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A Concise Review on Summability Techniques and its Applications in Different Fields

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Abstract: ***Summability techniques are widely applicable as a powerful tool for filtering the signals in the form of series (Infinite Series, Fourier series, Wavelets etc.) using various summability methods such as Matrix Summability, Nörlund and Generalized Nörlund Summability, Hölder Summability, Cesàro and Generalized Cesàro Summability, Riesz Summability ,Indexed Summability, Abel Summability, Euler Summability, Borel Summability, Hausdorff Summability, Banach Summability, Riesz-Banach Summability . In this paper a review has been done on Summability Techniques and their Applications. This paper will be useful for the researchers in selecting their research work on Summability Techniques and their Applications.***

Keywords: Nörlund and Generalized Nörlund Summability, Cesaro and Generalized Cesàro Summability, Riesz Summability Method, Indexed Summability, Abel Summability

I. INTRODUCTION

A Summability method is such an assignment of a limit to a subset of the set of divergent series which properly extends the classical notion of convergence. Summability methods include Cesàro summation, (C, k) summation, Abel summation and Borel summation, in increasing order of generality (and hence applicable to increasingly divergent series). A variety of general results concerning possible summability methods are known. The Silverman–Toeplitz theorem characterizes matrix summability methods, which are methods for summing a divergent series by applying an infinite matrix to the vector of coefficients. The most general method for summing a divergent series is non-constructive and concerns limits. Summation methods usually concentrate on the sequence of partial sums of the series. While this sequence does not converge, we may often find that when we take an average of larger and larger initial terms of the sequence, the average converges, and we can use this average instead of a limit to evaluate the sum of the series. So in evaluating $a = a_0 + a_1 + ... + a_n + ...$, we work with the sequence $\{s_n\}$, where $s_0 = a_0$ *and* $s_{n+1} = s_n + a_{n+1}$ for all $n \in N$. In the convergent case, the sequence $\{s_n\}$ approaches to the limit a. The basic technique of summability methods is to transform a given infinite series or sequence of partial sums into another series or sequences on which Cauchy's method is applicable.

II. LITERATURE REVIEW

Summability theory is the theory of the assignment of limits, which has a vital role in analysis, function theory, topology and functional analysis. In mathematical Analysis, Summability is an alternative formulation of convergence of a series which is divergent in the conventional sense. Summability is a field in which we study non-convergent series/integrals and assigns a value (number) to it.

Few interesting cases are given below, for which the assignment of limits can be considered,

1) Real or complex sequences for the limit process '$n \to \infty$',
2) Series (convergence of series),
3) Sequences and series of functions like Power series, Fourier series, etc.
4) Limit of a function at a point (continuity, continuous extension),
5) Differentiation of functions,
6) Integration of functions.

A. On the basis of Applications in Different Fields

The theory of summability has many uses throughout analysis and applied mathematics, for example, in Analytic Continuation, Quantum Mechanics, Probability Theory, Fourier Analysis, Theory of Approximation, and Fixed Point Theory. Engineers and physicist working with Fourier series or analytic continuation will also find the concepts of summability theory valuable to their

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research. Investigation and developments of the summability methods have been made possible in various fields of mathematics; such as, Applied Analysis, Function Theory, Theory of Fourier Series, Signal Processing, Digital Image Processing, Speech Processing and Control Theory etc. The theory of summability has played a very important role in Fourier analysis. The inadequacy of assigning sum to infinite series by Cauchy's method was vividly revealed in their applications to Fourier series. This marked the beginning of the realization of summability theory. Whenever a Fourier series of a continuous function converged, it did so to the value of the generating function. This was confirmed by the results of Fejer and Lebesgue that the Fourier series of any summable functions is $(C,1)$summable almost everywhere. This result settled the anomalous situation of the concept of convergence to which unfortunately Cauchy's method could not rise.

This realization gave impetus to be speedy development of the theory of summability. In this way Fourier analysis derived benefits from the applications of summability methods in resolving several anomalous situations it faced with both ordinary and absolute summability methods. Side by side summability methods received great impetus and refinements from their application to infinite series.

B. On the basis of Main Theorem/Results proved on Summability

Dr B.P.Padhy et al. have published a research paper on product summability of Fourier series *using Matrix-Euler method.* The main objective of this research paper is to study that Product Summability of Fourier series by Matrix-Euler method generalizes the (*N, p)(E, z)* -Product Summability of Fourier series and (*N, p, q*) (*E ,z)* - Product Summability of Fourier series [1].

Dr B.P.Padhy et al. have established a Study on the Local Property of indexed Summability of a factored Fourier series. In this paper they have established a theorem on the local property of $|\overline{N}, p_{n,}\alpha_n;\delta|_k$summability of a factored Fourier series. They have dealt with $|\overline{N}, p_n|_k$- Summability of an infinite series Bor [1] to establish the proof [2].

