iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 7 Issue: VII Month of publication: July 2019

DOI: http://doi.org/10.22214/ijraset.2019.7155

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
*ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.177*
*Volume 7 Issue VII, July 2019- Available at www.ijraset.com*

# Critical Loads in Reinforced Concrete Beams and Columns from Force Equilibrium Approach

Ibearugbulem. Owus M.[1], Ibearugbulem. C. N.[2], Nwachukwu. A. N.[3]
[1]*Senior Lecturer, Department of Civil Engineering, Federal University of Technology, Owerri Nigeria*
[2]*Research Scholar, Department of Civil Engineering, Federal University of Technology, Owerri Nigeria*
[3] *Lecturer, Department of Civil Engineering, Federal University of Technology, Owerri Nigeria*

***Abstract: This paper presents Critical loads in reinforced concrete beams and columns from force equilibrium approach (FEA). It modified the stress block given by BS 8110 to obtain the FEA stress block, whose neutral axis is always half of the effective depth. Limits of stresses on concrete and steel as provided by BS 8110 were adhered to. By making use of the limits of moment coefficient, k, lever arm, z and neutral axis provided by BS 8110, this paper obtains a stress factor, s which is to be used on the FEA stress block. Formulas for calculating areas of compression and tension reinforcements for beams and columns are determined based on the axiom that at all times, before failure, total force in the compression zone is in equilibrium with total force in tension zone. Formulas for calculating the critical imposed loads on beams are determined. Any imposed load more than the critical value will result to violation of the stress and deflection limits. Numerical problems are solved. The values of areas of steel reinforcement from FEA and BS 8110 are compared. It is observed that FEA values are always upper bound to BS 8110 value with average percentage difference that is less than 10%. For a beam with effective depth of 450mm, the span up of 7960mm can only support self weight without deflection exceeding 15mm limit. Above this span beam without imposed load will deflect more than 15mm.***
***Keywords: Critical load; reinforced concrete; beam; column; stress block; imposed load ; reinforcement; deflection***

## I. INTRODUCTION

Earlier works on reinforced concrete design are based on equilibrium bending moments in the cross section, hereinafter referred to moment equilibrium approach. This is approach used by British, Europe, America, India and South Africa standards ([1], [2], [3], [4] and [5]). Reference [6] presented a study titled “alternative method for flexural ultimate limit state design of reinforced concrete”, which is is based on stress equilibrium. Even though there merit in their study, it however presents the same problem of iterative approach that characterizes the moment equilibrium approach. The present study is trying to overcome this iterative design algorithm, which is based on trial and error. A robust algorithm with optimization mechanism is sought. In reinforced concrete design, the sought parameters include quantity of reinforcement needed per cross section dimensions, and the imposed load that will not led to stresses and deflections, which exceed the limit values. Most of these earlier approaches require experience on the side of the designer and iteration to optimize the design. Hence, the evolution of a robust design procedure and equations that can optimize the reinforced concrete design of rectangular cross section beams and columns is the primary objective of the present study.

## II. METHODOLOGY

### *A. Singly Reinforced Beam Of Rectangular Cross Section*

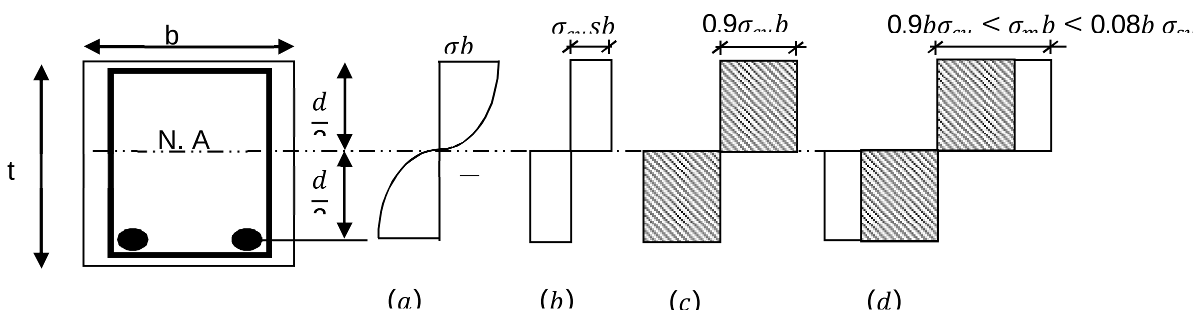



Figure 1: FEA strength diagram of a reinforced concrete beam

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.177
Volume 7 Issue VII, July 2019- Available at www.ijraset.com

The flexural strength of concrete is given as a function of compressive cube strength by BS 8110 - 1 (1997) in clause 2.5.3 and Figure 2.1 of the same code. The flexural strength of concrete is defined as:

$$\sigma_{cu} = 0.67 \frac{f_{cu}}{\gamma_m} \qquad 1$$

In Equation 1, 0.67 in Equation 13 is a coefficient, which is used to convert concrete cube strength to bending strength of a concrete member. γm is the concrete material factor of safety given on Table 2.2 of the code as 1.5. Substituting the safety factor of 1.5 into Equation 13 gives:

$$\sigma_{cu} = 0.447\, f_{cu} \qquad 2$$

For steel reinforcement bar used in concrete in flexure, the allowable stress is defined as:

$$\sigma_{su} = \frac{f_y}{\gamma_m} \qquad 3$$

The material safety factor for steel reinforcement is given on Table 2.2 of the code as 1.05. Substituting this value of factor of safety into Equation 15 gives:

$$\sigma_{su} = 0.952\, f_y \qquad 4$$

The stress in a rectangular section under flexure (bending) is defined mathematically as:

$$\sigma = \frac{6M}{bd^2} \qquad 5$$

Moment coefficient is given by BS 8110 – 1 (1997) in clause 3.4.4.4 as:

$$k = \frac{M}{f_{cu} bd^2} = \frac{0.447\, M}{\sigma_{cu} bd^2} \qquad 6$$

Rearranging Equation 6 gives:

$$f_{cu} = \frac{M}{kbd^2} \qquad 7$$

In the same clause 3.4.4.4 of BS 8110 – 1 (1997), the lever (moment) arm to depth ratio is defined as:

