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The Effect of Hall Parameter on MHD Fluid Flow of a Periodically Accelerated Plate

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***Abstract:** The current study Focus on the effect of Hall current on an unsteady Magneto hydrodynamic fluid flow of a periodically accelerated plate relative to a rotating fluid. The Laplace Transform technique is used to determine the analytical solutions of the derived mathematical model. The study has also presented graphically, the axial and transverse velocity profiles for different values of the parameters using symbolic Math software. Variation of the skin friction components are presented in the form of graphs.*
***Keywords:** MHD flow, Hall current, periodically accelerated plate, Laplace Transforms.*

Nomenclature

A	Constant Acceleration
H_0	Applied magnetic field
H_z^z	Component of magnetic field H
J_z^z	Component of current density J
μ_z	Magnetic permeability
m	Hall parameter
M	Hartmann number
υ	Kinematic viscosity
Ω_z^z	Component of angular velocity
Ω	Non dimensional angular velocity
ρ	Fluid density
σ	Electric conductivity
t	Time
T	Non dimensional time
(u,v,w)	Components of velocity field q
(U,V,W)	Non dimensional velocity components
(x,y,z)	Cartesian coordinates

I. INTRODUCTION

Magneto hydrodynamics (MHD) is the study of the relationship between magnetic fields and moving conducting fluids. In recent years MHD flow problems have become in view of its significant applications in industrial manufacturing processes such as plasma studies, petroleum industries, Magneto hydrodynamics power generators, boundary layer control in aerodynamics.

The flow due to a single infinite plate was first discussed by Vonkarman and Biot [1]. Batchelor [2] discussed a particular problem in MHD fluid flow of a moving plate. Stokes [3] observed viscous incompressible flow through an infinite that was at rest and then suddenly started the motion powerful with constant velocity in its own plane. Rossow [4] discussed MHD flow to start off an infinite flat plate. It was shown by Cowling [5] that when the magnetic field strength is sufficiently high then Ohm's law is adapted to Hall current.

The procedure of conduction in gases which are ionized, bearing a powerful magnetic field and in metallic substance are different from each other. Watanable and Pop [6] both discussed the results of steady MHD flow through a uniformly moving plate and Hall current. Pop [7] studied the Hall current on MHD flow because of self-generated movement of the plate that is binding only for a short time. Many other researchers have considered MHD fluid flow with rotational frame of reference for different fluid conditions

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[8-10]. Hayat and Abbas [11] observed the rotating flow of a second grade fluid past a porous heated plate with variable suction and Hall current. While Takhar et al [12] studied the combined effects of Hall current and free stream velocity on the MHD flow over a moving plate in a rotating fluid.

The purpose of this study is to explain rotational unsteady Magneto hydrodynamic fluid flow of a periodically accelerated plate with Hall current by using Laplace Transform approach. It is observed that the phenomenon discussed in this research work occurs in different situations, such as in tornadoes or in moving blades of a helicopter, blood flow through the heart, water turbines and many other situations. In this analysis the axial velocity, transverse velocity and skin friction components are presented graphically with the effect of the Hall parameter (m), Hartmann number (M) and the rotational parameter (Ω) using symbolic math software.

II. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

Consider incompressible fluid flow through an infinite plate occupying on a plane at z=0. Initially, the fluid and the plate are rotating with uniform angular velocity Ω_z^z about z-axis. The x-axis is in the direction of motion of the plate and the y-axis lies on the plate normal to the x and z-axis. H is the applied magnetic field to the system. H_0 is the uniform magnetic field is parallel to z-axis and the plate is electrically non conducting since $\nabla H = 0$ this implies that H_z is constant which further implies that $H_z = H_0$,every where in the flow. Here (u, v, w) are components of the velocity vector **q** , From the equation of continuity $\nabla q = 0$, gives w=0 everywhere in the flow such that the boundary condition w=0 is satisfied. Because of the horizontal homogeneity of the problem, the flow quantities depend only on z and t. Here u is the axial velocity and v is the transverse velocity and the problem so defined suggests that for uniform pressure $\nabla P = 0$. Under these considerations, the momentum equation and generalized Ohm's law for unsteady case of rotational flow with MHD effect are given by

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial z^2} + 2\Omega_z \nu - \frac{\sigma\mu_e{}^2 H_0{}^2}{\rho(1+m^2)}(m\nu + u) \tag{1}$$

$$\frac{\partial \nu}{\partial t} = \nu\frac{\partial^2 u}{\partial z^2} - 2\Omega_z u - \frac{\sigma\mu_e{}^2 H_0{}^2}{\rho(1+m^2)}(mu - \nu) \tag{2}$$

Where ν is Kinematic velocity, μ_e is magnetic permeability, H_0 is applied magnetic field, m is Hall parameter and ρ is fluid density.