Dr.H. M. Srivastava et al. have published a research paper on Generalized equi-statistical convergence of the deferred Nörlund summability and its applications to associated approximation theorems. In this paper, they have used the notion of equi-statistical convergence, statistical point-wise convergence and statistical uniform convergence in conjunction with the deferred Nörlund statistical convergence in order to establish several inclusion relations between them. They have also applied presumably new concept of the deferred Nörlund equi-statistical convergence to prove a Korovkin type approximation theorem and demonstrated that theorem is a non-trivial extension of some well-known Korovkin type approximation theorems which were proven by earlier authors. Finally, they have considered a number of interesting cases in support of definitions and results [3].

S. K. Paikray et al. have published a research paper on Degree of Approximation by Product Means of Conjugate Series of a Fourier series. In this paper a theorem on Degree of Approximation of a function $f \in Lip(\alpha, r)$ by Product Summability $(E, q)(\bar{N}, p_n)$ of Conjugate Series of Fourier series associated with f has been established [4].

Dr B.P.Padhy et al. have published a research paper on "A note on the use of quasi-f-power Increasing sequence in absolute indexed Riesz Summability ". They have extended the result of Bor [1](2016), and established a result concerning absolute indexed Riesz Summability factors $|\bar{N}, p_n, \theta_{n,}, \mu|_k, k \geq 1, \mu > 0$,using quasi – f - power increasing sequence [5].

Bidu Bhusan Jena et al. have published a paper on "Statistical Deferred Cesaro Summability and Its Applications to Approximation Theorems". In this paper, they applied statistical deferred Cesaro summability method to prove a Korovkin type approximation theorem for the set of functions $1; e^{-x}$and e^{-2x} defined on a Banach space $C[0,\infty)$ and demonstrated that their theorem is a non-trivial extension of some well-known Korovkin type approximation theorems. They also established a result for the rate of statistical deferred Cesaro summability method [6].

H.K. Nigam et al. have published a paper" On Approximation of Conjugate of Functions Belonging to Different Classes by Product Means". Many researchers have obtained the degree of approximation of functions belonging to $Lip\alpha, Lip\ (\alpha, r), Lip(\xi\ (t), r)$ *and* $W\ (L\ r\ , \xi\ (t))$ classes using Cesaro, Norlund and generalized Norlund single summability methods. In this paper, they introduce $(C,1)(E,q)$ product summability method and prove a quite new theorem on degree of approximation of conjugate of function $f \in W\ (Lr, \xi\ (t))$ *class using* $(C,1)(E,q)$ product summability means of conjugate Fourier series [7]

Mahendra Misra et al. have published a paper "on degree of approximation of conjugate series of a Fourier series by Product Summability".In this paper a theorem on degree of approximation of a function $f \in Lip(\alpha, r)$ by product summability $(E, q)(\bar{N}, p_n)$ of conjugate series of Fourier series associated with f has been proved[8].

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Shyam Lal has published a paper on "Approximation of functions belonging to the generalized Lipschitz class by $C1 \cdot Np$ summability method of Fourier series". In this paper two new theorems on the degree of approximation of the function $f \in Lip\alpha$ and $f \in W(Lr, \xi(t))$ have been established [9].

Shyam Lal has published a paper "On the degree of approximation of conjugate of a function belonging to weighted $W\left(L^p, \xi(t)\right)$ class by matrix summability means of conjugate series of a Fourier series". In this paper a new theorem on the degree of approximation of conjugate of a function belonging to weighted class by Matrix summability means of conjugate series of a Fourier series has been established. The main theorem is a generalization of several known and unknown results [10].

Kalpana Saxena et al. have published a paper on" A Study on $(E,Q)(E,Q)$ Product Summability of Fourier Series and Its Conjugate Series". In this paper, they have introduced the concept of (E,q) (E,q) product operators and established two new theorems on $(E,q)(E,q)$ product Summability of Fourier series and its conjugate series. The results obtained in the paper further extend several known result on linear operators [11].

Smita Sonker et al. have proved a theorem" On Generalized Absolute Riesz Summability Factor of Infinite Series". The objective of this manuscript is to obtain a moderated Theorem proceeding with absolute Riesz summability $|\bar{N}, p_n, \gamma; \delta|_k$ by applying almost Increasing sequence for infinite series. Also, a set of reduced and well-known factor theorems have been obtained under suitable conditions [12].

Dr.Banitamani Mallik et al. have proved a theorem on' 'Summability Factor of Improper Integrals''. In this paper, they have defined the summability for improper integrals generalizing results of Ozgen and Mishra et al. They have established a theorem on indexed absolute Norlund Summability factors of improper integral under sufficient conditions. Some auxiliary results (well-known) have also been deduced from the main result under suitable conditions. However, they established the main result on $|N, p, \delta; \mu|_k$ [13].

Dr.Banitamani Mallik et al. have proved a theorem on ''Equivalence of Two Summability Methods for Improper Integral". In this paper, they have introduced the concept of $|N, p; \delta, \mu|_k, k \geq 1$ summability of improper integrals and by this definition proved a theorem that generalizes theorems of Orhan, Ozgen and Acharya [14].