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{k}{0.9}} \qquad 8$$

Rearranging Equation 8 and making k the subject gives:

$$k = 0.9\left[0.25 - \left(\frac{z}{d} - 0.5\right)^2\right] \qquad 9$$

The domain of lever (moment) arm allowed by clause 3.4.4.4 of BS 8110 – 1 (1997) is:

$$0.775 \le \frac{z}{d} \le 0.95 \qquad 10$$

Substituting the limits of Equation 10 into Equation 9 gives the domain of k as:

$$0.0428 \le k \le 0.15694 \qquad 11$$

Dividing Equation 5 by Equation 7 gives"

$$\frac{\sigma}{f_{cu}} = 6k \qquad 12$$

Substituting the limits of Equation 11 into Equation 12 gives the following limits:

$$0.2565 \le \frac{\sigma}{f_{cu}} \le 0.9416 \qquad 13$$

Substituting Equation 2 into Equation 13 gives:

$$0.574 \le \frac{\sigma}{\sigma_{cu}} \le 2.107 \qquad 14$$

From Equations 13 and 14, the allowable stress in concrete is 2.107 $\sigma_{cu}$ or 0.9416 $f_{cu}$ (depending on the parameter one wants to use). When the applied stress is more than the allowable stress, then the excess must be borne by reinforcement steel. Furthermore, the clause 3.4.4.4 of BS 8110 – 1 (1997) give the neutral axis depth as function of lever arm as:

$$x = \frac{d - z}{0.45} \qquad 15$$

Substituting Equation 10 into Equation 15 give the domain of the neutral axis as:

$$0.1111 \leq \frac{x}{d} \leq 0.5 \qquad 16$$

Figure 3.3 of BS 8110 – 1 (1997) gives the depth of stress block as:

$$g \leq 0.9x \qquad 17$$

Substituting Equation 17 into Equation 16 gives:

$$0.1 \leq \frac{g}{d} \leq 0.45 \qquad 18$$

However, the present study is taking the ratio of g to d (g/d) as half the stress factor, s. Thus, the domain of stress factor for concrete is:

$$0.1 \leq \frac{s}{2} \leq 0.45 \qquad 19a$$

Multiplying Equation 19a by two gives:

$$0.2 \leq s \leq 0.9 \qquad 19b$$

When the stress factor (s) is more than 0.9, then the section has reached the maximum stress the concrete can bear. The excess of the stress shall be borne by the reinforcement steel. Using Equations 14 and 19b, linear relationships between stress factor and stress ratio are obtained as:

$$s = 0.457 \frac{\sigma}{\sigma_{cu}} - 0.062 \qquad 20a$$

$$\frac{\sigma}{\sigma_{cu}} = 2.188\, s + 0.136 \qquad 20b$$

When the beam is loaded laterally, flexural stress, σ is developed. The stress block is as shown on Figure 1 (a). This is parabolic stress block since the material is concrete. In this study, the parabolic stress block is converted to equivalent rectangular stress block as shown on Figure 1 (b). The modified stress, $\sigma_m$ due to change of stress block from parabolic to rectangular block is the product of the flexural strength and the stress factor given as:

$$\sigma_m = \sigma_{cu}\, s \qquad 21a$$

Replacing the stress factor with 0.9 in Equation 21a gives the limiting modified stress, beyond which reinforcement is needed. This limiting modified stress is:

$$\sigma_{m\ allow} = 0.9\, \sigma_{cu} \qquad 21b$$

Total compressive force above the neutral axis (N. A.) as shown on Figure 1 (b) is the area of the modified stress block given as:

$$F_c = \sigma_{cu} sb \times \frac{d}{2} = 0.5\, \sigma_{cu} bds \qquad 22$$

The cross section is at all times in equilibrium. Thus, the total tensile force below the neutral axis (N. A.) as shown on Figure 1 (b) is the area of the stress block given as:

$$F_t = 0.5\, \sigma_{cu} bds \qquad 23$$

The total force resisted by the steel rods as a result of the force below the neutral axis (as presented on Equation 23) is the product of area and allowable stress of reinforcement:

$$F_{ts} = F_t = \sigma_{su} A_{ts} \qquad 24$$

The maximum area of reinforcement allowed in either of compression or tension reinforcement is giving in clause 3.12.6 BS 8110 – 1 (1997) as 4% percent of gross area of the cross section. However, the present study is limiting the maximum area of reinforcement allowed in either of compression or tension reinforcement to be 4% of the net area of the cross section giving as:

$$A_{ts} \leq 0.04\, bd \qquad 25$$

Substituting Equation 23 into Equation 24 and making area of steel the subject gives:

$$A_{ts} = 0.5 \frac{\sigma_{cu}}{\sigma_{su}} bds \qquad 26$$

Comparing Equations 25 and 26 gives limiting stress factor as:

$$s_{max} = 0.08 \frac{\sigma_{su}}{\sigma_{cu}} \qquad 27$$

Doubly reinforced beam of rectangular cross section

Figure 1 (c) shows the stress below which compression reinforcement is not needed and above which compression reinforcement is needed. When compression reinforcement is needed, figure 1(d) is used. In this case, the force ($F_{cc}$) resisted by the concrete in the compression zone is less than the applied force ($F_c$) in the zone. Substituting Equation 21b into Equation 22 gives the maximum force concrete can resist:

$$F_{cc} = 0.5\,\sigma_{cu} \times 0.9bd = 0.45\,\sigma_{cu}bd \qquad 28$$

Subtracting Equation 28 from Equation 22 gives the force to be resisted by compression reinforcement as:

