iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 3 Issue: V Month of publication: May 2015

DOI:

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International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Synthesis of Planar Mechanisms; Part IV: Four-bar Mechanism for Four Coupler Positions Generation

Galal Ali Hassaan
Emeritus Professor, Department of Mechanical Design & Production,
Faculty of Engineering, Cairo University, Egypt

Abstract— The objective of the paper is to propose an approach relying on forming a mathematical model for a planar four-bar mechanism position incorporating four coupler positions. The model consists of 10 nonlinear equations considering the transmission angle of the mechanism in the four coupler positions. The equations are solved using the command 'fsolve' of MATLAB. A case study is presented as a justification for the proposed approach. Exact coupler positions are attained with transmission angles not more than 18.5 % of the optimum value of 90 degrees.
Keywords— Planar mechanisms synthesis, four-bar mechanisms, four-coupler position generation, nonlinear kinematic equations, computer-aided mechanism synthesis

I. INTRODUCTION

Mechanism synthesis techniques range from simple graphical techniques going through analytical approaches with many assumptions and trials to sophisticated techniques using optimization application.

The subject of mechanism synthesis has occupied the attention of researchers over decades. Only some publications are reviewed over the last one and half decades to highlight some of the efforts focused on mechanism synthesis.

Russel (2001) presented several methods for synthesizing adjustable spatial mechanisms. He synthesized spatial 4 and 5-bar mechanisms for different phases of prescribed rigid body positions. He extended his approach to incorporate rigid body tolerance problems [1]. Cabrera, Simon and Prado (2002) used a searching procedure applying genetic algorithms to the problem of synthesis of 4-bar planar mechanisms. They outlined the possibility of extending their method to other mechanisms [2]. Smaili and Zeineddine (2003) presented a software package based on Simulink and Matlab for the synthesis and analysis of linkage mechanisms. They coded precision point synthesis methods and optimization synthesis techniques to yield a mechanism for a specific task [3].

Bultovic and Djordjevic (2004) studied the optimal synthesis of a 4-bar linkage by method of controlled deviation. They used the Hooke-Jeeves optimization technique without dependence on the initial selection of the projected variables [4]. Shiakolas, Koladiya and Kebrle (2005) presented a methodology combining different evolution, an evolutionary optimization and geometric control of precision positions for mechanism synthesis. They employed two penalty functions, one for constraint violation and one for relative accuracy [5]. Damangir, Jafarijashemi, Mamduhi and Zohoor (2006) proposed a curvature path description method for path generation of planar mechanisms. The objective function was independent of rotation and translation transformations [6]. Xi and Chen (2007) proposed an approach for the kinematic synthesis of a crank-rocker mechanism to generate a coupler motion passing through a prescribed set of positions [7]. Schrocker, Juttler and Agner (2008) presented an evolution based method for optimal mechanism synthesis. They used curve and surface evolution techniques from computer-aided design and image processing [8].

Al-Smadi (2009) calculated the mechanism parameters required to achieve a set of prescribed rigid body positions [9]. Peng (2010) developed an optimal synthesis method based on link length structural error for the kinematic synthesis of adjustable planar mechanisms. He developed the optimal synthesis method for adjustable planar 4-bar mechanisms for three typical synthesis tasks [10]. Mutawe, Al-Smadi and Sodhi (2011) discussed the path generation of 4-bar mechanism with position tolerance variations due to joint running tolerance [11]. Hwang and Wang (2012) presented a synthesis technique for the planar Watt-I six-bar mechanism with a coupler point passing through 3 or 4 acceleration poles. They provided examples to illustrate the feasibility of their proposed method [12].

Larochelle (2013) presented a dimensional synthesis technique for solving the mixed exact and approximate motion synthesis problem for planar RR kinematic chains. His algorithm did not require the use of any optimization algorithm [13]. Kamat, Hoshing, Pawar, Lokhande, Patankar and Hatawalane (2014) synthesized an adjustable planar 4-bar mechanism for different angles. They adjusted the length of different links to obtain different paths accurately [14]. Shete and Kulkarni (2015) used genetic algorithm to achieve a desired trajectory. They analyzed three problems having different curvature [15].

