iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 7 Issue: X Month of publication: October 2019

DOI: http://doi.org/10.22214/ijraset.2019.10100

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.177
Volume 7 Issue X, Oct 2019- Available at www.ijraset.com

Mathematical Preliminaries in Mathematical Operation

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***Abstract:* This work presents analysis of Rolle's theorem, generalized Rolle's theorem and mean value theorem. These theorems are used to calculate the values and their accuracy is observed. Accuracy is analyzed by comparing the actual value with the values obtained by the theorems.**

I. INTRODUCTION

Rolle's theorem was discovered by an Indian mathematician Bhaskara II (1114-1185).[1] However, the theorem is named after Michel Rolle, Rolle's 1691 proof covered only the case of polynomial functions. This proof did not used the methods of differential calculus. This was considered a false idea at a point of time in his life. It was first proved by Cauchy in 1823 which was used as a corollary of the proof of mean value theorem. The name of Rolle's theorem was first used by Moritz Wilhelm Drobisch from Germany in 1834 and by Giusto Belavitis from Italy in 1846. Mean value theorem's special case was first explained by Parmeshvara (1370-1460)[2] . This was described from the Kerala School of Astronomy and mathematics in India. The modern form of Mean Value Theorem was stated and proved by Augustin Louis Cauchy in 1823.[5] In this both the Rolle's theorem and the mean value theorem are compared.

A. Rolle's Theorem

Rolle's Theorem or Rolle's lemma states that for any continuous, differentiable function that has two equal values, at two distinct points, then the function must have a point on the function where the first derivative is zero, i.e.

1) If f(x) is continuous in a≤x≤b.
2) f'(x) exists in a<x<b and
3) f(a)= f(b)=0.

Then there exists at least one value of x, say 'c', such that, f' (0) =0, a<0<b.

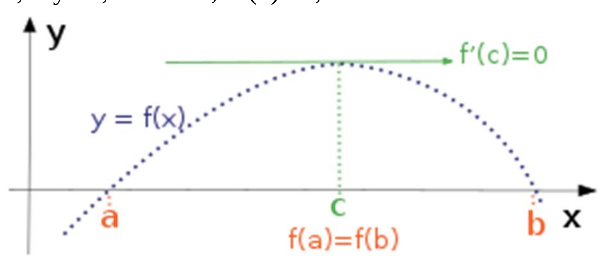

Fig 1.1. Graphical representation for Rolle's Theorem

B. Generalized Rolle's Theorem

Let f(x) be a function which is n times differentiable in [a, b]. If f(x) vanishes at (n+1) distinct points x_0, x_1, x_2, ……., x_n in (a, b), then there exists a number c in (a, b) such that $f^n(c)$ =0. It follows that between any two zeroes of the polynomial f(x) of degree ≥2, there lies at least one zero of the polynomial f'(x).[4]

$\frac{f(c+h)-f(c)}{h} \leq 0$,

Hence,

$f'(c) = \lim_{h\to 0} f(c+h) - \frac{f(c)}{h} \leq 0$

similarly, for h<0 the inequality sign gets reversed because now the denominator becomes negative and we get

$\frac{f(c+h)-f(c)}{h} \geq 0$,

Hence,

F'(c) = $\lim f(c+h) - f(c)/h \geq 0$
Where the limit tends to plus infinity.
Thus, when the right infinity limit and left infinity limits agree then derivative of function (f) at c must be zero. It follows that between any two zeroes of the polynomial f(x) of degree ≥ 2, there lies at least one zero of the polynomial f'(x).[3]

C. *Mean Value Theorem*

If a function is continuous on the closed interval [a,b] and differentiable on the open interval (a,b) then there exists a point c in the interval (a,b) such that f'(c) is equal to the functions average rate of change over (a,b) i.e. if f(x) is continuous in [a,b] and f'(x) exists in (a,b) then there exists at least one value of function say c between a and b such that
$f'(c)=\frac{f(b)-f(a)}{b-a}$ a<c<b

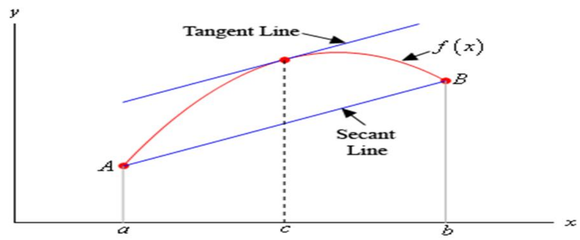

Fig2.1. Graphical representation of mean value theorem.