$F_{cs} = 0.5\,\sigma_{cu}sbd - 0.45\,\sigma_{cu}bd$. That is:

$$F_{cs} = 0.5\sigma_{cu}bd(s - 0.9) \qquad 29$$

The total force exerted on the compression steel rods is the product of area and allowable stress of reinforcement:

$$F_{cs} = \sigma_{su}A_{cs} \qquad 30$$

Equating Equations 29 and 30 and making the area of compression reinforcement the subject gives:

$$A_{cs} = 0.5\frac{\sigma_{cu}}{\sigma_{su}}bd(s - 0.9) \qquad 31$$

Reinforced column of rectangular cross section

A column is a member subject to both axial and flexural loads. This is called combined stress. That is combination of axial compressive stress and flexural stress on the column cross section. The equations that support flexural load have been determined in Equations 22, 23, 24 and 30. The next thing to be done is to determine the equations that support the axial load. This is done by assuming the cross section is in pure axial compression. The code (BS 8110 – 1, 1997) in clause 3.8.4 provided two equations (Equations 38 and 39), which are reproduced here as Equations 32 and 33.

$$N = 0.4\,f_{cu}A_c + 0.8f_yA_s \qquad 32$$

$$N = 0.35\,f_{cu}A_c + 0.7f_yA_s \qquad 33$$

For more conservativeness, Equation 33 is adopted to the present design. The cross section is assumed to be symmetric such that compressive zone is the mirror image of tensile zone. Equation 33 is written more precisely in terms of material areas in compression and tension as;

$$N = 0.35\,f_{cu}(A_{cc} + A_{tc}) + 0.7f_y(A_{cs} + A_{ts}) \qquad 34$$

If each zone is a mirror image of the other, then two equations are obtained from Equation 34 as:

$$0.5N = 0.35\,f_{cu}A_{cc} + 0.7f_yA_{cs} = 0.175\,f_{cu}bd + 0.7f_yA_{cs} \qquad 35$$

$$0.5N = 0.35\,f_{cu}A_{tc} + 0.7f_yA_{ts} = 0.175\,f_{cu}bd + 0.7f_yA_{ts} \qquad 36$$

Rearranging Equations 35 and 36 gives:

$$0.7f_yA_{cs} = 0.5N - 0.175\,f_{cu}bd \qquad 37$$

$$0.7f_yA_{ts} = 0.5N - 0.175\,f_{cu}bd \qquad 38$$

The implication of Equation 37 is:

$$F_{Ncc} = 0.5N - 0.175\,f_{cu}bd \qquad 39$$

$$F_{Ncs} = 0.7f_yA_{cs} \qquad 40$$

In the same way, implication of Equation 38 is:

$$F_{Ntc} = 0.5N - 0.175\,f_{cu}bd \qquad 41$$

$$F_{Nts} = 0.7f_yA_{st} \qquad 42$$

The combined force in the compression zone is obtained by adding Equations 22 and 39. That is:

$$F_c + F_{Ncc} = 0.5\,\sigma_{cu}bds + 0.5N - 0.175\,f_{cu}bd \qquad 43a$$

Substituting Equation 2 into Equation 43a gives:

$$F_c + F_{Ncc} = 0.5\sigma_{cu}bd(s - 0.783) + 0.5N \qquad 43b$$

Similarly, the combined force in the tension zone is obtained by subtracting Equation 23 from Equation 41. That is:

$$F_{Ntc} - F_t = 0.5N - 0.175\,f_{cu}bd - 0.5\,\sigma_{cu}bds \qquad 44a$$

Substituting Equation 2 into Equation 44a gives:

$$F_{Ntc} - F_t = 0.5N - 0.5\,\sigma_{cu}bd(s + 0.783) \qquad 44b$$

The resistant force in compression zone is the average of Equations 30 and 40. That is:

$$F_{csa} = \frac{\sigma_{su}A_{cs} + 0.7f_yA_{cs}}{2} \qquad 45$$

In the same way, the resistant force in tension zone is the average of Equations 24 and 42. That is:

$$F_{tsa} = \frac{\sigma_{su}A_{ts} + 0.7f_yA_{st}}{2} \qquad 46$$

Substituting Equation 4 into Equations 45 and 46 gives:

$$F_{csa} = \frac{\sigma_{su}A_{ts} + (0.7/0.952)\sigma_{su}A_{cs}}{2} = 0.8676\sigma_{su}A_{cs} \quad 47$$

$$F_{tsa} = 0.8676\sigma_{su}A_{ts} \quad 48$$

For equilibrium of force in the compression zone, Equations 43a and 47 must be equal as:

$$0.8676\sigma_{su}A_{cs} = 0.5[N + \sigma_{cu}bd(s - 0.783)] \quad 49$$

For equilibrium of force in the tension zone, Equations 44a and 47 must be equal as:

$$0.8676\sigma_{su}A_{ts} = 0.5[N - \sigma_{cu}bd(s + 0.783)] \quad 50$$

Rearranging Equation 49 and making the area of reinforcement steel the subject gives:

$$A_{cs} = 0.5763\left[\frac{N}{\sigma_{su}} + \frac{\sigma_{cu}}{\sigma_{su}}bd(s - 0.783)\right] \quad 51$$

Negative result indicates that reinforcement is not needed. Similarly, rearranging Equation 50 and making the area of reinforcement steel the subject gives:

$$A_{ts} = 0.5763\left[\frac{N}{\sigma_{su}} - \frac{\sigma_{cu}}{\sigma_{su}}bd(s + 0.783)\right] \quad 52$$

