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The Non-Homogeneous Quintic Equation with Six Unknowns $x^4 - y^4 = 109(z+w)P^3Q$

[1]Dr. N. Thiruniraiselvi, [2] Dr. M. A. Gopalan
[1]*Assistant Professor, Dept of Mathematics, Nehru Memorial College, Trichy- 621 007*
[2]*Professor, Dept of Mathematics, Shrimati Indira Gandhi College, Trichy-620 002*

Abstract: The non-homogeneous quintic equation with six unknowns given by $x^4 - y^4 = 109(z+w)P^3Q$ is analyzed for its patterns of non-zero distinct integer solutions.
Keywords: Non - homogeneous Quintic, Quintic with six unknowns, Diophantine equations, Integral solutions, Special numbers.

Notations

Special numbers	***Notations***
Regular Polygonal Number	$t_{m,n}$
Pronic Number	Pr_n
Pyramidal number	P_n^m

I. INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, Quintic equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity [1-2]. For illustration, one may refer [3-5] for Quintic equation with three unknowns and [6-8] for Quintic equation with five unknowns. This paper concerns with the problem of the non-homogeneous Quintic equation with six unknowns given by $x^4 - y^4 = 109(z+w)P^3Q$. A few relations among the solutions are presented.

II. METHOD OF ANALYSIS

The non-homogeneous quintic equation with six unknowns to be solved for its distinct non-zero integral solution is

$$x^4 - y^4 = 109(z+w)P^3Q \tag{1}$$

Assume

$$x = u+v, y = u-v, z = 2u+v, w = 2u-v, Q = 2v \tag{2}$$

Substituting (2) in (1), it leads to

$$u^2 + v^2 = 109P^3 \tag{3}$$

Different methods of solutions of the above equation are given below.

1) Method 1

Assume $P = a^2 + b^2$ (4)

where a and b are non-zero distinct integers.

Write 109 as $109 = (10+i3)(10-i3)$ (5)

Substituting (4) and (5) in (3) and applying the method of factorization, define

$$(u+iv)(u-iv) = (10+i3)(10-i3)(a+ib)^3(a-ib)^3$$

Equating positive and negative factors, we get

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$u+iv=(10+i3)(a+ib)^3$

$u-iv=(10-i3)(a-ib)^3$

Equating the real and imaginary parts in either of the above two equations, we get

$u=10a^3-9a^2b-30ab^2+3b^3$

$v=3a^3+30a^2b-9ab^2-10b^3$

Hence, in view of (2) and (4), we have

$x=x(a,b)=13a^3+21a^2b-39ab^2-7b^3$

$y=y(a,b)=7a^3-39a^2b-21ab^2+13b^3$

$z=z(a,b)=23a^3+12a^2b-69ab^2-4b^3$

$w=w(a,b)=17a^3-48a^2b-51ab^2+16b^3$

$Q=Q(a,b)=6a^3+60a^2b-18ab^2-20b^3$

$P=P(a,b)=a^2+b^2$

which satisfy (1).

a) Properties

1) $x^3-3xyQ=y^3+Q^3$

2) $x(a,1)-26P_5^a-16t_{3,a-1}+7\equiv 0(\mathrm{mod}31)$

3) $x(a,1)-26P_5^a-t_{18,a}+7\equiv 0(\mathrm{mod}32)$

4) $x(a,1)-26P_5^a-16t_{3,a}+7\equiv 0(\mathrm{mod}47)$

Note

In addition to (5), 109 may also be represented as

$109=(3+i10)(3-i10)$

Proceeding as in method-1, another set of solutions to (1) is exhibited below:

$x=x(a,b)=13a^3-21a^2b-39ab^2+7b^3$

$y=y(a,b)=-7a^3-39a^2b+21ab^2+13b^3$

$z=z(a,b)=16a^3-51a^2b-48ab^2+17b^3$

$w=w(a,b)=-4a^3-69a^2b+12ab^2+23b^3$

$Q=Q(a,b)=20a^3+18a^2b-60ab^2-6b^3$

$P=P(a,b)=a^2+b^2$

2) Method 2

Rewrite (3) as $u^2+v^2=109P^3*1$ (6)

Assume $1=\frac{(3+4i)(3-4i)}{25}$ (7)

Following the analysis presented in method-1,

$u=\frac{1}{5}[18a^3-147a^2b-54ab^2+49b^3]$ (*)