II. NOMENCLATURE

$f_1, f_2, \ldots, f_{10}$: nonlinear mechanism functions.

Volume 3 Issue V, May 2015
ISSN: 2321-9653

r_1, r_2, r_3, r_4:	lengths of links 1, 2, 3 and 4.
r_{1n}, r_{3n}, r_{4n}:	normalized lengths of links 1, 3 and 4.
x_{A1},y_{A1}:	coordinates of point A_1.
x_{A2},y_{A2}:	coordinates of point A_2.
x_1, x_2,…., x_{10}:	mechanism unknown parameters.
μ_1, μ_2, μ_3, μ_4:	mechanism transmission angles in the four coupler positions.
θ_1:	orientation of link 1 (frame).
θ_{21}, θ_{31}, θ_{41}:	orientation of links 2, 3 and 4 in the first mechanism position.
θ_{22}, θ_{32}, θ_{42}:	orientation of links 2, 3 and 4 in the second mechanism position.
θ_{23}, θ_{33}, θ_{43}:	orientation of links 2, 3 and 4 in the third mechanism position.
θ_{24}, θ_{34}, θ_{44}:	orientation of links 2, 3 and 4 in the fourth mechanism position.

III.METHODOLOGY

The proposed methodology is applied to standard 4-bar mechanisms having fixed lengths. The approach is applied as follows:

The desired 4 positions of the coupler are assigned in the motion plane.
Closed loops are formed for the mechanism in the 4 positions.
2 equations are written for each loop in the x and y directions.
4 equations are written for the 4 transmission angles (one per mechanism position).
The 10 equations are written in a normalized form by dividing each link dimension by r_2.
The equations are written such that the right hand side is zero.
The model in its final form consists of 6 nonlinear equations in 6 unknowns.
The model is solved using MATLAB for the mechanism unknowns.

A. Requirements

It is desired to have a coupler of a known length in 4 positions: A_1B_1 , A_2B_2 , A_3B_3 and $A_4B_4$with known orientations θ_{31} , θ_{32} , θ_{33} and θ_{34} (Fig.1).

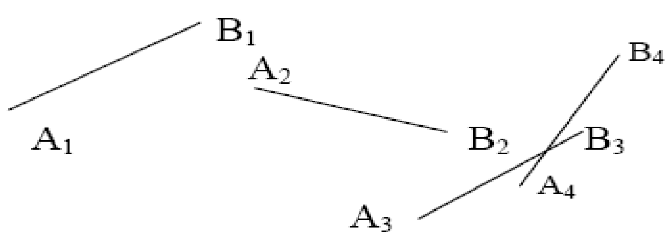

Fig.1 Desired coupler 4 positions

B. Mechanism

Fig.2 shows a 4-bar mechanism in 4 positions corresponding to the desired 4 coupler positions.

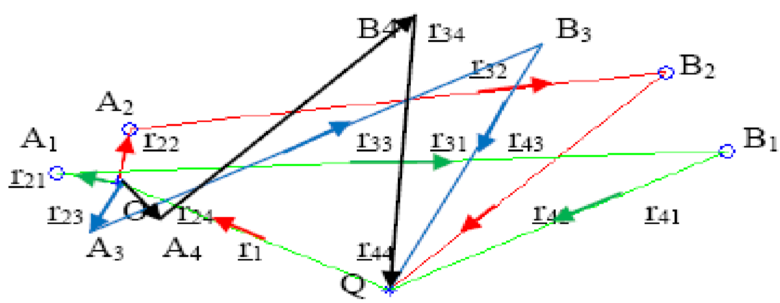

Fig.2 4-bar mechanism in 4 positions

4 polygons are closed which are required for displacement analysis in each mechanism position.

C. Analysis

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The 4 coupler positions are: A_1B_1 , A_2B_2 , A_3B_3 and A_4B_4.