Negative result indicates that flexural load is more that axial load and does not mean that reinforcement is not needed. Hence, the absolute value is taken. Moment stress resultant for beam is commonly defined as:

$$\sigma_x = E\varepsilon_x \quad 53$$

From the work of Reference [7], the deflection and normal strain for RBT3 (one of the two refined beam theories they presented) is:

$$w = A_1h \quad 54$$

$$\varepsilon_x = A_3.z\frac{d^2h}{dx^2} \quad 55a$$

$$\varepsilon_R = A_3.\frac{St}{L^2}.\frac{d^2h}{dR^2} \quad 55b$$

Where:

$$A_1 = \frac{k_3\left(k_1 + \frac{6(L/t)^2}{1+\mu}k_2\right)}{\frac{6(L/t)^2}{1+\mu}k_1k_2}.\frac{qL^4}{D_1} \quad 56$$

$$A_3 = -\frac{k_3}{k_1}.\frac{qL^4}{D_1} \quad 57$$

$$D_1 = \frac{Ebt^3}{12} \quad 58$$

Where h is shape function taken from the work of Reference [8]. The shape functions and their maximum numerical values are presented on Table 1. On the other hand the numerical values of stiffness coefficient, $k_1$, $k_2$ and $k_3$ for beams of various boundary conditions are presented on Table 2. Substituting Equation 58 into Equation 57 gives:

$$A_3 = -12\frac{k_3}{k_1}.\frac{qL^4}{Ebt^3} = -\frac{1}{2}.\frac{qL^4}{Ebt^3} \quad 59$$

Where $k_1/k_3$ =24 for all flexural boundary conditions of beam (see Table 2). Substituting Equation 55b into Equation 53 gives:

$$\sigma_R = E.A_3.\frac{St}{L^2}.\frac{d^2h}{dR^2} \quad 60$$

Substituting Equation 59 into Equation 60 gives:

$$\sigma_R = -\frac{S}{2b}.q(L/t)^2.\frac{d^2h}{dR^2} \quad 61$$

Maximum stress occurs at either the bottom surface or the top surface where absolute numerical value of R is 0.5. Substituting S = 0.5 into Equation 61 gives:

$$\sigma_R = -\frac{1}{4b}.q(L/t)^2.\frac{d^2h}{dR^2} \quad 62$$

Substituting Equation 58 into Equation 56 and rearranging gives:

$$A_1 = 2(1+\mu).(L/t).\left(\frac{k_1k_3}{k_1k_2} + \frac{6(L/t)^2}{1+\mu}\frac{k_2k_3}{k_1k_2}\right).\frac{qL}{Eb} = 2(1+\mu).\left(\frac{k_1k_3}{k_1k_2} + \frac{(L/t)^2}{4(1+\mu)}\right).\frac{qL(L/t)}{Eb} \quad . \text{ That is:}$$

$$A_1 = \left[2(1+\mu).\left(\frac{k_1k_3}{k_1k_2}\right) + \frac{(L/t)^2}{2}\right].\frac{qL(L/t)}{Eb} \qquad 63$$

Substituting Equation 63 into Equation 54 and making load the subject gives:

$$q = \frac{1}{(L/t).k_w.h}.\frac{wEb}{L} \qquad 64$$

$$w = \frac{q(L/t).k_w.hL}{Eb} \qquad 64$$

Where:

$$k_w = 2(1+\mu).\left(\frac{k_1k_3}{k_1k_2}\right) + 0.5(L/t)^2 \qquad 65$$

Let the allowable deflection be denoted as wall. Also let the imposed and self weight be denoted as $q_i$ and $q_s$ respectively. With these denotations, a formula for critical imposed load, $q_{iw}$ on the beam before allowable deflection is reached is obtained using Equation 64:

$$q_{icw} = \frac{1}{(L/t).k_w.h_{max}}.\frac{w_{all}.E_c.b}{L} - q_s \qquad 66$$

Where:

$$q_s = \gamma.b.t \qquad 67$$

Gamma $\gamma$ is the unit weight. In a similar manner, the yield stress is denoted as $\sigma_{cu}$. By rearranging Equation 62, a formula for critical imposed load, $q_{if}$ on the beam before yield stress is reached is obtained as:

$q_{icf}$ is difference between absolute value of $\left[\frac{4\,b\,\sigma_{cu}}{(L/t)^2.h''_{max}}\right]$ and self weight. That is:

$$q_{icf} = \left|\frac{4\,b\,\sigma_{cu}}{(L/t)^2.h''_{max}}\right| - q_s = \left|\frac{4\,b\,\sigma_{cu}}{(L/t)^2.h''_{max}}\right| - \gamma.b.t \qquad 68$$

Rearranging Equations 27 and making flexural strength of concrete subject gives:

$$\sigma_{cu} = 0.08\,\frac{\sigma_{su}}{s_{max}} \qquad 69$$

Substituting Equation 69 into Equation 68 and making some rearrangements gives:

$$q_{icf} = b\left(\left|\frac{0.32\ \sigma_{su}}{(L/t)^2.h''_{max}.s_{max}}\right| - \gamma.t\right) \qquad 70$$

Rearranging Equation 53 and making the strain the subject, allowable strains in steel and concrete can be written as:

$$\varepsilon_{su} = \frac{\sigma_{su}}{E_s} \qquad 71$$

$$\varepsilon_{cu} = \frac{\sigma_{cu}}{E_c} \qquad 72$$

Since there is no relative movement between the concrete and steel in the reinforced concrete beam before failure then the total strain in concrete is the same with that of steel. Hence, Equation 71 is the same as Equation 72. That is:

$$\frac{\sigma_{su}}{E_s} = \frac{\sigma_{cu}}{E_c} \qquad 73$$

Rearranging Equation 73 gives:

$$E_c = \frac{\sigma_{cu}}{\sigma_{su}}E_s \qquad 74$$

Substituting Equation 74 into Equation 66 and making some rearrangements gives:

$$q_{icw} = b\left(\frac{w_{all}.E_s.b}{(L/t).k_w.h_{max}.L}.\frac{\sigma_{cu}}{\sigma_{su}} - \gamma.t\right) \qquad 75$$

The least value among the critical loads on Equations 70 and 75 is the desired critical load. These two equations were used to graphs critical L/t (span to thickness ratios) versus beam thickness. This critical L/t is the L/t above which deflection based critical imposed load equation (Equation 75) is desired and below which stress based critical imposed load equation (Equation 70) is desired. The graphs are presented on Figures 2, 3, 4 and 5. For allowable deflections of 10mm, 15mm, 20mm and 25mm the respective critical L/t equations are:

