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The Non-Homogeneous Quintic Equation with Six Unknowns $x^4 - y^4 = 109(z + w)P^3Q$

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Abstract: The non-homogeneous quintic equation with six unknowns given by $x^4 - y^4 = 109(z + w)P^3Q$ is analyzed for its patterns of non-zero distinct integer solutions.

Keywords: Non - homogeneous Quintic, Quintic with six unknowns, Diophantine equations, Integral solutions, Special numbers.

Notations

Special numbers

Notations

Regular Polygonal Number

$t_{m,n}$

Pronic Number

Pr_n

Pyramidal number

P_n^m

I. INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, Quintic equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity [1-2]. For illustration, one may refer [3-5] for Quintic equation with three unknowns and [6-8] for Quintic equation with five unknowns. This paper concerns with the problem of the non-homogeneous Quintic equation with six unknowns given by $x^4 - y^4 = 109(z + w)P^3Q$. A few relations among the solutions are presented.

II. METHOD OF ANALYSIS

The non-homogeneous quintic equation with six unknowns to be solved for its distinct non-zero integral solution is

$$x^4 - y^4 = 109(z + w)P^3Q \quad (1)$$

Assume

$$x = u + v, y = u - v, z = 2u + v, w = 2u - v, Q = 2v \quad (2)$$

Substituting (2) in (1), it leads to

$$u^2 + v^2 = 109P^3 \quad (3)$$

Different methods of solutions of the above equation are given below.

1) *Method 1*

$$\text{Assume } P = a^2 + b^2 \quad (4)$$

where a and b are non-zero distinct integers.

$$\text{Write 109 as } 109 = (10 + i3)(10 - i3) \quad (5)$$

Substituting (4) and (5) in (3) and applying the method of factorization, define

$$(u + iv)(u - iv) = (10 + i3)(10 - i3)(a + ib)^3(a - ib)^3$$

Equating positive and negative factors, we get

$$u + iv = (10 + i3)(a + ib)^3$$

$$u - iv = (10 - i3)(a - ib)^3$$

Equating the real and imaginary parts in either of the above two equations, we get

$$u = 10a^3 - 9a^2b - 30ab^2 + 3b^3$$

$$v = 3a^3 + 30a^2b - 9ab^2 - 10b^3$$

Hence, in view of (2) and (4), we have

$$x = x(a, b) = 13a^3 + 21a^2b - 39ab^2 - 7b^3$$

$$y = y(a, b) = 7a^3 - 39a^2b - 21ab^2 + 13b^3$$

$$z = z(a, b) = 23a^3 + 12a^2b - 69ab^2 - 4b^3$$

$$w = w(a, b) = 17a^3 - 48a^2b - 51ab^2 + 16b^3$$

$$Q = Q(a, b) = 6a^3 + 60a^2b - 18ab^2 - 20b^3$$

$$P = P(a, b) = a^2 + b^2$$

which satisfy (1).

a) Properties

$$1) x^3 - 3xyQ = y^3 + Q^3$$

$$2) x(a, 1) - 26P_5^a - 16t_{3,a-1} + 7 \equiv 0 \pmod{31}$$

$$3) x(a, 1) - 26P_5^a - t_{18,a} + 7 \equiv 0 \pmod{32}$$

$$4) x(a, 1) - 26P_5^a - 16t_{3,a} + 7 \equiv 0 \pmod{47}$$

Note

In addition to (5), 109 may also be represented as

$$109 = (3 + i10)(3 - i10)$$

Proceeding as in method-1, another set of solutions to (1) is exhibited below:

$$x = x(a, b) = 13a^3 - 21a^2b - 39ab^2 + 7b^3$$

$$y = y(a, b) = -7a^3 - 39a^2b + 21ab^2 + 13b^3$$

$$z = z(a, b) = 16a^3 - 51a^2b - 48ab^2 + 17b^3$$

$$w = w(a, b) = -4a^3 - 69a^2b + 12ab^2 + 23b^3$$

$$Q = Q(a, b) = 20a^3 + 18a^2b - 60ab^2 - 6b^3$$

$$P = P(a, b) = a^2 + b^2$$

2) Method 2

$$\text{Rewrite (3) as } u^2 + v^2 = 109P^3 \quad (6)$$

$$\text{Assume } 1 = \frac{(3 + 4i)(3 - 4i)}{25} \quad (7)$$

Following the analysis presented in method-1,

$$u = \frac{1}{5}[18a^3 - 147a^2b - 54ab^2 + 49b^3] \quad (*)$$

$$v = \frac{1}{5}[49a^3 + 54a^2b - 147ab^2 - 18b^3]$$

Since our interest is on finding integer solutions, replacing a by 5A and b by 5B in (*), (4) and using (2), the corresponding integer solutions to (1) are found to be