Polygon 1: OA_1B_1QO. The displacement equation across the polygon is:

$$\underline{r}_1 + \underline{r}_{21} + \underline{r}_{31} + \underline{r}_{41} = 0$$

Working with the vectors components in the x-direction ; $\sum r_x = 0$ gives:

$$r_1\cos\theta_1 + r_2 \cos\theta_{21} + r_3 \cos\theta_{31} + r_4 \cos\theta_{41} = 0 \qquad (1)$$

Working with the vectors components in the y-direction ; $\sum r_y = 0$ gives:

$$r_1\sin\theta_1 + r_2 \sin\theta_{21} + r_3 \sin\theta_{31} + r_4 \sin\theta_{41} = 0 \qquad (2)$$

Polygon 2: OA_2B_2QO. The displacement equation across the polygon is:

$$\underline{r}_1 + \underline{r}_{22} + \underline{r}_{32} + \underline{r}_{42} = 0$$

Working with the vectors components in the x-direction ; $\sum r_x = 0$ gives:

$$r_1\cos\theta_1 + r_2 \cos\theta_{22} + r_3 \cos\theta_{32} + r_4 \cos\theta_{42} = 0 \qquad (3)$$

Working with the vectors components in the y-direction ; $\sum r_y = 0$ gives:

$$r_1\sin\theta_1 + r_2 \sin\theta_{22} + r_3 \sin\theta_{32} + r_4 \sin\theta_{42} = 0 \qquad (4)$$

Polygon 3: OA_3B_3QO. The displacement equation across the polygon is:

$$\underline{r}_1 + \underline{r}_{23} + \underline{r}_{33} + \underline{r}_{43} = 0$$

Working with the vectors components in the x-direction ; $\sum r_x = 0$ gives:

$$r_1\cos\theta_1 + r_2 \cos\theta_{23} + r_3 \cos\theta_{33} + r_4 \cos\theta_{43} = 0 \qquad (5)$$

Working with the vectors components in the y-direction ; $\sum r_y = 0$ gives:

$$r_1\sin\theta_1 + r_2 \sin\theta_{23} + r_3 \sin\theta_{33} + r_4 \sin\theta_{43} = 0 \qquad (6)$$

Polygon 4: OA_4B_4QO. The displacement equation across the polygon is:

$$\underline{r}_1 + \underline{r}_{24} + \underline{r}_{34} + \underline{r}_{44} = 0$$

Working with the vectors components in the x-direction ; $\sum r_x = 0$ gives:

$$r_1\cos\theta_1 + r_2 \cos\theta_{24} + r_3 \cos\theta_{34} + r_4 \cos\theta_{44} = 0 \qquad (7)$$

Working with the vectors components in the y-direction ; $\sum r_y = 0$ gives:

$$r_1\sin\theta_1 + r_2 \sin\theta_{24} + r_3 \sin\theta_{34} + r_4 \sin\theta_{44} = 0 \qquad (8)$$

Unknowns in Eqs.33-40: r_1, r_2, , r_4, θ_1, θ_{21}, θ_{41}, θ_{22} , θ_{42} , θ_{23} , θ_{43} , θ_{24} and θ_{44}.

Number of unknowns: 12.

Number of equations so far: 8

The number of design parameters is reduced through:

1. Assigning the ground length, r_1.
2. Using normalized dimensions by referring all the dimensions to r_2.

In this case, the unknown design parameters are: $x_1 = r_{4n}$, $x_2 = \theta_1$, $x_3 = \theta_{21}$, $x_4 = \theta_{41}$, $x_5 = \theta_{22}$, $x_6 = \theta_{42}$, $x_7 = \theta_{23}$, $x_8 = \theta_{43}$, $x_9 = \theta_{24}$ and $x_{10} = \theta_{44}$.

Number of unknowns is reduced to 10.

Two more equations may be written for the transmission angle in 4 positions of the mechanism.