$L/t = 147.6674\,(t^{-0.5017})$ 76

$L/t = 180.2782(t^{-0.5013})$ 77

$L/t = 209.2818\,(t^{-0.5021})$ 78

$L/t = 232.8318\,(t^{-0.5014})$ 79

Table 1: Shape functions and its derivatives and their maximum numerical values

| Line continuum | h | h' | h'' | h''' |
|---|---|---|---|---|
| S – S | $R - 2R^3 + R^4$<br>$h_{max} = 0.3125$ | $1 - 6R^2 + 4R^3$<br>$h'_{max} = 1\ or\ -1$ | $12(R^2 - R)$<br>$h''_{max} = -3$ | $12(2R - 1)$<br>$h'''_{max} = 12 or\ -12$ |
| C - C | $R^2 - 2R^3 + R^4$<br>$h_{max} = 0.0625$ | $2R - 6R^2 + 4R^3$<br>$h'_{max} = 0.19245$ | $2 - 12R + 12R^2$<br>$h''_{max} = 2$ | $12(2R - 1)$<br>$h'''_{max} = 12\ or\ -12$ |
| C - S | $1.5R^2 - 2.5R^3 + R^4$<br>$h_{max} = 0.12999$ | $3R - 7.5R^2 + 4R^3$<br>$h'_{max} = -0.5$ | $3 - 15R + 12R^2$<br>$h''_{max} = 3$ | $3(8R - 5)$<br>$h'''_{max} = -15$ |
| C – F (bending) | $6R^2 - 4R^3 + R^4$<br>$h_{max} = 3$ | $12R - 12R^2 + 4R^3$<br>$h'_{max} = 4$ | $12 - 24R + 12R^2$<br>$h''_{max} = 51$ | $24(R - 1)$<br>$h'''_{max} = -24$ |
| C – F (buckling) | $-8R^2 + 2/3\,R^3 + R^4$<br>$h_{max} = $ n.a. | $-16R + 2R^2 + 4R^3$<br>$h'_{max} = $ n.a. | $-16 + 4R + 12R^2$<br>$h''_{max} = $ n.a. | $4(8R - 1)$<br>$h'''_{max} = $ n.a. |

n.a. means not applicable

Table 2: Values of stiffness coefficient for beams of various boundary conditions

| Line continuum | $k_1$ | $k_2$ | $k_3$ | $k_1/k_2$ | $k_1/k_3$ | $k_1.k_2$ | $k_1.k_3$ | $k_2.k_3$ | $\frac{k_1.k_3}{k_1.k_2}$ | $\frac{k_2.k_3}{k_1.k_2}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| S – S | 4.8 | 17/35 | 1/5 | 9.8824 | 24 | 2.33143 | 0.96 | 0.09714 | 0.411765 | 1/24 |
| C - C | 4/5 | 2/105 | 1/30 | 42 | 24 | 0.01524 | 0.026667 | 0.00063 | 1.75 | 1/24 |
| C - S | 9/5 | 3/35 | 3/40 | 21 | 24 | 0.15429 | 0.135 | 0.00643 | 0.875 | 1/24 |
| C – F (bending) | 144/5 | 72/7 | 6/5 | 104/45 | 24 | 296.229 | 34.56 | 12.3429 | 0.116667 | 1/24 |
| C – F (buckling) | 1832/15 | 1732/35 | n.a. | 2.4681 | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |

n.a. means not applicable

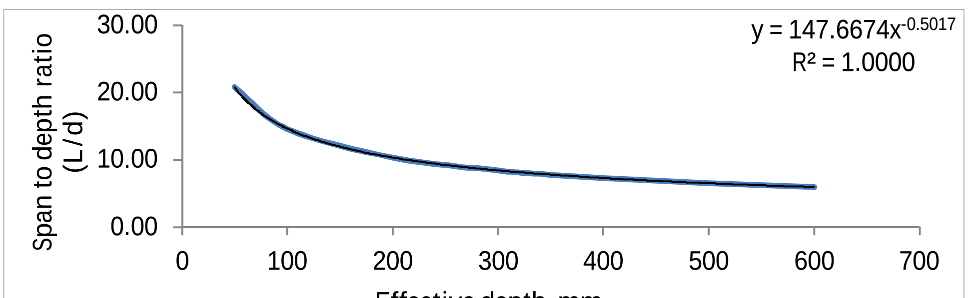


Figure 2: Critical L/d for 10mm allowable deflection

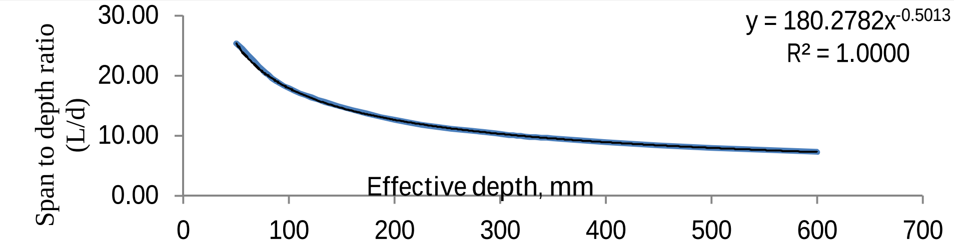



Figure 3: Critical L/d for 15mm allowable deflection

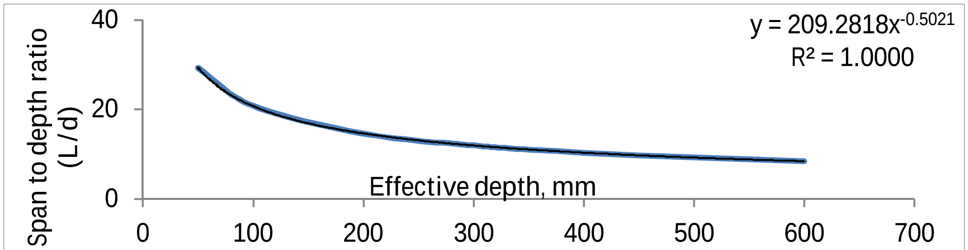



Figure 4: Critical L/d for 20mm allowable deflection

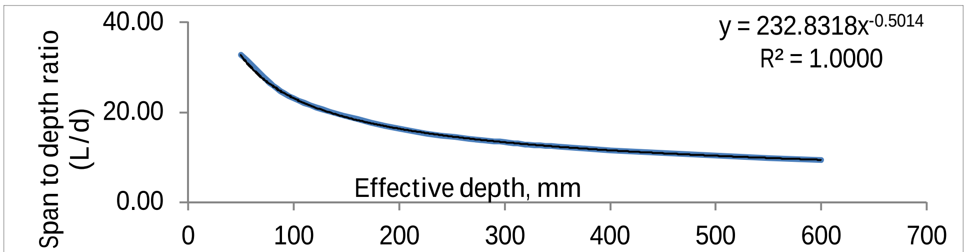



Figure 5: Critical L/d for 25mm allowable deflection

### *B. Numerical Problems*

It is required to determine the quantity of compression and tension reinforcement steel bars for rectangular cross sectional beams of various properties and loads using Force Equilibrium Approach (FEA) presented in this paper and the Moment Equilibrium Approach given by BS 8110 – part 1 (1997). It is also required to determine the compression and tension reinforcement steel bars for rectangular cross sectional columns of various properties and loads using Force Equilibrium Approach (FEA).

It is required to determine the maximum imposed load on a simply supported reinforced rectangular cross section beam such that neither the allowable stress in concrete nor allowable deflection is exceeded. The modulus of elasticity of steel is 200, 000N/mm$^2$. The allowable stresses in concrete and steel are $\sigma_{cu}$ = 0.447 $f_{cu}$ and $\sigma_{su}$ = $f_y$/1.05 respectively, and allowable deflection is 15mm. unit weight of reinforced concrete is 24 kN/m$^3$.

It is required to critical imposed loads on reinforced concrete beams whose spans ranges from 1000mm to 7000mm, effective depth is 450mm and width is 225mm. The allowable deflection is 15mm and the reinforcement steel high yield steel (with 460MPa strength and Es = 200 kN/mm$^2$). The compressive cube strength of the concrete is 25 kN/mm$^2$ and unit weight of reinforced concrete is 24 kN/m$^3$.

## III. RESULTS AND DISCUSSIONS

The results of beam design are presented on Tables 3, 4 and 5. It is observed from the tables that the quantities of steel rods from FEA are upper bound to those from MEA. This implies that FEA does not underestimate quantity of reinforcement required in beams for various loads. The average percentage differences between the results from FEA and MEA as seen on Tables 3, 4 and 5 are 6.32, 7.21 and 8.76 respectively. One can say that MEA is more economical in terms of cost of steel where as FEA presents higher factor of safety in terms of cost of steel. Although economy and safety are veritable parameters in design, safety take

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
*ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.177*
*Volume 7 Issue VII, July 2019- Available at www.ijraset.com*

precedence over economy. Another good feature of FEA is the ease simplicity of its calculations with very simple formulas when compared with MEA.

The results for column design are presented on Tables 6 and 7. It is observed that when either the axial load or the bending moment is kept constant and the other is allowed to vary, compression reinforcement always increases as the varying load increases. On the other hand, the tension reinforcement decreases as the varying load increases to a point (where axial forces from axial load becomes equal to axial force from bending moment) and starts to increase.

A good observation here is the simplicity of FEA in rectangular section column design. This makes designing rectangular cross sectional column very easy. This is unlike using MEA which most times requires the use design charts. Charts are not amenable to computer; hence, it will be difficult to program design of rectangular column whose design is based on design charts.

The critical imposed loads on the beams are presented on the fifth column of Table 8. This critical imposed load is the smaller of the values presented on the third and fourth columns of the table. With these critical loads, the stresses and deflections of the beams are at worst equal to the allowable values. For span to depth ratios less than 7.8, any imposed load more than the critical value will result into stresses more than the allowable stress of 11.175 MPa. On the other hand, for L/d up to or more than 8.9, any imposed load more than the critical imposed load will result into deflections exceeding the allowable limit of 15mm. An observation that is worthy of note is that for this beam whose effective depth is 450mm, the tolerable length and L/d are respectively 7960mm and is 17.69. When the span exceeds 7960mm, the beam deflection will exceed 15mm under only self weight.

Table 3: Quantities of steel rods for rectangular section beam with b = 250mm, h = 350mm, c = 25mm, rebar = Y16, link = R8, $f_{cu}$ = 30 Mpa, $A_{smin}$ = 100.1mm$^2$, $A_{smax}$ = 2719.2mm$^2$

| | FORCE | | BS 8110 | |
|---|---|---|---|---|
| M | Ast | Asc | Ast | Asc |
| 26.94 | 208.3336 | 0 | 210.0072 | 0 |
| 30 | 239.3243 | 0 | 235.3562 | 0 |
| 50 | 441.8781 | 0 | 410.3758 | 0 |
| 70 | 644.4319 | 0 | 605.7556 | 0 |
| 90 | 846.9857 | 0 | 830.8637 | 0 |
| 120 | 1150.816 | 214.4326 | 1124.61 | 185.2246 |
| 160 | 1555.924 | 619.5403 | 1466.152 | 526.7663 |
| 200 | 1961.032 | 1024.648 | 1807.694 | 868.308 |
| 240 | 2366.139 | 1429.756 | 2149.235 | 1209.85 |
| 270 | 2669.97 | 1733.586 | 2405.392 | 1466.006 |

Legend: b = width of section, h = height of section, c = concrete cover to reinforcement, rebar = reinforcement bar, $A_{smin}$ = minimum area of reinforcement, $A_{smax}$ = maximum area of reinforcement