The initial and boundary conditions are given by

u=0 ;	v=0	at	$t \le 0$	for all	(3)
u=$c_1 sint$;	v=0	at	z=0,	$t > 0$	(4)
u→ 0 ;	v → 0	as	z → ∞,	$t > 0$	(5)

where $c_1 > 0$ is a constant.

By introducing the dimensionless quantities

$$U = \frac{u}{(c_1\nu)^{1/3}} \qquad V = \frac{\nu}{(c_1\nu)^{1/3}} \qquad Z = z\,(\frac{c_1}{\nu^2})^{1/3}$$

$$T = t\,(\frac{c_1{}^2}{\nu})^{1/3} \qquad \Omega = \Omega_z\,(\frac{\nu}{c_1{}^2})^{1/3} \qquad M^2 = \frac{\sigma\mu_e{}^2 H_0{}^2 \nu^{1/3}}{2\rho c_1{}^{2/3}} \qquad K = (\frac{c_1{}^2}{\nu})^{-1/3} \tag{6}$$

From the above transformations; Eqs. (1) and (2) in dimensionless form as

$$\frac{\partial U}{\partial T} = \frac{\partial^2 U}{\partial Z^2} + 2V\left(\Omega - \frac{M^2 m}{1+m^2}\right) - 2U(\frac{M^2}{1+m^2}) \tag{7}$$

$$\frac{\partial V}{\partial T} = \frac{\partial^2 U}{\partial Z^2} - 2U\left(\Omega - \frac{M^2 m}{1+m^2}\right) - 2V(\frac{M^2}{1+m^2}) \tag{8}$$

The transformed boundary conditions are

U = 0,	V=0	at	$T \le 0$	for all Z	(9)
U = A sin KT,	V = 0	at	Z = 0,	T > 0	(10)
U → 0	V → 0	as	Z → ∞,	T> 0	(11)

Where $A = (\frac{c_1{}^2}{\nu})^{1/3}$

Now introduce F = U+ i V, which is substituted into (7)-(11), we have

$$\frac{\partial F}{\partial T} = \frac{\partial^2 F}{\partial Z^2} - 2F\left\{i\left(\Omega - \frac{M^2 m}{1+m^2}\right) + (\frac{M^2}{1+m^2})\right\} \tag{12}$$

$$\frac{\partial F}{\partial T} = \frac{\partial^2 F}{\partial Z^2} - aF \tag{13}$$

Where $a = 2i\left(\Omega - \frac{M^2 m}{1+m^2}\right) + 2(\frac{M^2}{1+m^2})$ (14)

With boundary conditions

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F = 0 at $T \leq 0$ for all Z (15)

F = A sin KT at Z = 0 , T > 0 (16)

$F \rightarrow$ as $Z \rightarrow \infty$, T> 0 (17)

Taking Laplace Transform of Equ. (14) along with the initial and boundary conditions (15) – (17) give rise to

$$\overline{F(Z,s)} = \frac{AK}{s^2+K^2}\, e^{-Z\sqrt{(s+a)}} \tag{18}$$

Where $\overline{F(Z,s)}$ is the Laplace transform of F(Z, T)

$$\overline{F(Z,s)} = \frac{Ai}{2}\left[\frac{e^{-Z\sqrt{(s+a)}}}{s+ik} - \frac{e^{-Z\sqrt{(s+a)}}}{s-ik}\right] \tag{19}$$