The transmission angle is related to links 3 and 4 orientation angles through:

$$\mu_1 = \theta_{41} - \pi - \theta_{31}$$
$$\mu_2 = \theta_{42} - \pi - \theta_{32}$$
$$\mu_3 = \theta_{43} - \pi - \theta_{33}$$
$$\mu_4 = \theta_{44} - \pi - \theta_{34}$$

Now, the kinematical model equations are written in the normalized form as:

$$f_1 = r_{1n}\cos x_2 + \cos x_3 + r_{3n}\cos\theta_{31} + x_1\cos x_4 \qquad (9)$$
$$f_2 = r_{1n}\sin x_2 + \sin x_3 + r_{3n}\sin\theta_{31} + x_1\sin x_4 \qquad (10)$$
$$f_3 = r_{1n}\cos x_2 + \cos x_5 + r_{3n}\cos\theta_{32} + x_1\cos x_6 \qquad (11)$$
$$f_4 = r_{1n}\sin x_2 + \sin x_5 + r_{3n}\sin\theta_{32} + x_1\sin x_6 \qquad (12)$$
$$f_5 = r_{1n}\cos x_2 + \cos x_7 + r_{3n}\cos\theta_{32} + x_1\cos x_8 \qquad (13)$$
$$f_6 = r_{1n}\sin x_2 + \sin x_7 + r_{3n}\sin\theta_{32} + x_1\sin x_8 \qquad (14)$$
$$f_7 = r_{1n}\cos x_2 + \cos x_9 + r_{3n}\cos\theta_{34} + x_1\cos x_{10} \qquad (15)$$
$$f_8 = r_{1n}\sin x_2 + \sin x_9 + r_{3n}\sin\theta_{34} + x_1\sin x_{10} \qquad (16)$$
$$f_9 = (TA_1 - x_4 + \pi + \theta_{31})^2 + (TA_2 - x_6 + \pi + \theta_{32})^2 \qquad (17)$$
$$f_{10} = (TA_3 - x_8 + \pi + \theta_{33})^2 + (TA_4 - x_{10} + \pi + \theta_{34})^2 \qquad (18)$$

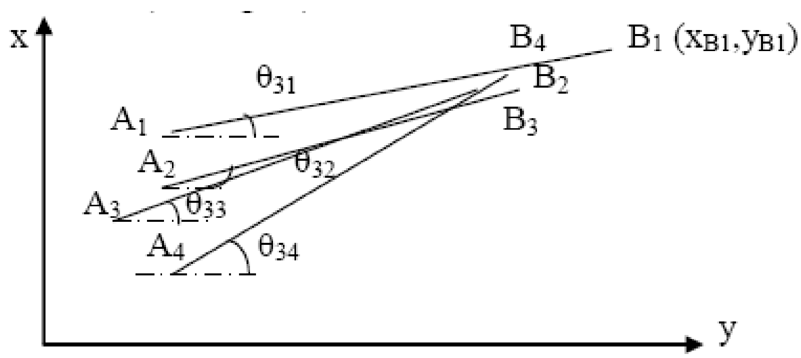
Equations 17 and 18 are functions of the 4 transmission angles of the 4-bar mechanism in its 4 positions corresponding to each coupler position. It's a novel use of those relations in such formulation.

D. Mechanism Synthesis:

The synthesis equations are equations 9-18 (10 equations).
The equations are nonlinear in 10 unknowns.
The 10 equations are in the form: $f = 0$
The 10 equations may be solved with MATLAB using its command "fsolve" or any other numerical technique [16].

E. Case Study

It is required to design a 4-bar planar mechanism to move the coupler AB from position A_1B_1 to A_2B_2 to A_3B_3 to A_4B_4 as shown in Fig.3.



$r_3 = AB = 200$ mm , $\theta_{31} = 20^\circ$, $\theta_{32} = 25^\circ$,
$\theta_{33} = 30^\circ$, $\theta_{34} = 43^\circ$
$x_{B1} = 500$, $y_{B1} = 400$ mm

Fig.3 Desired coupler 4 positions with location of x_{B1},y_{B1}.

F. Mechanism Synthesis

A MATLAB code is written to solve Eqs.9-18 satisfying the right hand side which is zero for the 10 equations.

