components in the x and z directions are given by

$$v_{x1} = \frac{e}{im\gamma(\omega_1 - k_z v_b)}\left[E_{x1} - \frac{k_z v_b E_{x1}}{\omega_1} + \frac{k_{x1} v_b E_{z1}}{\omega_1}\right]. \qquad (5)$$

$$v_{z1} = \frac{eE_{z1}}{im(\omega_1 - k_z v_b)\gamma^3}. \qquad (6)$$

On linearizing and solving equation of continuity, we obtain density perturbation

$$n_1 = n_{b0}\frac{\vec{k1}.\vec{v_1}}{\left(\omega_1 - k_z v_b\right)}. \qquad (7)$$

Using the value of $v_{x1}$ and $v_{z1}$ from equations
 (5) and (6) in equation (7), we get

$$n_1 = \frac{en_{b0}}{im\left(\omega_1 - k_z v_b\right)^2}\left[\frac{E_{x1} k_{x1}}{\gamma}(1 - \frac{k_z v_b}{\omega_1}) + \frac{k_{x1}^2 v_b E_{z1}}{\gamma \omega_1} + \frac{k_{z1}E_{z1}}{\gamma^3}\right]. \qquad (8)$$

The perturbed current density is given by

$$\vec{J_1} = -n_{b0}e\vec{v_1} - n_1 e\vec{v_b}. \qquad (9)$$

Substituting the values of $\vec{V_1}$ and $n_1$ from equations (5), (6) and (8) in equation (9), and keeping the value in the wave equation, we obtain

$$k_1^2\vec{E_1} - \vec{k_1}\left(\vec{k_1}.\vec{E_1}\right) - \frac{\omega_1^2}{c^2}\varepsilon\vec{E_1} = \frac{4\pi i \omega_1 \vec{J_1}}{c^2} \qquad (10)$$

and writing x and z components of the latter, we obtain

$$(k_{z1}^2 - \frac{\omega_1^2}{c^2}\varepsilon + \frac{\omega_{pb}^2}{\gamma c^2})E_{x1} = (k_{x1}k_z - \frac{\omega_{pb}^2}{\gamma c^2}\frac{k_{x1}v_b}{\left(\omega_1 - k_z v_b\right)})E_{z1}, \qquad (11)$$

$$\text{where } \omega_{pb}^2 = \frac{4\pi n_{bo}}{m}e^2.$$

 Equation (11) gives the dispersion relation and

can be further rearranged by taking $\omega_{pb}^2$ terms to the right hand side and retaining only those terms which have a resonance

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denominator $(\omega_1 - k_z v_b)^2$, we get

$$(\omega_1^2 - \frac{k_1^2 c^2}{\varepsilon})(\omega_1 - k_z v_b)^2 = \frac{\omega_{pb}^2}{\gamma^3 \varepsilon}(\omega_1^2 + k_{x1}^2 v_b^2 \gamma^2). \qquad (12)$$

The two factors on the left–hand side of equation (12) when equated to zero $\omega_1 - \frac{k_1 c}{\sqrt{\varepsilon}} = 0, \omega_1 - k_z v_b = 0$, give radiation and beam modes, respectively. To determine the growth rate of the CFEL instability, we use the first order perturbation techniques .In the presence of the right hand side terms $\left(i.e., n_{b0} \neq 0\right)$, we assume that the eigen functions are not modified but their eigen value are.

We expand $\omega_1$ as

$$\omega_1 = \omega_{1r} + \delta = k_z v_b + \delta = k_{z0} v_b + \delta,$$

where $\delta$ is the small frequency mismatch and $\omega_{1r} = \frac{k_1 c}{\sqrt{\varepsilon}}$.

On further solving equation (12) we obtain

$$\delta = \left[\frac{\omega_{pb}^2 (\omega_{1r}^2 + k_{x1}^2 v_b^2 \gamma^2)}{2\omega_{1r} \gamma^3 \varepsilon}\right]^{1/3} e^{i\frac{2n\pi}{3}}, n=0,1,2,3,\dots\dots \qquad (13)$$

Hence the growth rate, i.e., the imaginary part of $\delta$ is given as

$$\Gamma = \left[\frac{\omega_{pb}^2 (\omega_{1r}^2 + k_{x1}^2 v_b^2 \gamma_0^2)}{2\omega_{1r} \gamma^3 \varepsilon}\right]^{1/3} \frac{\sqrt{3}}{2} \qquad (14)$$

where $\gamma = \gamma_0 (1 + \Delta \sin \omega_0 \tau)$.

For maximum gain it is assumed that all electrons are bunched in the decelerating zone, i.e., $\omega_0 \tau = -\pi/2$. This gives $\gamma = \gamma_0 (1 - \Delta)$, where $\Delta$ is the modulation index, its value lies between 0 to 1 and $\Delta \neq 1$.

Using Equation (13) the real part of $\delta$ is given as

$$\left|\delta_r\right| = \frac{\Gamma}{\sqrt{3}}, \qquad i.e., \omega_1 = k_z v_b - \frac{\Gamma}{\sqrt{3}}$$

$$or \quad v_b = \frac{\omega_1}{k_z} + \frac{\Gamma}{\sqrt{3}} k_z \quad i.e., v_b > \frac{\omega_1}{k_z}.$$

This is the necessary condition for electron bunching and net energy transfer from beam electrons to the radiation wave.