We now apply the inverse Laplace transform on $\overline{F(Z,s)}$ using the following results by the Hetnarski [14];

$$L^{-1}\left[\frac{e^{-c\sqrt{s+b}}}{s-a}\right] = \frac{e^{at}}{2}\left[e^{-c\sqrt{a+b}}\, erfc\left(\frac{c}{2\sqrt{t}} - \sqrt{(a+b)t}\right) - e^{c\sqrt{a+b}}\, erfc\left(\frac{c}{2\sqrt{t}} + \sqrt{(a+b)t}\right)\right] \tag{20}$$

Thus we obtain the following solution for F (Z, T) using the Laplace Transform approach

$$F(Z,T) = \frac{iA}{2}\left\{\frac{e^{iKT}}{2}\left[e^{-Z\sqrt{a-iK}}\, erfc\left(\frac{Z}{2\sqrt{T}} - \sqrt{(a-iK)T}\right) - e^{Z\sqrt{a-iK}}\, erfc\left(\frac{Z}{2\sqrt{T}} + \sqrt{(a-iK)T}\right)\right] - \frac{e^{iKT}}{2}\left[e^{-Z\sqrt{a+iK}}\, erfc\left(\frac{Z}{2\sqrt{T}} - \sqrt{(a+iK)T}\right) - e^{Z\sqrt{a+iK}}\, erfc\left(\frac{Z}{2\sqrt{T}} + \sqrt{(a+iK)T}\right)\right]\right\} \tag{22}$$

It is observed that when $\Omega = \frac{M^2m}{1+m^2}$ then the velocity component V disappears in F= U + i V. So that the transverse component of velocity V=0 everywhere in the flow field and thus the flow is reduced to unidirectional flow which is along the direction of the plate only.

We find the numerical calculations of Equ. (22), by separating F into real and imaginary parts to obtain the axial and transverse velocity components U and V. Since the arguments of the complementary error functions (erfc) are complex, we have separated the functions into real and imaginary parts using results by Abramowitz and Stegun [15].The dimensionless skin friction at the plate Z=0 is derived from Equ. [22] gives rise to

$$\left(\frac{dF}{dZ}\right)_{Z=0} = \frac{iA}{2}\left\{\frac{e^{iKT}\sqrt{a+iK}}{2}\left[erfc(-\sqrt{(a+iK)T} + erfc(\sqrt{(a+iK)T}\right] - \frac{e^{-iKT}\sqrt{a-iK}}{2}\left[erfc(-\sqrt{(a-iK)T} + erfc(\sqrt{(a-iK)T}\right]\right\}$$

Seperate the real and imaginary parts of $\left(\frac{dF}{dZ}\right)_{Z=0}$ and the dimensionless skin friction components $\tau_x = \left(-\frac{dU}{dZ}\right)_{Z=0}$ and $\tau_y = \left(-\frac{dV}{dZ}\right)_{Z=0}$ can be calculated.

III. RESULTS AND DISCUSSION

The effect of Hall parameter on the axial and transverse velocity components are presented in Fig 1 and Fig 2 .It is observed that due to an increase in the Hall parameter there is a growth in both the axial and transverse velocity components.

Variation of the Hartmann number M for both velocity components is also investigated. Fig 3 and Fig 4 display that the amplitude of both the axial and transverse components of velocity decrease more rapidly as the Hartmann number increases.

Fig 5 and Fig 6 show clearly that the behaviour of the velocity components with various rotational parameters Ω on the axial and transverse components of velocity. With increase in the rotational parameter the values of the velocity components behave in an oscillatory manner.

In Fig 7 and Fig 8, we note that the behaviour of velocity components with variation in the amplitude parameter A. As this parameter is directly proportional to both the velocities, we observe that increasing its value leads to increase in both velocities.

Finally we plot the effect of increasing in Hall parameter m, over dimension less shear stress for both velocity components in Fig 9 and Fig 10.It is observe that for increasing values of m, the values of dimensionless skin friction components behave in an oscillatory manner.

IV. CONCLUSIONS

The study of Hall current on an unsteady magneto hydrodynamic fluid flow of a periodically accelerated plate relative to a rotating fluid reveals the phenomenon of reducing the two dimensional flow to a one dimensional flow. In the absence of rotation this type of phenomenon does not occur. Thus here we investigate that when the rotational parameter $\Omega = \frac{M^2m}{(1+m^2)}$ then the flow is in the direction of the plate only. The skin friction components along the plate have oscillatory behaviour .The transverse component of the skin friction increases and then decreases steadily for higher values of Hall parameter.