$v=\frac{1}{5}[49a^3+54a^2b-147ab^2-18b^3]$

Since our interest is on finding integer solutions, replacing a by 5A and b by 5B in (*), (4) and using (2), the corresponding integer solutions to (1) are found to be

$$x = x(A,B) = 1675A^3 - 2325A^2B - 5025AB^2 + 775B^3$$
$$y = y(A,B) = -775A^3 - 5025A^2B + 2325AB^2 + 1675B^3$$
$$z = z(A,B) = 2125A^3 - 6000A^2B - 6375AB^2 + 2000B^3$$
$$w = w(A,B) = -325A^3 - 8700A^2B + 975AB^2 + 2900B^3$$
$$Q = Q(A,B) = 2450A^3 + 2700A^2B - 7350AB^2 - 900B^3$$
$$P = P(A,B) = 25(A^2 + B^2)$$

a) Properties

1) $Q(A,1) - 4900\ p_5^A - 500\ t_{3,A} + square\ \ number\ \equiv 0 \pmod{7600}$
2) $y(1,B) - 3350\ P_5^B - t_{1300,B} + 775 \equiv 0 \pmod{4376}$
3) $4[z(A,1) - 4250\ P_5^A + 6375\ \Pr_A + 1750\ t_{4,A}]$ *is a cubical integer* .
4) $w(1,B) - 5800\ P_5^B + 1925\ t_{4,A} + 325 \equiv 0 \pmod{8700}$

Note: In addition to (7), 1 may also be represented by $1 = \dfrac{(1+i)^{2n}(1-i)^{2n}}{2^{2n}}$

For this choice, the corresponding integer solutions to (1) are found to be

$$x = x(a,b) = \cos \frac{n\pi}{2}[13a^3 - 39ab^2 - 21a^2b + 7b^3]$$
$$- \sin \frac{n\pi}{2}[7a^3 - 21ab^2 + 39a^2b - 13b^3]$$
$$y = y(a,b) = \cos \frac{n\pi}{2}[-7a^3 + 21ab^2 - 39a^2b + 13b^3]$$
$$- \sin \frac{n\pi}{2}[13a^3 - 39ab^2 - 21a^2b + 7b^3]$$
$$z = z(a,b) = \cos \frac{n\pi}{2}[16a^3 - 48ab^2 - 51a^2b + 17b^3]$$
$$- \sin \frac{n\pi}{2}[17a^3 - 51ab^2 + 48a^2b - 16b^3]$$
$$w = w(a,b) = \cos \frac{n\pi}{2}[-4a^3 + 12ab^2 - 69a^2b + 23b^3]$$
$$- \sin \frac{n\pi}{2}[23a^3 - 69ab^2 - 12a^2b + 4b^3]$$
$$Q = Q(a,b) = \cos \frac{n\pi}{2}[20a^3 - 60ab^2 + 18a^2b - 6b^3]$$
$$- \sin \frac{n\pi}{2}[6a^3 - 18ab^2 - 60a^2b + 20b^3]$$
$$P = P(a,b) = a^2 + b^2$$

3) Method 3

Taking $u = 109^2U, v = 109^2V \ \&\ P = 109R$ (8)

in (3), it becomes $U^2 + V^2 = R^3$ (9)

which is satisfied by

$$U = m(m^2 + n^2)$$
$$V = n(m^2 + n^2) \qquad (10)$$
$$R = (m^2 + n^2)$$

Substituting (10) in (8) and using (2), the integer solutions to (1) are given by

$$x = 109^2(m+n)(m^2+n^2)$$
$$y = 109^2(m-n)(m^2+n^2)$$
$$z = 109^2(2m+n)(m^2+n^2)$$
$$w = 109^2(2m-n)(m^2+n^2)$$
$$q = 2(109^2)[n(m^2+n^2)]$$
$$p = 109[m^2+n^2]$$

a) Properties

1) $2[z(m,n)-x(m,n)-y(m,n)]-Q(m,n)=0$
2) $z(m,n)-w(m,n)-Q(m,n)=0$
3) $x(m,n)+y(m,n)-z(m,n)+109P=0$
4) $x(m,n)+y(m,n)-z(m,n)+w(m,n)-Q(m,n)\equiv 0 \pmod{109}$

Note: It is to be noted that (9) is also satisfied by

$$U=(m^3-3mn^2)$$
$$V=(3nm^2-n^3)$$
$$R=(m^2+n^2)$$

In this case, the integer solutions to (1) are seen to be

$$x=109^2[m^3+3m^2n-3mn^2-n^3]$$
$$y=109^2[m^3-3m^2n-3mn^2+n^3]$$
$$z=109^2[2m^3+3m^2n-6mn^2-n^3]$$
$$w=109^2[2m^3-3m^2n-6mn^2+n^3]$$
$$Q=2[109^2][3nm^2-n^3]$$
$$P=109[m^2+n^2]$$

III. CONCLUSION

In this paper, we have illustrated different methods of obtaining non-zero integer solutions to the quintic equation with six unknowns given by $x^4-y^4=109(z+w)P^3Q$. As the quintic Diophantine equation are rich in variety one may consider other forms of quintic equation with variable ≥ 6 and search for their corresponding integer solutions along with the corresponding properties.