D. *Proof of Mean Value Theorem*

Let g(x) =f(x) – rx, where r is a constant. As, f is continuous [a,b] and differentiable on (a,b) , the same is satisfied for function g. we now take r so that function g satisfies the Rolle's theorem.[7]
g(a)= g(b) <-> f(a) -ra = f(b) – rb
<-> r(b-a) = f(b)- f(a)
$r = \frac{f(b)-f(a)}{b-a}$
By Rolle's theorem as function g is differentiable and g(a) =g(b), there is some c in (a, b) for which g'(c) =0, and it satisfies the equality g(x)= f(x) – rx then,
g'(x)=f'(x) -r
g'(c) =0
g'(c) =f'(c) -r =0
$f'(c) = r = \frac{f(b)-f(a)}{b-a}$

E. *For Example*

f(x)= x^2-5x+4 [1,4]

F. *Rolle's Theorem*

Function f(x)= x^2-5x +4 is a polynomial and is therefore continuous for all values of x, also the function f(x) is differentiable[4].
f'(x) = 2x -5
here, f'(x) is also continuous function.
Now,
f(x)=x^2-5x+4
f (1) = $(1)^2$- 5(1) +4
f (1) = 0
f (4) = $(4)^2$- 5(4) +4
f (4) =0
here f (1) = f (4) =0
both points f (1) and f (4) are at same height. In order to find the slope, equate the derivative of the function equal to zero.
Thus, f'(x)= 2x-5
2x -5 = 0
X= 2.5 lies in the interval [1,4]

G. Mean Value Theorem

$f(x)= 4x^3-8x^2+7x-2$ in the interval (2,5)

$f(2) = 4(2)^3 - 8(2)^2 + 7(2) - 2$

$f(2) = 12$

$f(5) = 4(5)^3 - 8(5)^2 - 7(5) - 2$

$f(5) = 333$

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$f'(c)= 12c^2-16c+7$

$12c^2 - 16c + 7 = \frac{333-12}{5-2}$

$12c^2 - 16c + 7 = 321/3$

$12c^2 - 16c + 7 = 107$

$12c^2 - 16c - 100 = 0$

$$c = \frac{+16 \pm \sqrt{16^2 - 4(12)(-100)}}{2(12)}$$

C= -2.2961, 3.6294

Therefore, c=3.6294 lies in the interval (2,5)

II. CONCLUSION

Rolle's theorem has three hypotheses while the mean value theorem has only two hypotheses. In Rolle's theorem the slope of secant is not necessarily zero. Both the theorems i.e. the Rolle's theorem and the mean value theorem state that at some point the slope of secant is same as the slope of tangent connecting the points (a, f(a)) and (b , f(b)), also the mean value theorem is valid for continuous functions, over a closed interval [a,b], and differentiable over the open interval (a,b). The proofs are simplest to solve if we derive Rolle's theorem first and using this proof to prove mean value theorem. Beyond this Rolle's theorem is not useful which means that the mean value theorem can handle every other use. Though it is also true that the mean value theorem is more intuitive and perplexing as compared to the Rolle's theorem.[6]

III. ACKNOWLEDGEMENT

We would like to express our special thanks of gratitude to **Dr. D.S. Bankar**, Head, Department of Electrical Engineering for their able guidance and support for completing my research paper. I would also like to thank the faculty members of the department of electrical engineering who helped us with extended support.

