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A Topological Power Space

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Abstract: A Power Set is not only a container of all family of subsets of a set and the set itself, but , in topology, it is also a generator of all topologies on the defined set. So, there is a topological existence of power set, being the strongest topology ever defined on a set, there are some properties of it's topological existence. In this paper, such properties are being proved and concluded.
The following theorems stated are on the basis of the topological properties and separated axioms, which by satisfying, moves to a conclusion that, not only a power set is just a topology on the given defined set, but also it can be considered as a "Universal Topology" or a "Universal Topological Space" , that is the container of all topological spaces.
This paper gives a general understanding about what a power set is, topologically and gives us a new perceptive from a "power set" to a "topological power space".

I. INTRODUCTION

This paper is a slight modification to the manuscript "A View to Power Set: A Topological Discussion", a new separational property is being proved in this paper, although the results more evidently establish the conclusion, proving the separational existence of topological power space.

Let X be an infinite set. Since the number of topologies defined on a set depends on the cardinality of the power set, as for a finite set, power set is finite, so the number of topologies defined on the set are also finite. But, since the set X is assumed to be an infinite set, therefore the power set will be infinite, also the number of topologies defined on set X will be infinite. Therefore, (X, P(X)) will be a topological space, since P(X) is always a topology on the set X.

The following theorems proved below are stated to conclude the topological properties of the topological space (X,P(X)).

Theorem 1: (X,P(X)) is a totally disconnected space.

Proof: Since X is an infinite set. Therefore, the sub-families of the power set P(X), are the respective topologies defined on the set X, such that the topologies are of the type,

$\eta(1) = \{\phi, X, j(1)\}$, $\eta(2) = \{\phi, X, j(1), j(2)\}$, $\eta(3) = \{\phi, X, j(1), j(2), j(3)\}$,............................. $\eta(n) = \{\phi, X, j(1), j(2), \ldots\ldots, j(n)\}$, where $j(1) \neq j(2) \neq j(3) \neq \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \neq j(n)$, and $n < \infty$, and $\bigcup_{i=1}^{n} j(i) = X$.

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Since P(X) is a discrete topology by itself, Therefore, $\eta(1)$, $\eta(2)$, $\eta(3)$, ..., $\eta(n)$ are the respective discrete topologies defined on the infinite set X. But, since all the respective topologies are discrete and contains more than one point, therefore the space is disconnected, but, since every discrete space is a totally disconnected space. Therefore, (X,P(X)), is a union of all such discrete spaces, hence (X,P(X)) is a totally disconnected space.

A. *Theorem 2: (X,P(X)) is not Compact.*

Proof: As (X,P(X)) is a topological space in which X is infinite and P(X) is a discrete topology, therefore, (X,P(X)) is not a compact, as the topology is being a discrete one, so for a discrete topology, the topological space is not compact.

B. *Theorem 3: (X,P(X)) is a Hausdorff Space.*

Proof: By Theorem 1, it is proved that (X,P(X)) is a totally disconnected space, since every totally disconnected space is a Hausdorff space, therefore, (X,P(X)) is a Hausdorff space.

Also, since a discrete topological space is a Hausdorff Space, therefore, (X,P(X)) is a Hausdorff space.

C. *Corollary 1: (X,P(X)) is a T_0 - Space.*

Proof: Since every discrete topological space is a To - space, therefore, (X,P(X)) is a T_0 - space.

D. Corollary 2: (X,P(X)) is Regular.

Proof: Since every discrete topological space is regular. therefore,(X,P(X)) is a regular space.

E. Corollary 3: (X,P(X)) is a Frechet Space(or T_1 - Space).

Proof: By Theorem 3, (X,P(X)) is a Hausdorff Space, since every Hausdorff space is a Frechet Space, hence (X,P(X)) is a Frechet Space(or T_1 - Space).

F. Theorem 4: (X,P(X)) is a completely regular space.

Proof: Let F be the closed subset of X, $F \subset X$, such that for some $x \varepsilon X$, $x \varepsilon X - F$.

Let f be a map such that f: X $\rightarrow$ [0,1] , by Theorem 3, since (X, P(X)) is a Hausdorff space and also, since [0, 1] with relative topology is a Hausdorff space, that means, $\exists$ open sets G and H of [0, 1] such that 0 ε G, 1 ε H and G ∩ H = ϕ. For x ε G and y ε H, for all y ε F,

f(x) = 0 ε G,

f(y) = 1 ε H, ⇒ f(F) = 1 ε H, for all y ε F. As f(x) = 0 ε G, f(y) = 1 ε H, thus

x ε f^{-1}(G) , y ε f^{-1}(H),

Since x ≠ y, therefore,

III. A TOPOLOGICAL POWER SPACE $F^{-1}(G) \cap F^{-1}(H) = \Phi$,

Therefore, f^{-1}(G) and f^{-1}(H) are open subsets of X , hence f is continuous. Therefore, (X, P(X)) is a completely regular space.

A. Corollary 4: (X, P(X)) is a Tychonoff's Space.

Proof: By Theorem 3 and 4, it is proved that (X,P(X)) is a Hausdorff and a completely regular space, since a topological space which is completely regular is a Tychonoff's space, hence, (X,P(X)) is a Tychonoff's space.

B. Theorem 5: (X, P(X)) is a completely normal space.

Proof: From Theorem 1, it is proved that (X, P(X)) is a totally disconnected space, that means, every pair of disjoint sets can be separated by a disconnection of X. Let A and B be two such disjoint sets, such that no point is common in them.

Let p ε A and q ε B, such that the neighbourhood of p is a subset G of the topological space (X,P(X)) that consists of an open set C containing p, such that,

p ε C ⊆ G,

Also, q ε B , it's neighbourhood is a subset H of (X,P(X)) that consists an open set D containing q such that,

q ε D ⊆ H. Claim: G is contained in H.