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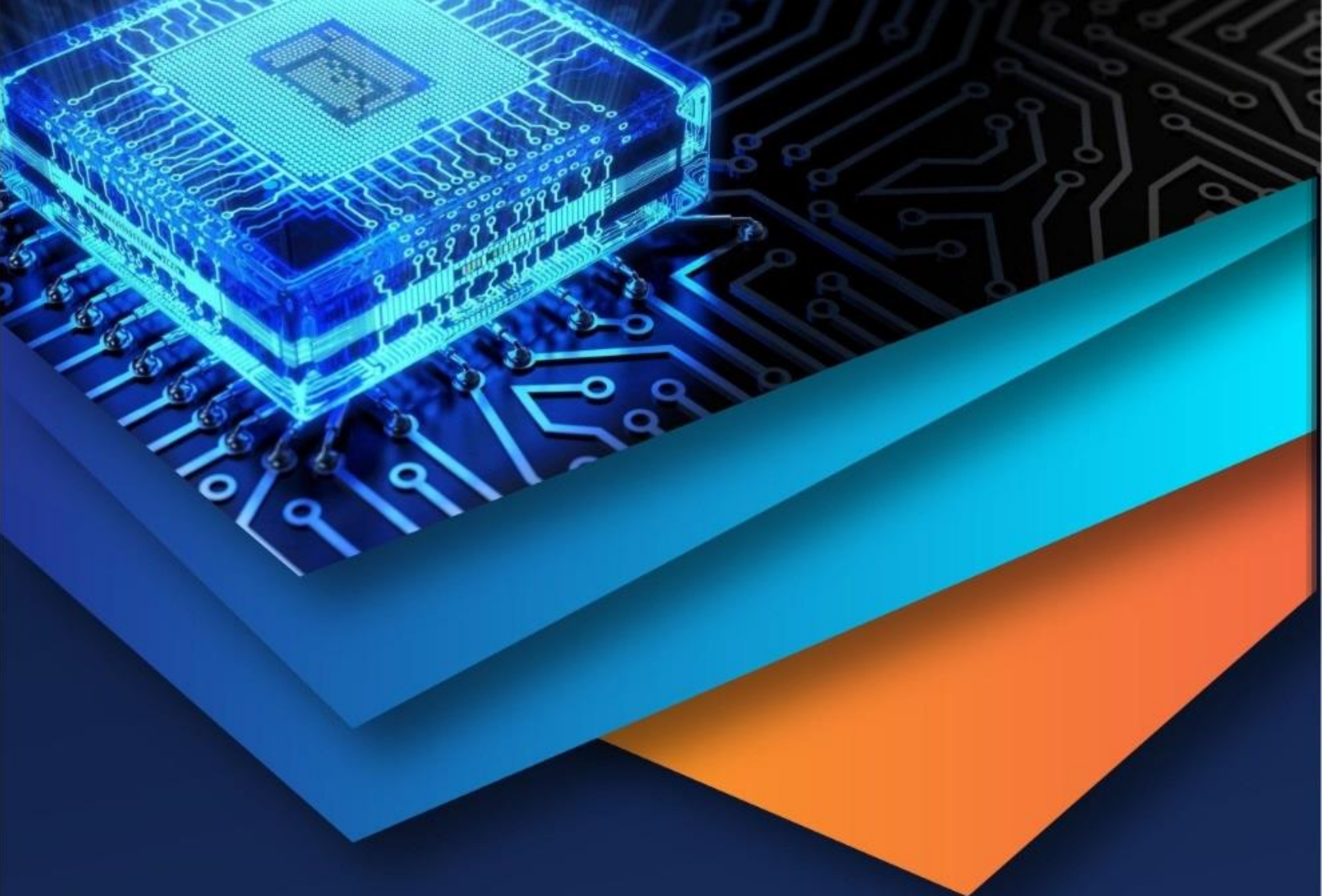

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