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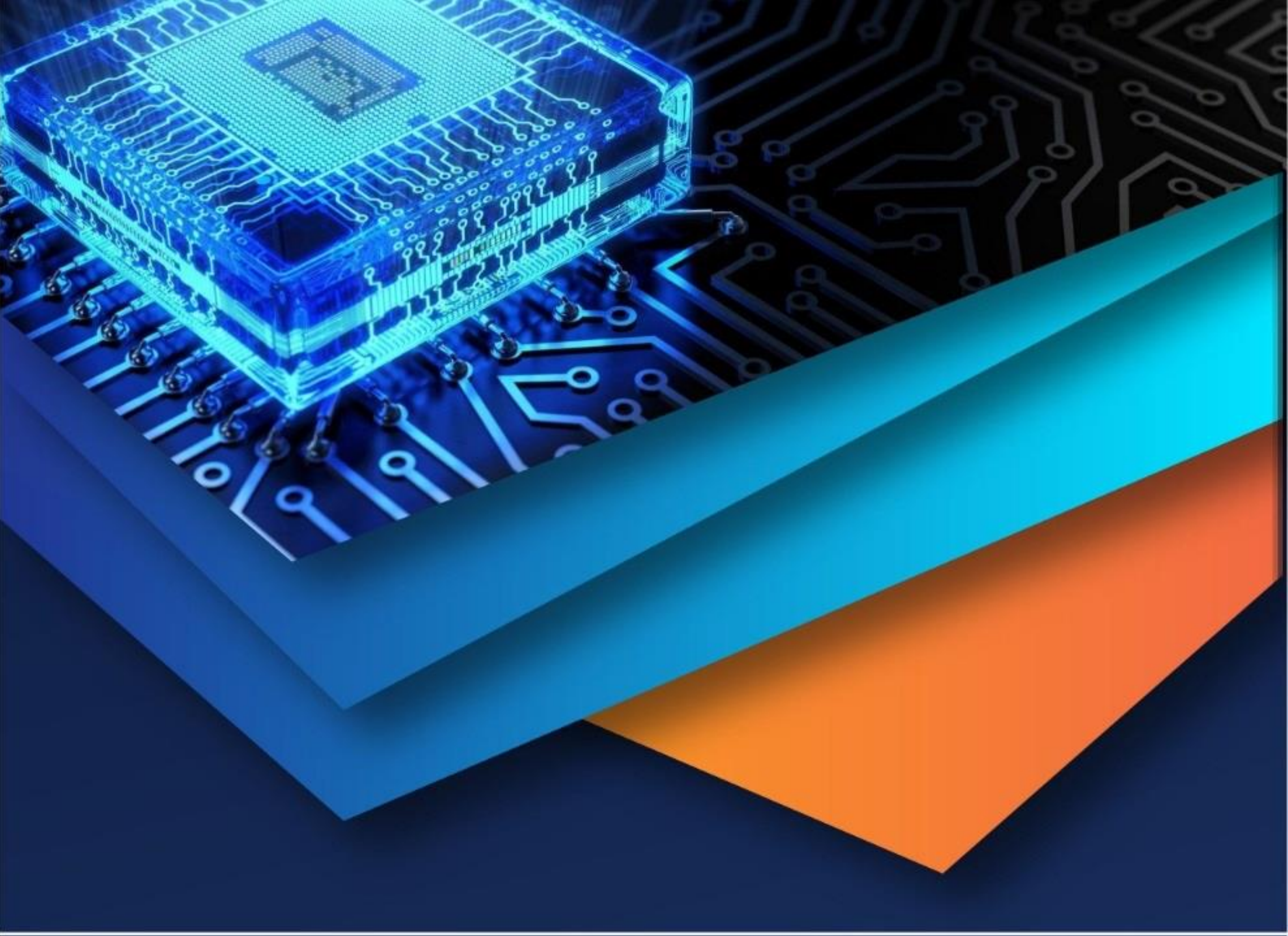

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