- Code inputs: $r_{3n} = 5$, $r_{1n} = 6$, $\theta_{31} = 20^o$, $\theta_{32} = 25^o$, $\theta_{33} = 30^o$ and $\theta_{34} = 43^o$
  $TA_1 = TA_2 = TA_3 = TA_4 = 90^o$
- Code output:

3.7011 (r_{4n})
-3.2306 (θ_1)
-4.3871 (θ_{21})
5.1740 (θ_{41})
-3.3370 (θ_{22})
5.4147 (θ_{42})
0.6475 (θ_{23})
4.9453 (θ_{43})
-1.5848 (θ_{24})
5.3833 (θ_{44})

Normalized model functions at convergence:

0.0509 -0.1228 -0.0348 0.0155 0.0056 0.0353 -0.0324
0.0447 0.0834 0.0909

Mechanism dimensions:

Coupler length:	$r_3 = 200$ mm
Crank length:	$r_2 = r_3/r_{3n} = 200/5 = 40$ mm
Rocker length:	$r_4= r_{4n}xr_2 = 148.04$ mm

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Ground length:	$r_1 = r_{1n} x r_2 = 240$ mm
Ground angle:	$\theta_1 = 174.9^o$
Crank orientation:	$\theta_{21} = 108.6^o$
	$\theta_{22} = 168.8^o$
	$\theta_{23} = 37.1^o$
	$\theta_{24} = 269.2^o$
Rocker orientation:	$\theta_{41} = 296.4^o$
	$\theta_{42} = 310.2^o$
	$\theta_{43} = 283.3^o$
	$\theta_{44} = 308.4^o$

The designed mechanism in its 4 positions is shown in Fig.4.

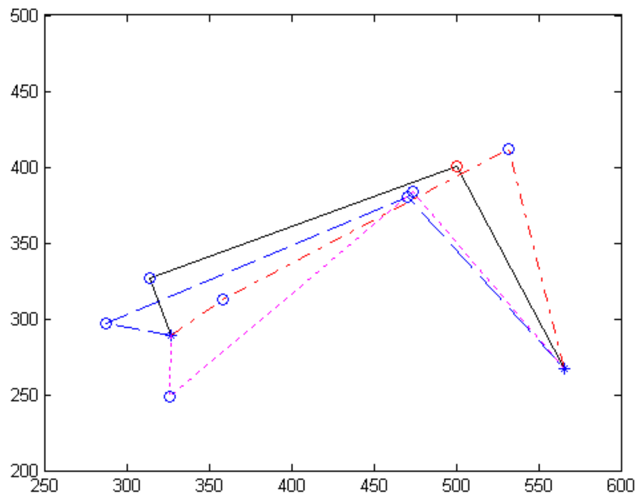

Fig.4 Synthesized mechanism in its 4 positions.

Transmission angles of the synthesized mechanism:

In the first position:	$\mu_1 = 96.4^o$.
In the second position:	$\mu_2 = 105.2^o$.
In the third position:	$\mu_3 = 73.3^o$.
In the third position:	$\mu_4 = 85.4^o$.

@ Mechanism type:

$L_{min} = 40$ mm (crank)
$L_{max} = 240$ mm
$L_a = 148$ mm
$L_b = 200$ mm
$L_{min} + L_{max} = 280$ mm
$L_a + L_b = 348$ mm
Then: $L_{min} + L_{max} < L_a + L_b$ and the crank is the minimum.

Therefore, the designed mechanism is a crank-rocker Grashof mechanism [17].

IV. CONCLUSIONS

The proposed approach is very accurate and reliable in synthesizing 4-bar planar mechanisms for 4 specific positions of its coupler.
The assumptions are only one dimension (r_1) giving easy and straight forward synthesis of the 4-bar mechanism.
The coupler traced exactly the desired 4-positions.
The deviation of the transmission angle of the mechanism from the ideal value of 90^o is:

7.1 % error in the first coupler position.

16.9 % error in the second coupler position.

-18.5 % error in the third coupler position.

-5% error in the fourth coupler position.

The transmission angle in all the 4 mechanism positions is within the recommended range of $45^o \leq \mu \leq 135^o$ [18] .