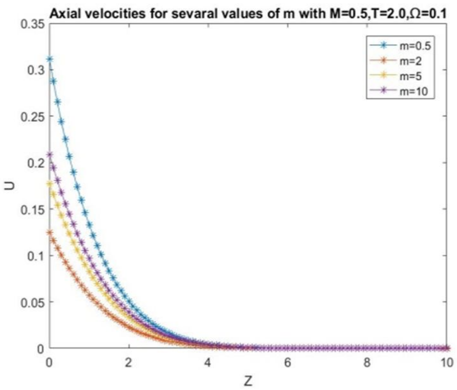

Fig 1

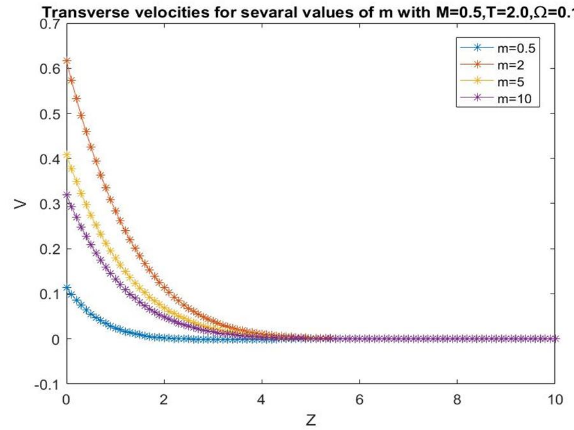

Fig 2

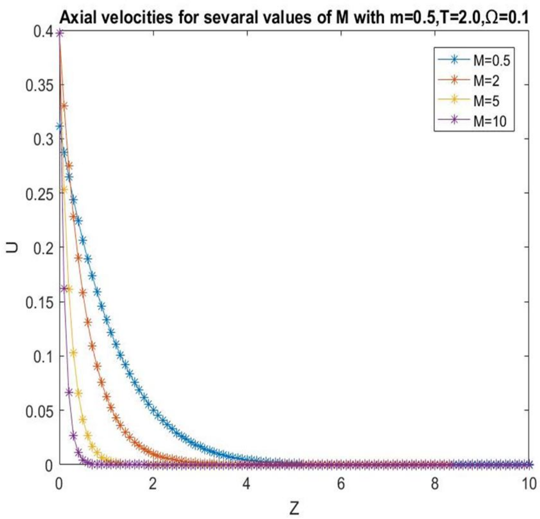

Fig 3

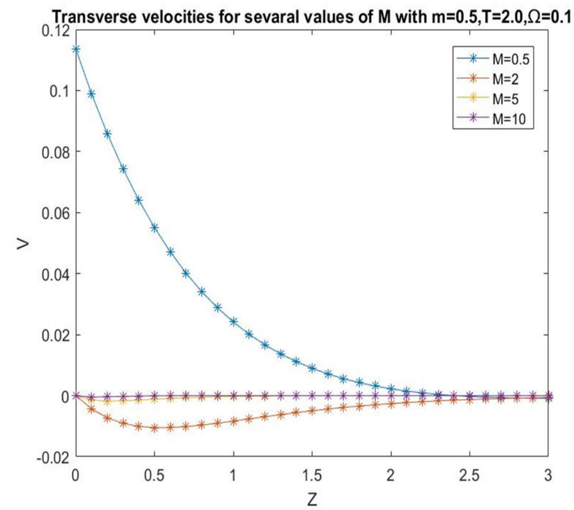

Fig 4