Table 4: Quantities of steel rods for rectangular section beam with b = 250mm, h = 300mm, c = 25mm, rebar = Y16, link = R8, $f_{cu}$ = 25 Mpa, $A_{smin}$ = 85.8mm$^2$, $A_{smax}$ = 2279.2mm$^2$

| | FEA | | BS 8110 | |
|---|---|---|---|---|
| M | Ast | Asc | Ast | Asc |
| 15.75 | 145.2477 | 0 | 146.4676 | 0 |
| 20 | 196.5998 | 0 | 188.8632 | 0 |
| 32.5 | 347.6353 | 0 | 322.6246 | 0 |
| 45 | 498.6709 | 0 | 474.2502 | 0 |
| 57.5 | 649.7064 | 0 | 653.6721 | 0 |
| 70 | 800.742 | 146.6874 | 786.7802 | 130.629 |
| 110 | 1284.056 | 630.0012 | 1206.657 | 550.506 |
| 150 | 1767.369 | 1113.315 | 1626.534 | 970.3829 |
| 190 | 2250.683 | 1596.629 | 2046.411 | 1390.26 |
| 192 | 2274.849 | 1620.794 | 2067.405 | 1411.254 |

Legend: as before

Table 5: Quantities of steel rods for rectangular section beam with b = 250mm, h = 400mm, c = 25mm, rebar = Y16, link = R8, $f_{cu}$ = 25 Mpa, $A_{smin}$ = 130 $mm^2$, $A_{smax}$ = 3590 $mm^2$

| | FEA | | BS 8110 | |
|---|---|---|---|---|
| M | Ast | Asc | Ast | Asc |
| 34.4 | 228.8999 | 0 | 230.7991 | 0 |
| 45 | 321.3017 | 0 | 307.2696 | 0 |
| 70 | 539.2304 | 0 | 500.3743 | 0 |
| 95 | 757.1591 | 0 | 716.7064 | 0 |
| 120 | 975.0878 | 0 | 967.4435 | 0 |
| 170 | 1410.945 | 380.7348 | 1352.591 | 319.0782 |
| 250 | 2108.317 | 1078.107 | 1928.272 | 894.7586 |
| 330 | 2805.689 | 1775.479 | 2503.952 | 1470.439 |
| 410 | 3503.061 | 2472.851 | 3079.632 | 2046.119 |
| 420 | 3590.233 | 2560.022 | 3151.592 | 2118.079 |

Legend: as before

Table 6: Quantities of steel rods for rectangular section column with b = 225mm, h = 225mm, c = 25mm, rebar = Y16, link = R8, $f_{cu}$ = 25 Mpa, $A_{smin}$ = 202.5 $mm^2$, $A_{smax}$ = 2484 $mm^2$

| M | N | Asc | Ast |
|---|---|---|---|
| 0 | 1000 | 838.94 | 838.94 |
| 10 | 1000 | 997.24 | 680.64 |
| 20 | 1000 | 1193.27 | 484.60 |
| 30 | 1000 | 1389.30 | 288.57 |
| 40 | 1000 | 1585.34 | 202.50 |
| 50 | 1000 | 1781.37 | 202.50 |
| 60 | 1000 | 1977.40 | 299.53 |
| 70 | 1000 | 2173.44 | 495.56 |
| 80 | 1000 | 2369.47 | 691.60 |
| 85 | 1000 | 2467.49 | 789.61 |

Legend: as before

Table 7: Quantities of steel rods for rectangular section column with b = 225mm, h = 225mm, c = 25mm, rebar = Y16, link = R8, $f_{cu}$ = 30 Mpa, $A_{smin}$ = 225 $mm^2$, $As_{max}$ = 2821.5 $mm^2$

| M | N | Asc | Ast |
|---|---|---|---|
| 50 | 50 | 227.73 | 1395.25 |
| 50 | 200 | 425.05 | 1197.93 |
| 50 | 350 | 622.37 | 1000.61 |
| 50 | 500 | 819.69 | 803.29 |
| 50 | 650 | 1017.01 | 605.97 |
| 50 | 800 | 1214.33 | 408.65 |
| 50 | 950 | 1411.65 | 225.00 |
| 50 | 1100 | 1608.97 | 225.00 |
| 50 | 1250 | 1806.29 | 225.00 |
| 50 | 1400 | 2003.61 | 380.63 |