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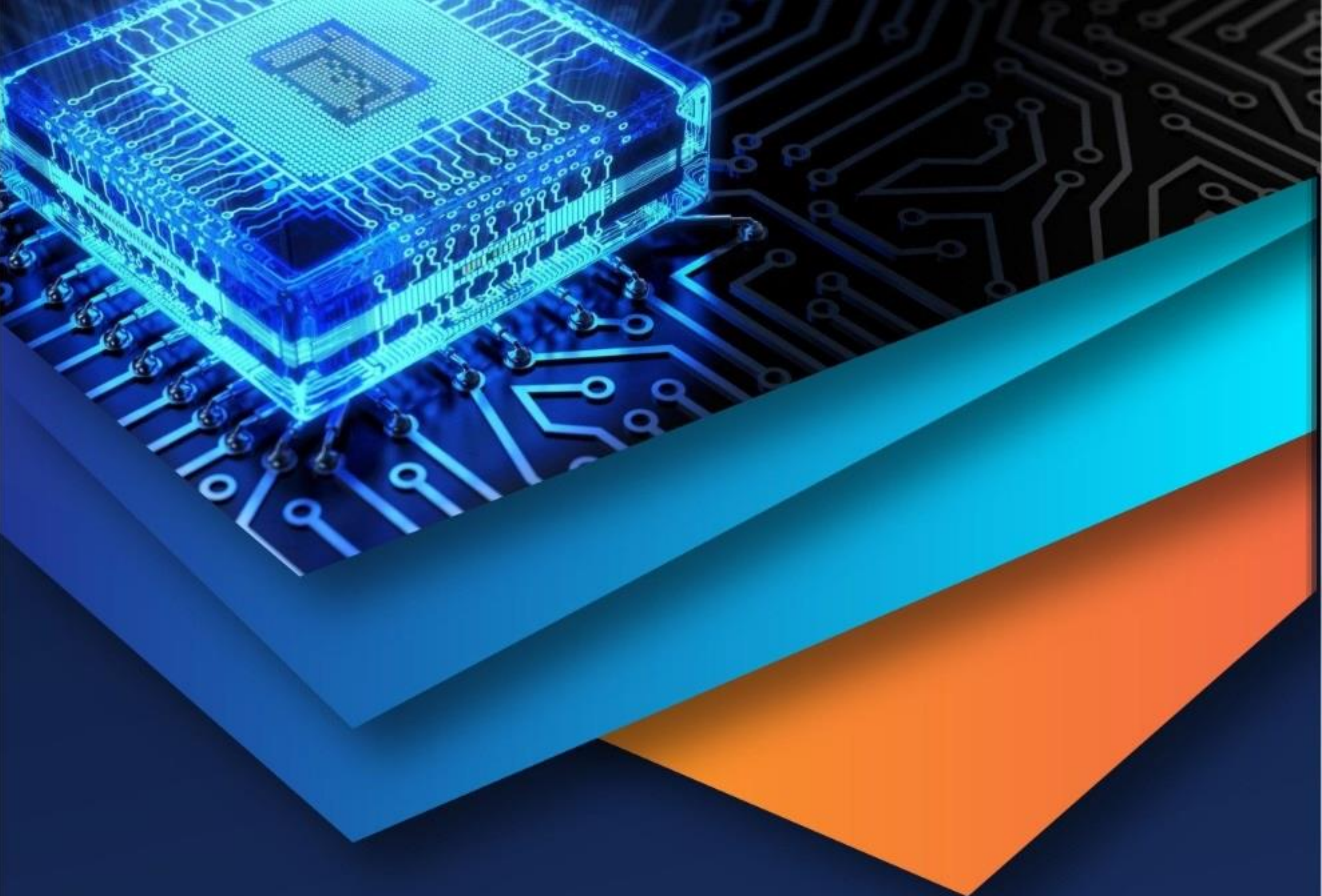
BIOGRAPHY

Galal Ali Hassaan

A. Emeritus Professor of System Dynamics and Automatic Control.
B. Has got his B.Sc. and M.Sc. from Cairo University in 1970 and 1974.
C. Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
D. Now with the Faculty of Engineering, Cairo University, EGYPT.
E. Research on Automatic Control, Mechanical Vibrations , Mechanism Synthesis and History of Mechanical Engineering.
F. Published 10’s of research papers in international journals and conferences.
G. Author of books on Experimental Systems Control, Experimental Vibrations and Evolution of Mechanical Engineering.
H. Editor-in-Chief of IJCT.
I. Reviewer of some international engineering and technological journals.



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