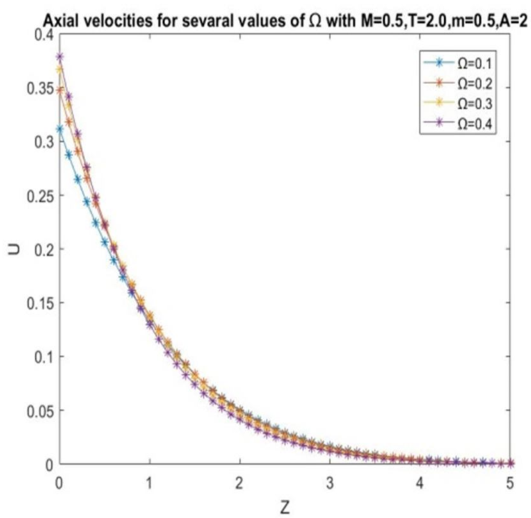

Fig 5

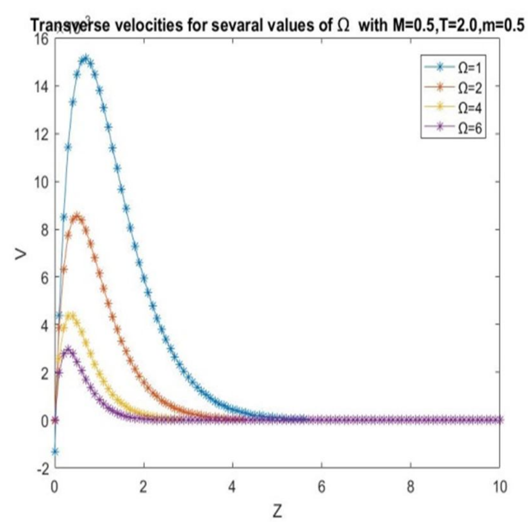

Fig 6

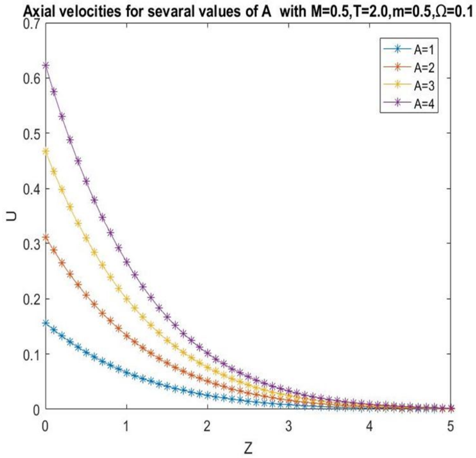

Fig 7

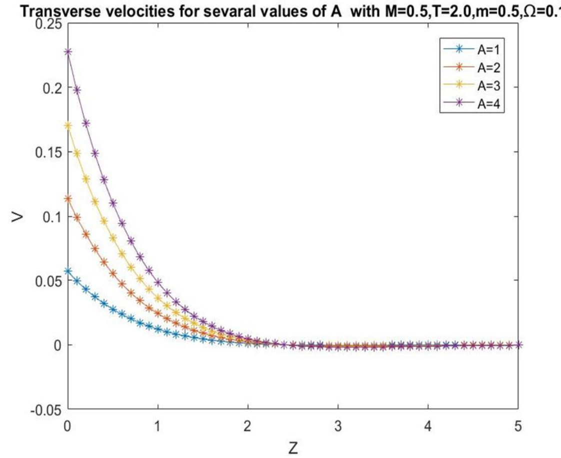

Fig 8

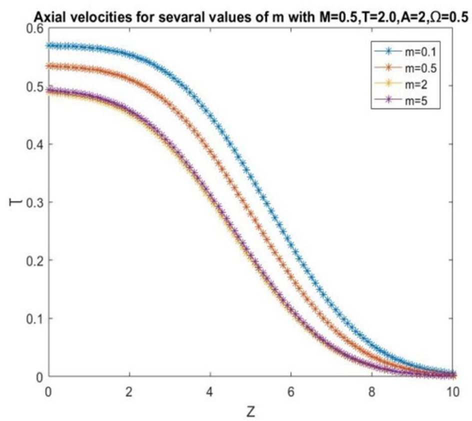

Fig 9

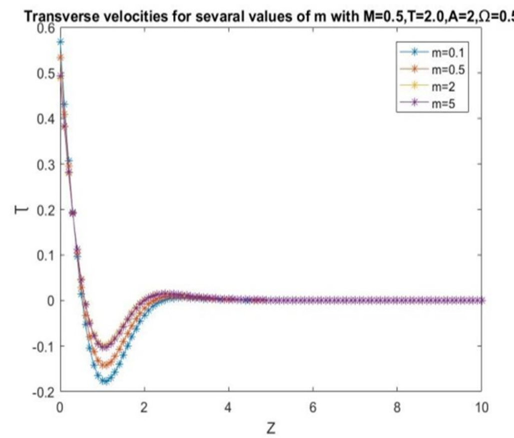

Fig 10

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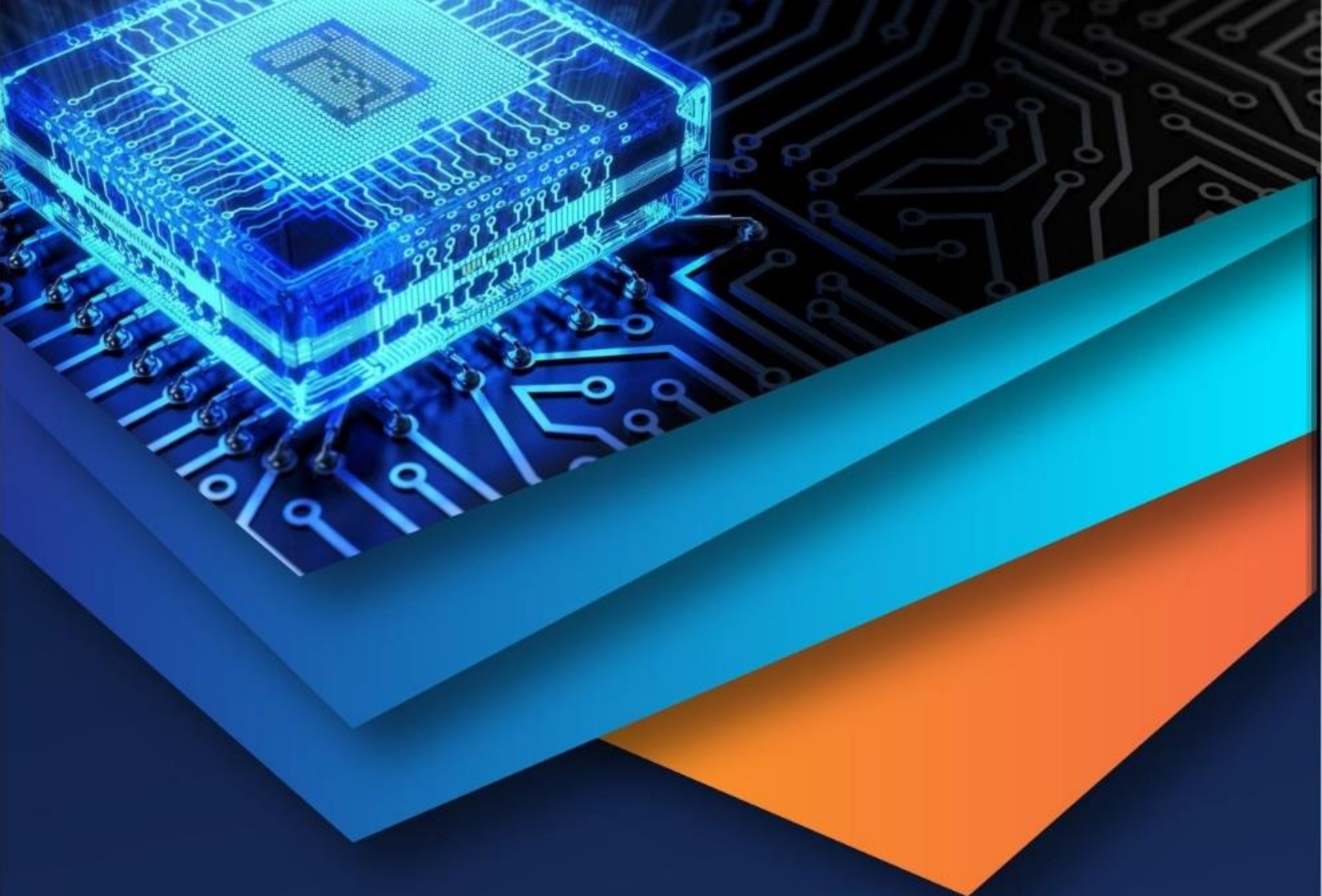

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