$$x = x(A, B) = 1675 A^3 - 2325 A^2 B - 5025 AB^2 + 775 B^3$$

$$y = y(A, B) = -775 A^3 - 5025 A^2 B + 2325 AB^2 + 1675 B^3$$

$$z = z(A, B) = 2125 A^3 - 6000 A^2 B - 6375 AB^2 + 2000 B^3$$

$$w = w(A, B) = -325 A^3 - 8700 A^2 B + 975 AB^2 + 2900 B^3$$

$$Q = Q(A, B) = 2450 A^3 + 2700 A^2 B - 7350 AB^2 - 900 B^3$$

$$P = P(A, B) = 25(A^2 + B^2)$$

a) Properties

$$1) Q(A, 1) - 4900 P_5^A - 500 t_{3,A} + \text{square number} \equiv 0 \pmod{7600}$$

$$2) y(1, B) - 3350 P_5^B - t_{1300,B} + 775 \equiv 0 \pmod{4376}$$

$$3) 4[z(A, 1) - 4250 P_5^A + 6375 Pr_A + 1750 t_{4,A}] \text{ is a cubical integer.}$$

$$4) w(1, B) - 5800 P_5^B + 1925 t_{4,A} + 325 \equiv 0 \pmod{8700}$$

Note: In addition to (7), 1 may also be represented by $1 = \frac{(1+i)^{2n}(1-i)^{2n}}{2^{2n}}$

For this choice, the corresponding integer solutions to (1) are found to be

$$x = x(a, b) = \cos \frac{n\pi}{2} [13a^3 - 39ab^2 - 21a^2b + 7b^3]$$

$$- \sin \frac{n\pi}{2} [7a^3 - 21ab^2 + 39a^2b - 13b^3]$$

$$y = y(a, b) = \cos \frac{n\pi}{2} [-7a^3 + 21ab^2 - 39a^2b + 13b^3]$$

$$- \sin \frac{n\pi}{2} [13a^3 - 39ab^2 - 21a^2b + 7b^3]$$

$$z = z(a, b) = \cos \frac{n\pi}{2} [16a^3 - 48ab^2 - 51a^2b + 17b^3]$$

$$- \sin \frac{n\pi}{2} [17a^3 - 51ab^2 + 48a^2b - 16b^3]$$

$$w = w(a, b) = \cos \frac{n\pi}{2} [-4a^3 + 12ab^2 - 69a^2b + 23b^3]$$

$$- \sin \frac{n\pi}{2} [23a^3 - 69ab^2 - 12a^2b + 4b^3]$$

$$Q = Q(a, b) = \cos \frac{n\pi}{2} [20a^3 - 60ab^2 + 18a^2b - 6b^3]$$

$$- \sin \frac{n\pi}{2} [6a^3 - 18ab^2 - 60a^2b + 20b^3]$$

$$P = P(a, b) = a^2 + b^2$$

3) Method 3

Taking $u = 109^2 U, v = 109^2 V$ & $P = 109R$ (8)

in (3), it becomes $U^2 + V^2 = R^3$ (9)

which is satisfied by

$$U = m(m^2 + n^2) \quad (10)$$

$$V = n(m^2 + n^2)$$

$$R = (m^2 + n^2)$$

Substituting (10) in (8) and using (2), the integer solutions to (1) are given by

$$x = 109^2 (m + n)(m^2 + n^2)$$

$$y = 109^2 (m - n)(m^2 + n^2)$$

$$z = 109^2 (2m + n)(m^2 + n^2)$$

$$w = 109^2 (2m - n)(m^2 + n^2)$$

$$q = 2(109^2)[n(m^2 + n^2)]$$

$$p = 109[m^2 + n^2]$$

a) *Properties*

- 1) $2[z(m, n) - x(m, n) - y(m, n)] - Q(m, n) = 0$
- 2) $z(m, n) - w(m, n) - Q(m, n) = 0$
- 3) $x(m, n) + y(m, n) - z(m, n) + 109P = 0$
- 4) $x(m, n) + y(m, n) - z(m, n) + w(m, n) - Q(m, n) \equiv 0 \pmod{109}$

Note: It is to be noted that (9) is also satisfied by

$$U = (m^3 - 3mn^2)$$

$$V = (3nm^2 - n^3)$$

$$R = (m^2 + n^2)$$

In this case, the integer solutions to (1) are seen to be

$$x = 109^2[m^3 + 3m^2n - 3mn^2 - n^3]$$

$$y = 109^2[m^3 - 3m^2n - 3mn^2 + n^3]$$

$$z = 109^2[2m^3 + 3m^2n - 6mn^2 - n^3]$$

$$w = 109^2[2m^3 - 3m^2n - 6mn^2 + n^3]$$

$$Q = 2[109^2][3nm^2 - n^3]$$

$$P = 109[m^2 + n^2]$$

III. CONCLUSION

In this paper, we have illustrated different methods of obtaining non-zero integer solutions to the quintic equation with six unknowns given by $x^4 - y^4 = 109(z + w)P^3Q$. As the quintic Diophantine equation are rich in variety one may consider other forms of quintic equation with variable ≥ 6 and search for their corresponding integer solutions along with the corresponding properties.

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