III. CONCLUSION

The aim of this review paper is to focus on the latest developments and achievements in summability theory such as sequence spaces and their geometry, statistical summability. Many researchers focused on various alternative methods based on the theory of infinite series. These methods have been combined under a single heading which is called Summability Methods. Approximation by trigonometric polynomials is at the heart of approximation theory. Much of the advances in the theory of trigonometric approximation are due to the periodicity of the functions. In recent years, these methods have been used in approximation by linear positive operators. In connection with the concept of statistical convergence and statistical summability, many useful developments have been used in various contexts such as approximation theory, probability theory, quantum mechanics, analytic continuation, Fourier analysis, the behaviors of dynamical systems, the theory of orthogonal series, and fixed point theory.Research is to focus on recent achievements in summability theory such as sequences. Space Spaces and their geometry, statistical summability, compact matrix operators between sequence spaces and infinite systems of differential and integral equations in sequence spaces, and various applications.

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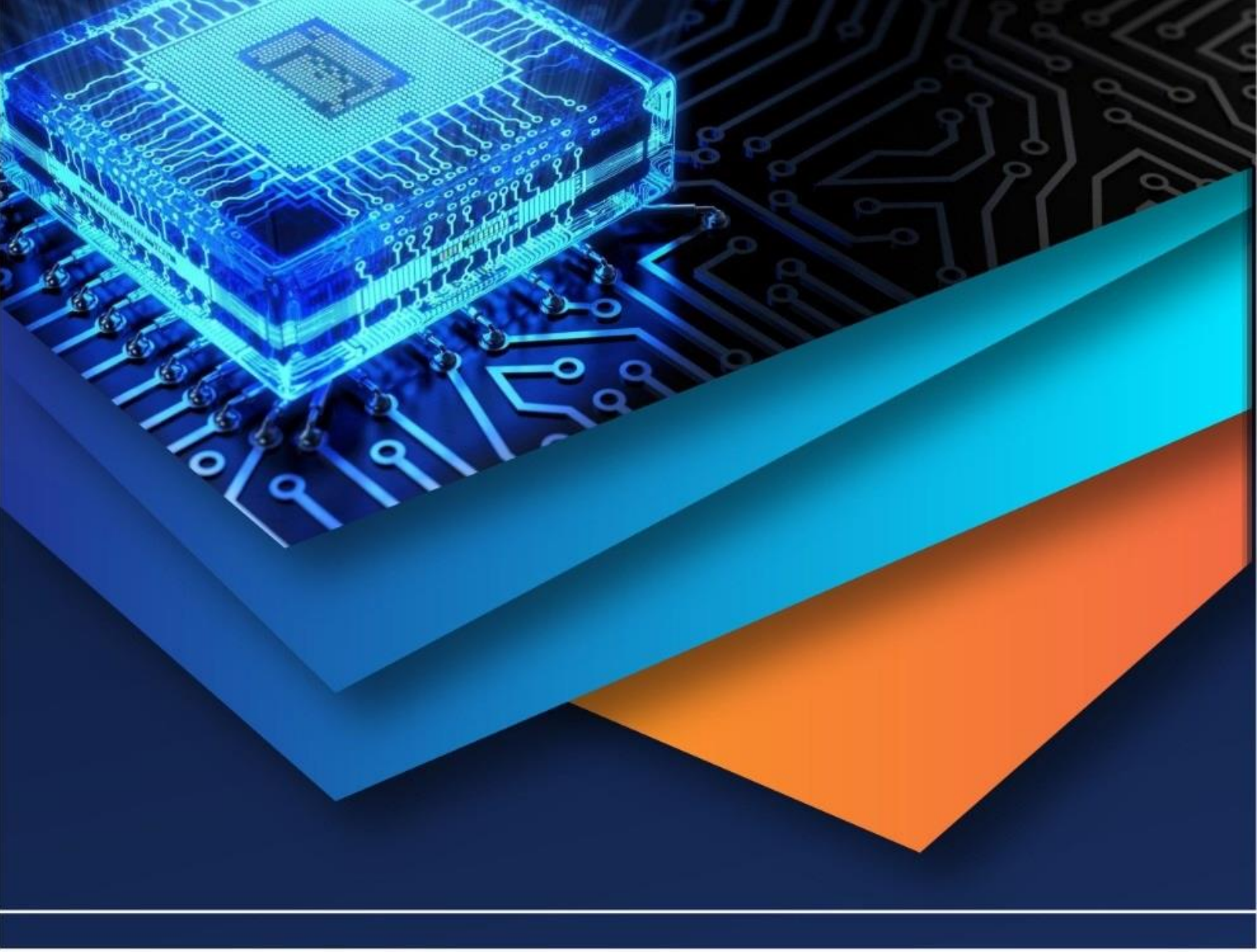
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