If G is contained in H, that means, C is contained in D, so possibly, p is contained in q or p is equal to q, but, since the given topological space is totally disconnected, and no disjoint sets have points common in them, therefore, neither p is contained in q nor p is equal to q, therefore, neighbourhood G of set A and neighbourhood H of sets B are disjoint. Therefore, the topological space (X,P(X)) is a normal space. Hence, since the neighbourhoods of any two disjoint sets are disjoint, therefore, (X, P(X)) is a completely normal space.

C. Corollary 5: (X, P(X)) is a T_5 - Space.

Proof: From Theorem 5 and Corollary 3, it is proved that the topological space (X, P(X)) is a completely normal and a T_1 - space(or Frechet Space) , hence a topological space which is completely normal and a T_1- space(or Frechet Space), is a T_5- space, hence the topological space (X, P(X)) is a T_5- Space.

D. Corollary 6: (X,P(X)) is a normal space.

Proof: From theorem 5, it is proved that (X,P(X)) is a completely normal space, and since every completely normal space is a normal space(completely normal ⇒ normal space), hence
(X, P(X)) is a normal space.

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IV. A TOPOLOGICAL POWER SPACE

In this discussion, topological properties of power space topology has been discovered, from all the theorems and corollaries, stated and proved in this paper, gives a new image of power set not only as a strongest topology on the defined set X, or a topological power space along with the defined set X, as satisfying most of the separation axioms, the topological power space itself contains separational topological spaces, which are all satisfied when defined on the infinite set X, thus, it proves the power set topology along with set X is a, as a container of all separational topological space, “Universal Topology” or “Universal Space”.

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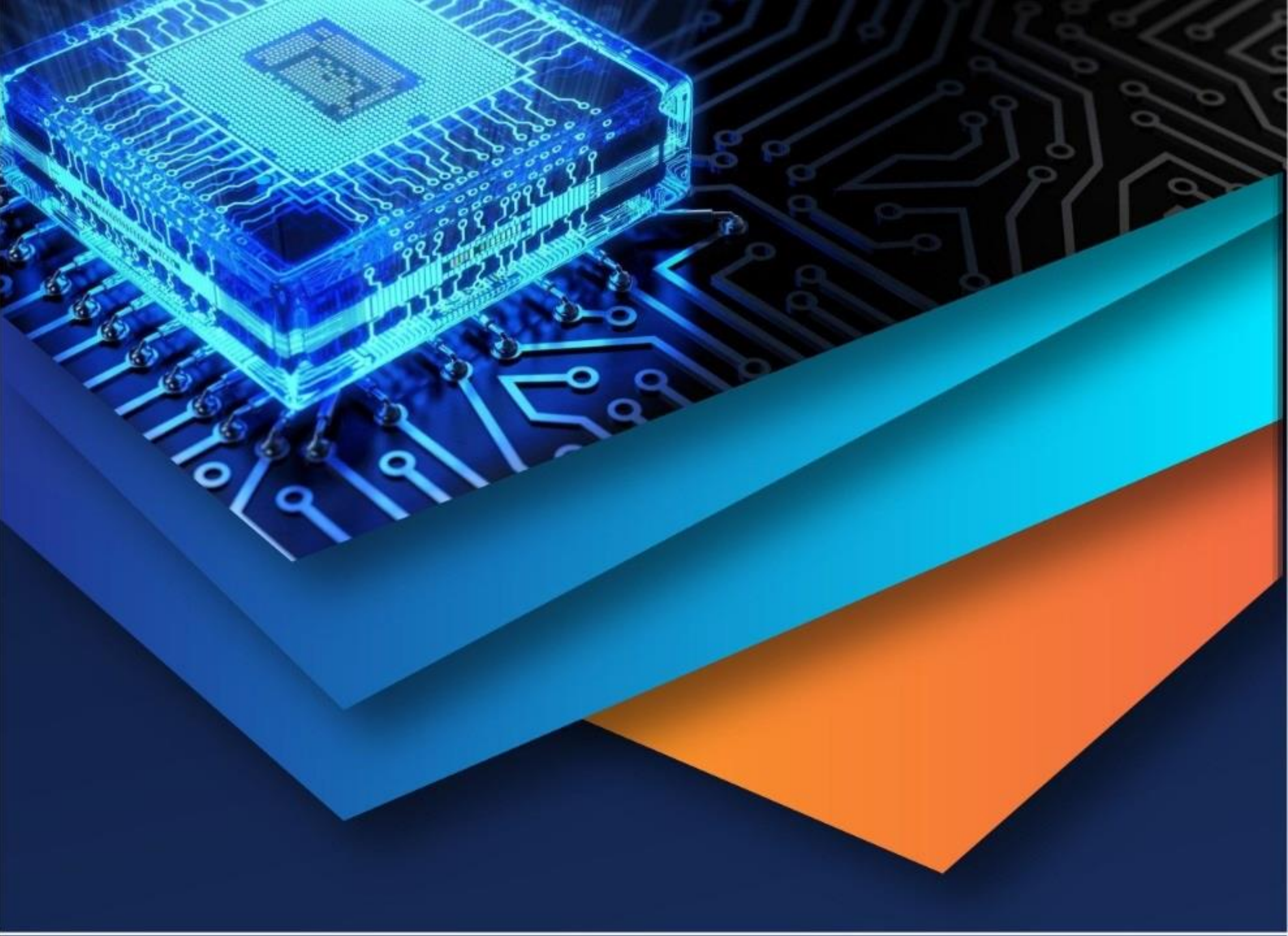

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