## A. Gain

The gain G in dB is defined by

$$G = 10 \log(\frac{A}{A_0}) = 10 \frac{\Gamma L}{c}, \qquad (15)$$

where L is the length of the interaction region, $A_0$ and A are the amplitudes of the wave at distance z=0 and z=L.
From equation (15) we can see that the gain, hence the efficiency of the CFEL device increases with the growth rate $\Gamma$.

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## III. RESULTS AND DISCUSSIONS

In the numerical calculations we have used typical parameters of Cerenkov free electron laser (CFEL). (e.g., beam energy = 1.30 MeV and beam-current $I_b$ =1KA, modulation frequency $\omega_0$ =4.87GHz and effective permittivity $\mathcal{E}$ = 1.7)

In Fig. 2, we have plotted the variation of the growth $\Gamma$ (in rad /sec) as a function of modulation index $\Delta$ when the phase of the pre-bunched beam is $-\pi/2$, i.e., when the electron beam is in the decelerating zone. From Fig. 2, it can be seen that the growth rate increases with the modulation index, when $\Delta \approx 0.80$ and beyond this value of modulation index, i.e., when $\Delta$ increases from 0.80 to 0.98, the growth rate increases by a factor of 9 for CFEL parameters. For modulation index $\Delta$=0, i.e., without modulated beam, the value of the growth is found to be 2.696x10$^{10}$ rad/sec for CFEL parameters. The growth rate of the CFEL instability scales as one third power of the beam current and minus one third power of the effective permittivity [cf. eq. (14)].
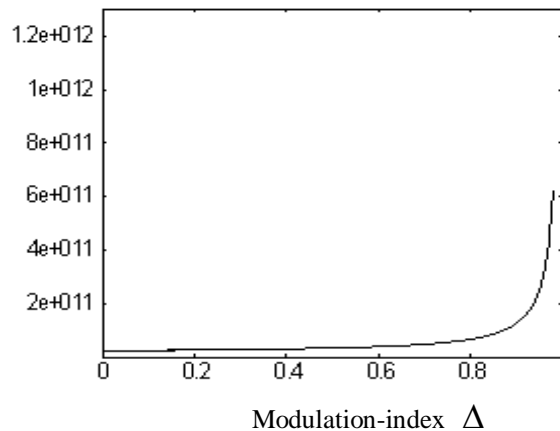


Fig.2.Growth rate $\Gamma$ (in rad /sec) as a function of modulation index $\Delta$ for (a) CFEL parameters with $E_b$ = 1.30 MeV, $I_b$= 1.0 KA and for $\sin \omega_0 \tau$ = -1

In Fig. 3, we have plotted the growth rate of the CFEL instability (in rad/sec) as a function of the phase of pre-bunched electron beam for the same parameters as Fig. 2 and for modulation index $\Delta$ = 0.80. From Fig. 2, we can observe that the growth rate reaches maximum when the phase of the pre-bunched beam is $-\pi/2$, i.e., when the beam electrons are in the retarding zone, and the growth rate becomes minimum when the phase of the pre-bunched beam is $+\pi/2$, i.e., when the beam electrons are in the accelerating zone.
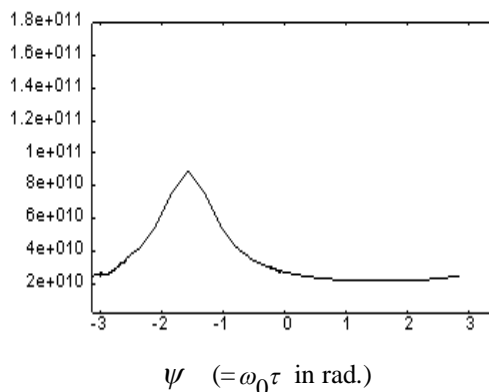


$\psi$ (= $\omega_0 \tau$ in rad.)

Fig.3.Growth rate $\Gamma$ (in rad /sec) as a function of phase angle $\psi \left(= \omega_0 \tau \right)$ of the pre-bunched electron beam for CFEL parameters for the same parameters as Fig. 2 and for modulation index $\Delta$ = 0.80.

If we introduce plasma in the interaction region of CFEL then further reducing the requirements on beam energy for generating shorter wavelength radiation. We have neglected the radial spread in the beam such as dc and ac space charge effects. We have also

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neglected the dc self generated magnetic field as we have chosen the beam parameters in such a way that dc self –generated magnetic field may not play an important role. The growth rate of the pre-bunched CFEL increases with the modulation index and consequently the gain and efficiency of the device also increases with the modulation index.

## IV. CONCLUSION

In conclusion, we can say that the beam pre-bunching on Cerenkov free electron laser offers considerable enhancement in gain and hence efficiency when the beam electrons are in the retarding zone and when the phase velocity of the radiation is comparable to the pre-bunched beam velocity. By increasing the modulation index the growth rate, gain, and efficiency of the pre-bunched CFEL increases, but at the same time the beam energy spread will also be increased, and consequently the efficiency and gain will be affected. In addition to this it is seen that by using pre-bunched electron beams, requirement for beam energy can be reduced drastically for generating high frequency radiations, for CFEL. The scheme seems to work well at millimeter and sub-millimeter wavelengths.

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