Legend: as before

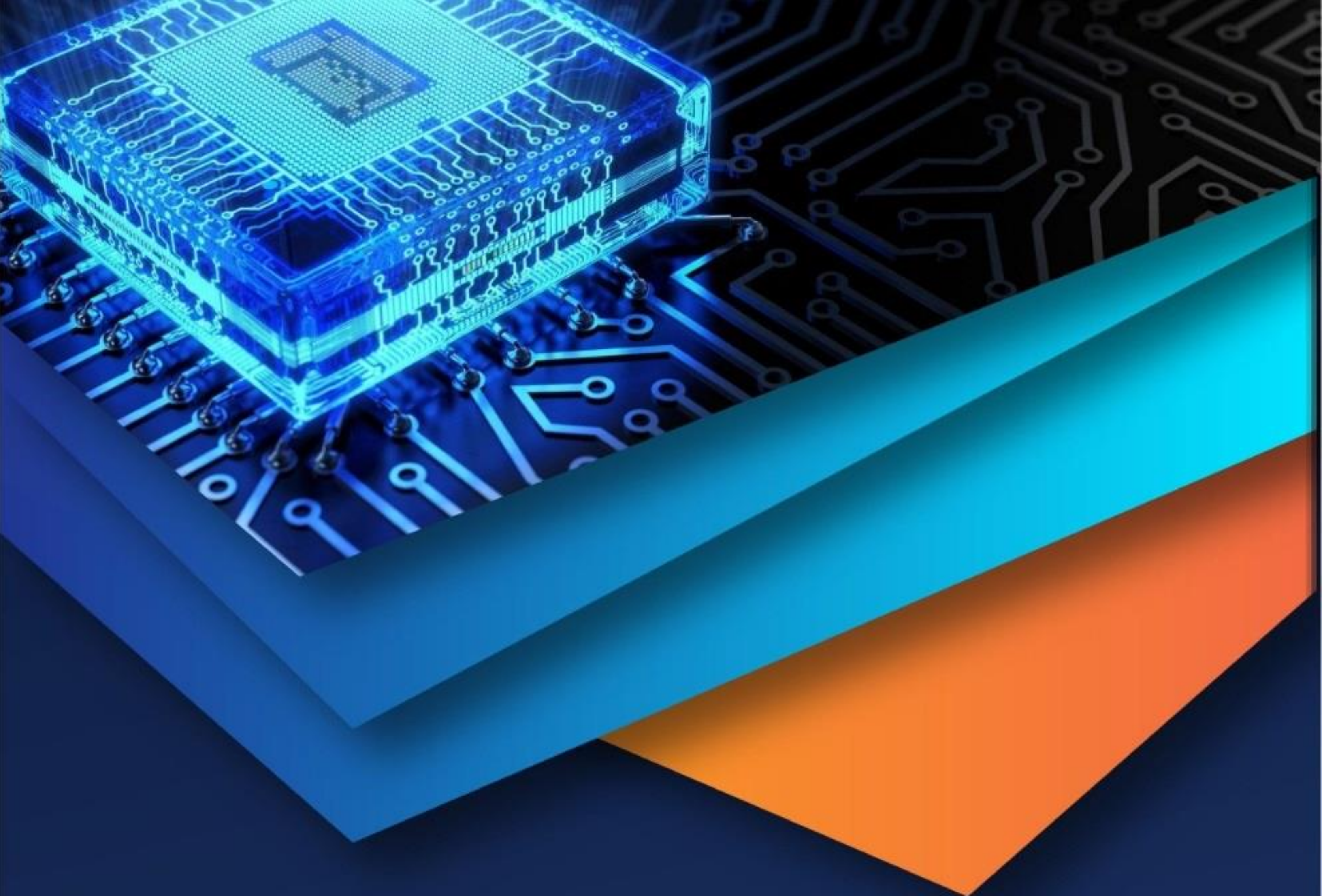
Table 8: Critical imposed loads on beams of rectangular section with b = 225mm, d = 450mm, self weight, qs = 2.43 kN/m, $f_{cu}$ = 30 Mpa, $f_y$ = 460 MPa, Es = 200kN/ $mm^2$

| L (mm) | L/d | $q_{icw}$ (kN/m) | $q_{icf}$ (kN/m) | Critical Load, $q_{ic}$ (kN/m) | Total load, q = qic + qs (kN/m) | stress, σ MPa | Deflection, w (mm) |
|---|---|---|---|---|---|---|---|
| 1000 | 2.222 | 9752.201 | 676.451 | 676.451 | 678.881 | 11.175 | 1.044 |
| 1500 | 3.333 | 1924.411 | 299.295 | 299.295 | 301.725 | 11.175 | 2.349 |
| 2000 | 4.444 | 607.234 | 167.290 | 167.290 | 169.720 | 11.175 | 4.176 |
| 2500 | 5.556 | 247.289 | 106.191 | 106.191 | 108.621 | 11.175 | 6.525 |
| 3000 | 6.667 | 117.998 | 73.001 | 73.001 | 75.431 | 11.175 | 9.395 |
| 3500 | 7.778 | 62.574 | 52.989 | 52.989 | 55.419 | 11.175 | 12.788 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4000 | 8.889 | 35.674 | 40.000 | 35.674 | 38.104 | 10.036 | 15 |
| 4500 | 10.000 | 21.358 | 31.095 | 21.358 | 23.788 | 7.929 | 15 |
| 5000 | 11.111 | 13.177 | 24.725 | 13.177 | 15.607 | 6.423 | 15 |
| 5500 | 12.222 | 8.230 | 20.012 | 8.230 | 10.660 | 5.308 | 15 |
| 6000 | 13.333 | 5.097 | 16.428 | 5.097 | 7.527 | 4.460 | 15 |
| 6500 | 14.444 | 3.035 | 13.638 | 3.035 | 5.465 | 3.800 | 15 |
| 7000 | 15.556 | 1.633 | 11.425 | 1.633 | 4.063 | 3.277 | 15 |
| 7500 | 16.667 | 0.653 | 9.639 | 0.653 | 3.083 | 2.855 | 15 |
| 7960 | 17.689 | 0.000 | 8.284 | 0.000 | 2.430 | 2.534 | 15 |
| 8000 | 17.778 | -0.048 | 8.178 | -0.048 | 2.382 | 2.509 | 15 |

## REFERENCE

[1] IS 456 (2000). Indian Standard . Code of Practice for Plain and Reinforced Concrete.
[2] BS 8110-1 (1997). British Standard. Structural use of concrete — Part 1: Code of practice for design and construction
[3] EN 1992-1-1 (1992). Eurocode 2: Design of concrete structures
[4] ACI 318 (2009). American Concrete Institute. “Building Code Requirements for Structural Concrete and Commentary,”
[5] SABS 0100-1 (2000).South African Standard Code of practice. The structural use of concrete Part 1: Design.
[6] Ibearugbulem, Owus M., Njoku, K. O., Anyanwu, T. U. And mbadike, E. S. (2015). Alternative method for flexural ultimate limit state design of reinforced concrete. International Journal of Scientific & Engineering Research, Volume 6, Issue 11, pp.1268-1271.
[7] Ibearugbulem, Owus M., Njoku, K. O., Ibearugbulem, C. N., Onuoha, I. (2015). Correct assumption approach to refined theory for analysis of thick. International Journal of Advance Research, IJOAR .org Volume 3, Issue 10, pp. 1-10. Online: ISSN 2320-9100
[8] O. M. Ibearugbulem, L. O. Ettu, J. C. Ezeh, and U. C. Anya (2013). A new shape function for analysis of line continuum by direct variational calculus. The International Journal Of Engineering And Science (Ijes),Vol. 2, Issue 3, Pp. 66-71




10.22214/IJRASET

INDEX COPERNICUS

45.98

IMPACT FACTOR: 7.129

TOGETHER WE REACH THE GOAL

IMPACT FACTOR: 7.429


# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY