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Domination Numbers, Chromatic Numbers and Total Domination Numbers of all Powers of Cycles using a Circular-Arc Graph G

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Abstract: ***Graph theory is the study of graphs, which are mathematical structures used to demonstrate pair wise relations between objects. A dominating set is used as a backbone for communication. Among various applications of the theory of domination, chromatic number and total domination the most often discussed is a communication network. The Circular-arc graphs have drawn the attention of research for over number of years. They are extensively studied and revealed their practical relevance for modeling problems arising in the real world. The aim of this paper is to find the domination numbers, Chromatic numbers and Total domination numbers of all powers of cycles using a Circular-arc graph G corresponding to a Circular-arc family A.***

Keywords: Domination number, Chromatic number, Total domination number, Cycle, power of graph, power of cycle, Circular-arc graph, Circular-arc family, neighborhood, maximum degree, Regular graph, Order of graph.

I. INTRODUCTION

Graphs are useful in enhancing the understanding of the organization and behavioral characteristics of computer system its rigor and mathematical elegance appear in problem solving. Problems in almost every conceivable discipline can be solved using graph models. Graph theory is the study of graphs, which are mathematical structures used to demonstrate pair wise relations between objects. Graph theory is too much old subject; even it is too much young due to its progressive applications in various fields like, Operations Research, Computer Science, Decision Theory, Game Theory, etc. Now-a-days the graph theory is used in communication system, internet, mobile, Computer design, Social networks, etc. And it has extensive applications in computer science, engineering science, mathematical science, physical science etc. So almost every real world problem can be composed by using graph theory.One of the beauties of Graph Theory is that it depends very little on the other branches of Mathematics. There are several reasons for the acceleration of interest in Graph Theory. One of the attractive features of Graph Theory is its inherent pictorial character.

Circular-arc graphs are introduced as generalization of interval graphs. The class of interval graphs is properly contained in the class of circular-arc graphs. In fact every interval graph is a Circular-arc graph and the converse need not be true. However both these classes of graphs have received considerable attention in the literature in recent years and have been studied extensively. Circular-arc graphs are rich in combinatorial structures and have found applications in several disciplines such as biology, ecology, psychology, traffic control, genetics, computer science and particularly useful in cyclic scheduling and computer storage allocation problems etc.

The main focus of the paper is structured on the theory of domination and coloring. For finding these various types of dominating sets in different circular-arcs we introduced the different types of algorithms. The theory of domination in graphs by Ore [1] and Berge [2] is an emerging area of research in graph theory. A vertex v in a graph G is said to dominate both itself and its neighbors, that is v dominates every vertex in its closed neighborhood N[v]. A dominating set is used as a backbone for communication. Among various applications of the theory of domination, chromatic number and total domination the most often discussed is a communication network. And also we have introduced a new graph coloring concept more exactly; we study the chromatic number on several classes of graphs as well as finding general parameters like domination number, chromatic number, and total domination.

Graph coloring problems are bottomed by frequency assignment in broadcast communication, traffic planning, register allocation problem, task assignment, fleet maintenance and much more. In the fundamental graph coloring problem two adjoining vertices are colored by various colors. This coloring problem is known as vertex coloring problem or optimal coloring problem. By imposing several conditions different graph coloring problems are raised by different researchers.

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We need to make explicit our assumptions about the kinds of computer we expect the algorithm to be executed on. The assumptions we make can have important consequences with respect to how fast a problem can be solved. This has given scope to consider faster computers and the need for faster computers has increased in recent years. As a consequence there has been considerable interest in demising parallel algorithms for solving various computational problems.

II. PRELIMINARIES

Let $A = \{A_1, A_2, \ldots, A_n\}$ be a circular-arc family on a circle, where each A_i is an arc. Without loss of generality assumes that the end points of all arcs are distinct and no arc covers the entire circle. Denote an arc i that begins at end point p and ends at endpoint q in the clock wise direction by (p, q). Define p to be head, denoted by h(i) of the arc i, q to be the tail, denoted by t(i) Thus i = (h(i), t(i)).

The continuous part of the circle that begins with an end point c and ends with d in the clockwise direction is referred to as segment (c, d) of the circle. We use "arc" to refer to a member of A and "segment" to refer to a part of the circle between two end points. A point on the circle is said to be an arc (p, q) if it is contained in segment (p, q). An arc (p, q) of A is also referred as the segment (p, q). An arc i = (a, b) is said to be contained in another arc j = (c, d) if segment (a, b) is contained in the segment (c, d). A circular-arc family A is said to be proper if no arc in A is contained in another arc.

Let G (V, E) be a graph. Let$A = \{A_1, A_2, \ldots, A_n\}$ be a family of arcs on a circle. Then G is called a Circular-arc graph if there is one-to-one correspondence between V and A such that two vertices in V are adjacent if and only if their corresponding arcs in A intersects. We denote this Circular-arc graph by G[A]. Circular-arc family is proper then the corresponding graph is called a proper Circular-arc graph.

A neighbor of a vertex v in G is a vertex adjacent to v. The open neighborhood of v, denoted $N_G(v)$, is the set of all neighbors of v in G, while the closed neighborhood of v is the set $N_G[v] = N_G(v) \cup \{v\}$. We denote the degree of a vertex v in G by $d_G(v) = |N_G(v)|$. The minimum and maximum degrees among all vertices of G are denoted by $\delta(G)$ and $\Delta(G)$, respectively.

In this paper we considered all graphs here are without loops and multiple edges, connected, finite and undirected graphs Let G= (V, E) be a graph the order of graph G is number of vertices in G that is |V|=n. A connected graph without self loops and multiple edges is called simple graph.

A simple graph G is said to be regular graph if degrees of all the vertices of the graph G are equal. If $d_G(v) = k$ for every vertex v of a given graph G, for some positive integer k, then the graph G is called a k-regular graph.A simple graph G of order $n \geq 2$, in which all vertices are adjacent to all other vertices of the graph is said to be a complete graph or a full graph. In other words a simple graph G is said to be complete graph if a vertex v_i in G is adjacent to all other vertices in that graph for $1 \leq i \leq n$. A complete graph with n vertices is denoted by k_n. A complete graph G of order n is (n-1)-regular graph and a (n-1)-regular graph G is a complete graph of order n. All complete graphs are regular graphs but all regular graphs are not complete graphs.

The power of graph [3] G is donated as $G^k$and defined as 'u' and 'v' are two vertices in G^k if the distance $d(u, v) \leq k$ then 'u', 'v' are adjacent vertices in G^k.

A graph G with 'n' vertices is said to be a cycle if v_i and v_{i+1} are adjacent for $1 \leq i \leq n-1$ and v_n and v_1 are adjacent, denoted as C_n. The k^{th}power of cycle of order 'n' is denoted as C_n^k and defined as v_i is adjacent to $v_{i+1}, v_{i+2}, \ldots v_{i+k}$ for $1 \leq i \leq n-k$ and v_{i+1} is adjacent to $v_{i+2}, v_{i+3}, \ldots v_{i+k}$, and v_1 for i=n-k, v_1, v_2, for i=n-k+1, so on $v_1, v_2, \ldots v_k$ for i=n-1.

A cyclic graph G is 2-regular graph because degree of each vertex in a cycle is two. Similarly k^{th} power of cycle is 2k-regular graph. We can construct a (n-1)-regular graph which is a complete graph of order n form a cycle of order n by constructing the k^{th} power of cycle by increasing the value of k up to $\frac{n}{2}$ if n is even $\frac{n-1}{2}$ if n is odd. If order of the cycle n, is even then we increase the power of the cycle k up to $\frac{n}{2}$ then the k^{th} power of cycle is (n-1)-regular graph in terms of k, (2k-1)-regular graph. If order of the cycle n, is odd then we increase the power of the cycle k up to $\frac{n-1}{2}$ then the k^{th} power of cycle is (n-1)-regular graph in terms of k, (2k)-regular graph.

Let G be a graph. A subset D of V is said to be a dominating set of G if every vertex in V\D is adjacent to a vertex in D. The domination number of the graph G is the minimum cardinality of the dominating set in G denoted by $\gamma(G)$.

For every graph G, a vertex coloring is a mapping $f:V(G) \rightarrow \{0,1,2,\ldots..,\}$ so that no two adjacent vertices get the same color and every vertex get one color. A p-coloring of a graph consist of p distinct colors and then the graph G is called p-colorable. For any graph G the lease number p which subsists a p-coloring of G is called the Chromatic number of the graph G and it is denoted by $\chi(G)$. If $\chi(G)=p$ then the graph is said to be p-chromatic.

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Let G be a graph. A set 'S' of vertices in G is said to be a total dominating set of G if every vertex of G is adjacent to some vertex in 'S' and the subgraph induced by 'S' has no isolated vertices. The smallest cardinality of a total dominating set is called total domination number which is denoted by $\gamma_t(G)$.

In this paper we are going to find Domination number, Chromatic number and Total Domination number of all powers of cycles using order of cycle 'n' and power of cycle 'k' for all powers of cycles of order 'n'.

III. MAIN THEOREMS

1) *Theorem 1:* If A= {A_1, A_2, ….. A_n} be a Circular-arc family and G=C_n^k is a Circular-arc graph corresponding to a Circular-arc family 'A'. We consider first 2k+1 consecutive circular-arcs as S_1 and another 2k+1 consecutive circular-arcs as S_2 and so on. If the remainder when $\frac{n}{2k+1}$ is zero then the domination number of G is $\gamma(G) = \frac{n}{2k+1}$ and if the remainder when $\frac{n}{2k+1}$ is nonzero then the domination number of G is $\gamma(G) = \left\lfloor \frac{n}{2k+1} \right\rfloor + 1$where 'n' is order of the cycle and 'k' is power of the cycle.

a) *Proof:* Let G=C_n^k be a circular-arc graph corresponding to a circular-arc family 'A' where A={A_1, A_2,….A_n}. Our aim to find the domination number of G=C_n^k using power of cycle 'k' and order of cycle 'n'. k^{th} power of cycle of order n is corresponding to a circular-arc family, circular-arcs in such a way that A_i circular-arc in A is adjacent to A_{i+1}, A_{i+2}, . . . A_{i+k} intervals. Vertex v_i in G is adjacent to v_{i+1}, v_{i+2}, . . .v_{i+k} vertices in G.

We divide 'n' (finite) circular-arcs into few disjoint sets. We consider first 2k+1 consecutive circular-arcs as S_1 and another 2k+1 consecutive circular-arcs as S_2 and so on. Each set contains 2k+1 consecutive circular-arcs where 'k' is power of cycle. $(k+1)^{th}$ circular-arc in first set A_{k+1} dominates A_1, A_2, . . A_k, A_{k+2}, . . .A_{2k+1} circular-arcs. Corresponding to this v_{k+1} vertex in G dominates v_1, v_2, . . .v_k, v_{k+2}, . . . v_{2k+1} vertices in G. Middle circular-arc in first set dominates all other circular-arcs in that set. Similarly A_{3k+2}, dominates in second set. Likewise in all other sets. V_{k+1}, v_{3k+2}, . . .∈D

In each set middle circular-arc∈D we are choosing the elements for D such a way that those elements are not connected. Since sets are disjoint. From each set we are taking one circular-arc namely middle circular-arc. If the remainder when $\frac{n}{2k+1}$ is zero then the domination number of G is $\gamma(G) = \frac{n}{2k+1}$ that means the domination number of the graph G is equal to the number of sets each contains 2k+1 circular-arcs. There is no circular-arc left we are selecting And if the remainder when $\frac{n}{2k+1}$ is nonzero then the domination number of G is $\gamma(G) = \left\lfloor \frac{n}{2k+1} \right\rfloor + 1$ that means the domination number of the graph G is equal to the number of sets each contains 2k+1 circular-arcs and one set with r circular-arcs where r is equal to the remainder when $\frac{n}{2k+1}$.

A cyclic graph G is 2-regular graph. Similarly k^{th} power of cycle is 2k-regular graph. So this theorem works for all even regular graphs. If order of the cycle n, is even then we increase the power of the cycle k up to $\frac{n}{2}$ then the k^{th} power of cycle is (n-1)-regular graph in terms of k, (2k-1)-regular graph which is a complete graph with order n. If order of the cycle n, is odd then we increase the power of the cycle k up to $\frac{n-1}{2}$ then the k^{th} power of cycle is (n-1)-regular graph in terms of k, (2k)-regular graph which is a complete graph of order n. So this theorem works for all complete graphs all complete graphs are regular graphs so theorem works for all odd regular graphs also. We can conclude that theorem works for all regular graphs.

b) *Algorithm*

i) Step 1 start
ii) Step 2 D=ϕ
iii) Step 3 v_{k+1}∈D
iv) Step 4 max{nbd[v_{k+1}]}=v_{2k+1}
v) Step 5 max{nbd[v_{2k+2}]}=v_{3k+2}∈D
vi) Step 6 repeat steps 4 and 5
vii) Step 7 in repeating if v_n in 4^{th} step, stop

If v_n in 5^{th} step v_n∈D and stop

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c) Experimental Problem 1: Find Domination number of G=C_{12}^2

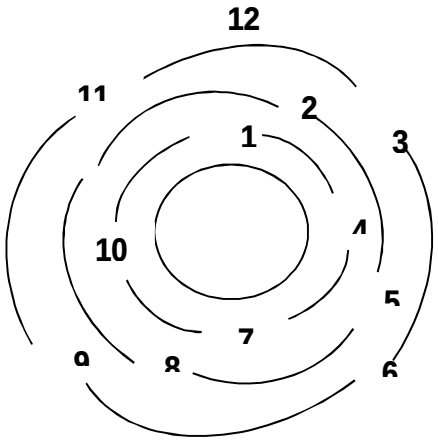


Fig. 1: Circular-Arc family

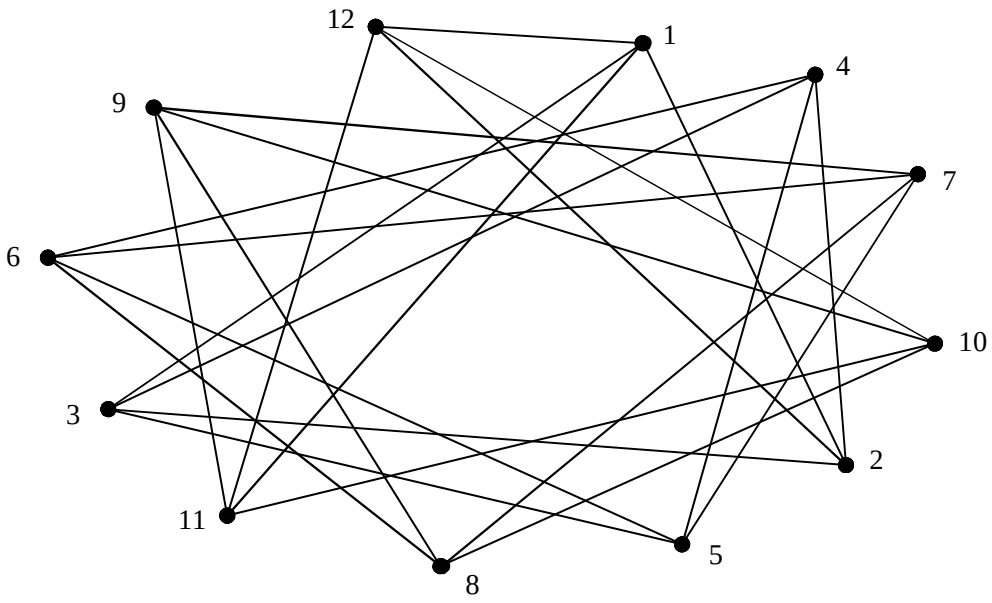


Fig.2: Circular-arc graph G=P_{12}^2

d) Domination number of G=C_{12}^2

Here order of the cycle n=12

Power of the cycle k=2

Remainder when $\frac{n}{2k+1}$

$\frac{12}{2(2)+1}=\frac{12}{5}$ here remainder when $\frac{n}{2k+1}$ is $2\neq 0$

Domination number of G is γ(G) =$\left\lfloor\frac{n}{2k+1}\right\rfloor+1$

$\left\lfloor\frac{12}{2(2)+1}\right\rfloor+1=\left\lfloor\frac{12}{5}\right\rfloor+1=2+1=3$

Domination number of G is γ(G) =3

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e) Find the closed neighborhood of $v_i$nbd[v_i] and Maximum of nbd[v_i] for $1 \leq i \leq 12$ to the graph $G=C_{12}^2$

nbd[v_1]={v_1, v_2, v_3, v_{11}, v_{12}} Max{nbd[v_1]}=v_{12}
nbd[v_2]={v_1, v_2, v_3, v_4, v_{12}} Max{nbd[v_2]}=v_{12}
nbd[v_3]={v_1, v_2, v_3, v_4, v_5} Max{nbd[v_3]}=v_5
nbd[v_4]={v_2, v_3, v_4, v_5, v_6} Max{nbd[v_4]}=v_6
nbd[v_5]={v_3, v_4, v_5, v_6, v_7} Max{nbd[v_5]}=v_7
nbd[v_6]={v_4, v_5, v_6, v_7, v_8} Max{nbd[v_6]}=v_8
nbd[v_7]={v_5, v_6, v_7, v_8, v_9} Max{nbd[v_7]}=v_9
nbd[v_8]={v_6, v_7, v_8, v_9, v_{10}} Max{nbd[v_8]}=v_{10}
nbd[v_9]={v_7, v_8, v_9, v_{10}, v_{11}} Max{nbd[v_9]}=v_{11}
nbd[v_{10}]={v_8, v_9, v_{10}, v_{11}, v_{12}} Max{nbd[v_{10}]}=v_{12}
nbd[v_{11}]={v_9, v_{10}, v_{11}, v_{12}, v_1} Max{nbd[v_{11}]}=v_{12}
nbd[v_{12}]={v_{10}, v_{11}, v_{12}, v_1, v_2} Max{nbd[v_{12}]}=v_{12}

f) Algorithm verification for Domination number of $G=P_{16}^3$

i) Step 1 start
ii) Step 2 D=ϕ
iii) Step 3 $v_3 \in D$
iv) Step 4 max{nbd[v_3]}=v_5
v) Step 5 max{nbd[v_6]}=$v_8 \in D$
vi) Step 6 max{nbd[v_8]}=v_{10}
vii) Step 7 max{nbd[v_{11}]}=$v_{12} \in D$
viii) Step 8 D= {v_3, v_8, v_{12}}
ix) Step 9 stop

Dominating set of Graph G is D= {v_3, v_8, v_{12}}
Domination number of G is $\gamma(G)$ =3
Our algorithm is verified true for Domination number of $G=C_{12}^2$

2) Theorem 2: If A= {A_1, A_2, ….A_n} be a Circular-arc family and $G=C_n^k$ is a Circular-arc graph corresponding to a Circular-arc family 'A'. We consider first k+1 consecutive circular-arcs as S_1 and another k+1 consecutive circular-arcs as S_2 and so on. If the remainder when $\frac{n}{k+1}$ is zero then the Chromatic number of G is $\chi(G)=k+1$ and If the remainder when $\frac{n}{k+1}$ is one then the Chromatic number of G is $\chi(G)=k+2$ and so on If the remainder when $\frac{n}{k+1}$ is k then the Chromatic number of G is $\chi(G)=2k+1$ where 'K' is power of the cycle.

a) Proof: Let $G=C_n^k$ be a circular-arc graph corresponding to a circular-arc family 'A' where A= {A_1,A_2,….A_n}. Our aim to find the Chromatic number of $G=C_n^k$ using power of cycle 'k'. k^{th} power of cycle of order n is corresponding to acircular-arc family A, circular-arcs in such a way that $A_i$circular-arc in A is adjacent to A_{i+1}, A_{i+2}, . . . A_{i+k} circular-arcs. Vertex v_i in G is adjacent to v_{i+1}, v_{i+2}, . . .v_{i+k} vertices in G.

We divide 'n' (finite) circular-arcs into few disjoint sets. We consider first k+1 consecutive circular-arcs as S_1 and another k+1 consecutive circular-arcs as S_2 and so on. Each set contains k+1 consecutive circular-arcs where 'k' is power of cycle. By the definition the power of graph G is donated as G^k and defined as 'u' and 'v' are two vertices in G^k if the distance $d(u,v) \leq k$ then 'u' and 'v' are adjacent vertices in G^k. A graph G with 'n' vertices is said to be a cycle if v_i and v_{i+1} are adjacent for $1 \leq i \leq n-1$ and v_n and v_1 are adjacent, denoted as C_n. The k^{th} power of cycle of order 'n' is denoted as C_n^k. In C_n^k give colors 1,2,…,k+1 to v_1, v_2. . . .v_{k+1} respectively in first set. We can repeat the colors in same order in all other sets. Because by definition of k^{th} power of cycle v_1 is adjacent to v_2, v_3, . . .v_{k+1} only for this reason we should give k+1 different colors and v_1 is not adjacent to v_{k+2} so we can repeat the color 1 similarly other colors in the same order. If the remainder when $\frac{n}{k+1}$ is zero that means each set with k+1 circular-arcs then we need minimum colors to color all the vertices of G is k+1 so the chromatic number of $G=C_n^k$ is $\chi(G)=k+1$.

If the remainder when $\frac{n}{k+1}$ is one that means each set with k+1 circular-arcs and one circular-arc left alone. For this we need to give different color because by the definition of cycle and power of cycle v_n is adjacent to v_1, v_2, . . v_k as well as v_{n-k}, $v_{n-(k+1)}$, . . . v_{n-1} so the colors whatever we given namely k+1 colors are there with adjacent vertices to v_n for this reason we should give different color other that k+1 colors which we already given then the Chromatic number of G is $\chi(G)=k+1+1=k+2$. Similarly If the remainder when

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$\frac{n}{k+1}$ is two that means each set with k+1 circular-arcs and two circular-arc left alone. For this two circular-arcs we need to give two different colors because by the definition of cycle, power of cycle and proper coloring and by the same above reason we should give two different colors other that k+1 colors which we already given then the Chromatic number of G is χ(G)=k+1+2=k+3. And so on

If the remainder when $\frac{n}{k+1}$ is k then the Chromatic number of G is χ(G)=k+1+k=2k+1.

If the remainder when $\frac{n}{k+1}$ is zero then the Chromatic number of G is χ(G)=k+1

If the remainder when $\frac{n}{k+1}$ is one then the Chromatic number of G is χ(G)=k+1+1=k+2

If the remainder when $\frac{n}{k+1}$ is two then the Chromatic number of G is χ(G)=k+1+2=k+3

If the remainder when $\frac{n}{k+1}$ is r then the Chromatic number of G is χ(G)=k+1+r for $0 \leq r \leq k$

If the remainder when $\frac{n}{k+1}$ is k then the Chromatic number of G is χ(G)=k+1+k=2k+1

A cyclic graph G is 2-regular graph. Similarly k^{th} power of cycle is 2k-regular graph. So this theorem works for all even regular graphs. If order of the cycle n, is even then we increase the power of the cycle k up to $\frac{n}{2}$ then the k^{th} power of cycle is (n-1)-regular graph in terms of k, (2k-1)-regular graph which is a complete graph of order n. If order of the cycle n, is odd then we increase the power of the cycle k up to $\frac{n-1}{2}$ then the k^{th} power of cycle is (n-1)-regular graph in terms of k, (2k)-regular graph which is a complete graph of order n. So this theorem works for all complete graphs. We can conclude that theorem works for all regular graphs.

b) Experimental Problem 2: Find Chromatic number of G=C_{16}^3

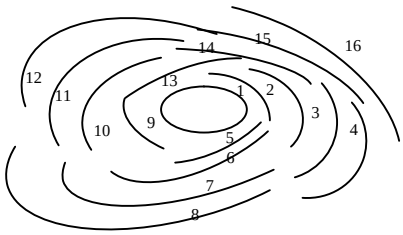


Fig. 3: Circular-Arc family

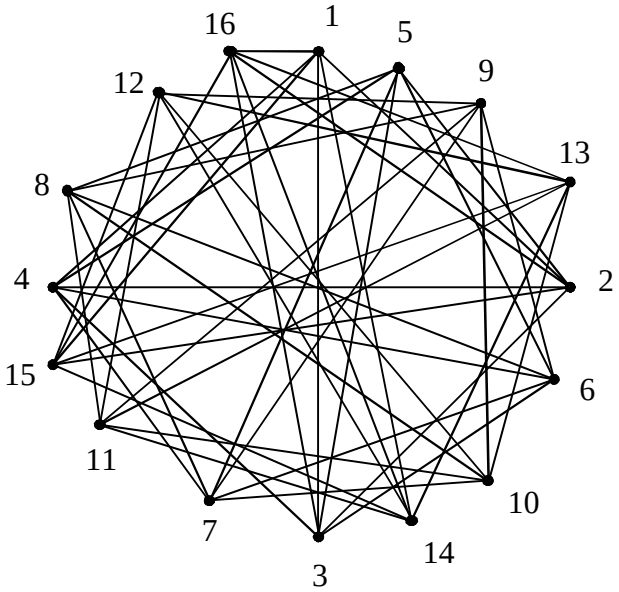


Fig.4: Circular-arc graph G=C_{16}^3

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c) *Chromatic Number of* $G=C_{16}^3$: Vertex v_1 is adjacent to vertices v_2, v_3 and $v_4$in the graph. So we need at least 4 different colors to color the graph such a way that no two adjacent vertices get the same color. We can repeat the 4 colors in the same order so that we will get different colors for adjacent vertices in the graph. By the definition of Chromatic number of a graph for every graph G, a vertex coloring is a mapping f: V(G) → {0, 1, 2, …..,} so that no two adjacent vertices get the same color and every vertex get one color. A p-coloring of a graph consist of p distinct colors and then the graph G is called p-colorable. For any graph G the least number p which subsists a p-coloring of G is called the Chromatic number of the graph G and it is denoted by χ(G). If χ(G)=p then the graph is said to be p-chromatic.

Give color 1 to vertex v_1Vertex v_1 is adjacent to vertices v_2, v_3 and $v_4$in the graph. Sogive colors 2, 3 and 4 to vertices v_2, v_3 and v_4 respectively. Vertex v_1 is not adjacent to vertex v_5 so we can repeat the color 1 and give to vertex v_5. Vertex v_2 is not adjacent to vertex v_6 so we can repeat the color 2 and give to vertex v_6. Vertex v_3 is not adjacent to vertex v_7 so we can repeat the color 3 and give to vertex v_7. Vertex v_4 is not adjacent to vertex v_8 so we can repeat the color 4 and give to vertex v_8. By repeating the same for other vertices v_9, v_{10}, v_{11} and v_{12} get 1, 2, 3 and 4 colors respectively. Vertex v_{13} is not adjacent to vertex v_9 and v_1 so we can repeat the color 1 and give to vertex v_{13}.Vertex v_{14} is not adjacent to vertex $v_{10}$and v_2 so we can repeat the color 2 and give to vertex v_{14}.Vertex v_{15} is not adjacent to vertex $v_{11}$and v_3 so we can repeat the color 3 and give to vertex v_{15}.Vertex v_{16} is not adjacent to vertex $v_{12}$and v_4 so we can repeat the color 4 and give to vertex v_{16}. Here no two adjacent vertices get the same color and every vertex get one color.

So the chromatic number of $G=C_{16}^3$ is χ(G)=4

d) *Verification of our formula for the Chromatic number of* $G=C_{16}^3$

Here order of the cycle n=16

Power of the cycle k=3

The remainder when $\frac{n}{k+1}$

$\frac{16}{3+1}=\frac{16}{4}$ remainder is Zero

Chromatic number of G is χ(G)=k+1 that is χ(G)=3+1=4

Our formula is verified true for the Chromatic number of $G=C_{16}^3$

3) *Theorem 3:* If A= {A_1, A_2, …..A_n} be a Circular-arc family and $G=C_n^k$ is a Circular-arc graph corresponding to a Circular-arc family 'A' We consider first k+1 consecutive intervals as S_1 and another k+1 consecutive intervals as S_2 and so on with a condition that last interval of S_i is first interval of S_{i+1}. If the remainder when $\frac{n-1}{k}$ is zero then the total domination number of G is $\gamma_t(G)=\frac{n-1}{k}-1$ and If the remainder when $\frac{n-1}{k}$ is nonzero then the total domination number of G is $\gamma_t(G)=\left\lfloor\frac{n-1}{k}\right\rfloor$ where n is order of the cycle and k is power of the cycle.

a) *Proof:* Let $G=C_n^k$ be a circular-arc graph corresponding to a circular-arc family 'A' where A= {A_1, A_2, ….A_n}. Our aim to find the Total domination number of $G=C_n^k$ using power of cycle 'k'. k^{th} power of cycle of order n is corresponding to a circular-arc family A, circular-arcs in such a way that A_i circular-arc in A is adjacent to A_{i+1}, A_{i+2}, . . . A_{i+k} circular-arcs. Vertex v_i in G is adjacent to v_{i+1}, v_{i+2}, . . .v_{i+k} vertices in G.

We divide 'n' (finite) circular-arcs into few sets. We consider first k+1 consecutive circular-arcs as S_1 and another k+1 consecutive circular-arcs as S_2 and so on Such a way that last circular-arc of one set is first circular-arc in next set. Each set contains k+1 consecutive circular-arcs where 'k' is power of cycle.By the definition of k^{th} power of cycle and our consideration of sets the circular-arcs in intersection of two setsare adjacent and dominate all other circular-arcs which satisfy total domination. Corresponding vertices in $G=C_n^k$ are adjacent and dominates all other vertices in G. v_{k+1}, v_{2k+1}, . . . which satisfies total domination.

The $(k+1)^{th}$ circular-arc in intersection of first two sets A_{k+1} dominates A_1, A_2, . . A_k, A_{k+2}, . . . A_{2k+1} circular-arcs. Corresponding to this v_{k+1} vertex in G dominates v_1, v_2, . . .v_k, v_{k+2}, . . . v_{2k+1} vertices in G. Similarly A_{2k+1} in intersection of second and third sets dominate all other circular-arcs in those sets and adjacent to A_{k+1} which is dominating for first two sets. Corresponding to this v_{2k+1} vertex in G dominates other vertices up to v_{3k+1} and adjacent to v_{k+1} which is previous dominating vertex.Likewise circular-arc in intersection of S_i and S_{i+1}sets dominates all other circular-arcs in those sets and adjacent to previous dominating circular-arc. Corresponding to this v_{3k+1}, v_{4k+1} . . dominates all other vertices and adjacent to previous dominating vertex. These dominating vertices satisfy total domination.

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A cyclic graph G is 2-regular graph. Similarly k^{th} power of cycle is 2k-regular graph. So this theorem works for all even regular graphs. If order of the cycle n, is even then we increase the power of the cycle k up to $\frac{n}{2}$ then the k^{th} power of cycle is (n-1)-regular graph in terms of k, (2k-1)-regular graph which is a complete graph of order n. If order of the cycle n, is odd then we increase the power of the cycle k up to $\frac{n-1}{2}$ then the k^{th} power of cycle is (n-1)-regular graph in terms of k, (2k)-regular graph which is a complete graph of order n. So this theorem works for all complete graphs all complete graphs are regular graphs so theorem works for all odd regular graphs also. We can conclude that theorem works for all regular graphs.

b) Algorithm

i) Step 1 start

ii) Step 2 D=ϕ

iii) Step 3 $v_{k+1} \in D$

iv) Step 4 $\max\{nbd[v_{k+1}]\}=v_{2k+1} \in D$

v) Step 5 repeat step 4

vi) Step 6 in repeating if v_n is adjacent to v_i where $v_i \in D$ stop.

c) Experimental Problem 3: Find Total domination number of$G=C_{12}^2$

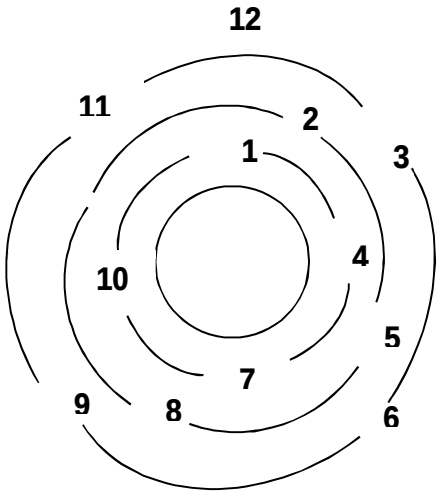


Fig. 5: Circular-Arc family

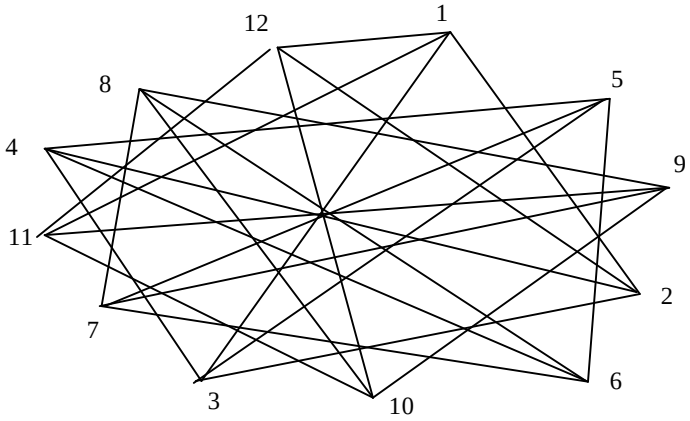


Fig.6: Circular-arc graph $G=C_{12}^2$

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d) Total Domination number of $G=C_{12}^2$

Here order of the cycle n=12

Power of the cycle k=2

Remainder when $\frac{n-1}{k}$

$\frac{12-1}{2} = \frac{11}{2}$ Remainder is $1 \neq 0$

Total domination number of $G=C_{12}^2$ is $\gamma_t(G)=\left\lfloor\frac{n-1}{k}\right\rfloor$ that is $\left\lfloor\frac{12-1}{2}\right\rfloor=\left\lfloor\frac{11}{2}\right\rfloor = 5$

$\gamma_t(G)=5$

e) Find the closed neighborhood of $v_i nbd[v_i]$ *and Maximum of* $nbd[v_i]$ *for* $1 \leq i \leq 12$ *to the graph* $G=C_{12}^2$

$nbd[v_1]=\{v_1, v_2, v_3, v_{11}, v_{12}\}$ $Max\{nbd[v_1]\}=v_{12}$

$nbd[v_2]=\{v_1, v_2, v_3, v_4, v_{12}\}$ $Max\{nbd[v_2]\}=v_{12}$

$nbd[v_3]=\{v_1, v_2, v_3, v_4, v_5\}$ $Max\{nbd[v_3]\}=v_5$

$nbd[v_4]=\{v_2, v_3, v_4, v_5, v_6\}$ $Max\{nbd[v_4]\}=v_6$

$nbd[v_5]=\{v_3, v_4, v_5, v_6, v_7\}$ $Max\{nbd[v_5]\}=v_7$

$nbd[v_6]=\{v_4, v_5, v_6, v_7, v_8\}$ $Max\{nbd[v_6]\}=v_8$

$nbd[v_7]=\{v_5, v_6, v_7, v_8, v_9\}$ $Max\{nbd[v_7]\}=v_9$

$nbd[v_8]=\{v_6, v_7, v_8, v_9, v_{10}\}$ $Max\{nbd[v_8]\}=v_{10}$

$nbd[v_9]=\{v_7, v_8, v_9, v_{10}, v_{11}\}$ $Max\{nbd[v_9]\}=v_{11}$

$nbd[v_{10}]=\{v_8, v_9, v_{10}, v_{11}, v_{12}\}$ $Max\{nbd[v_{10}]\}=v_{12}$

$nbd[v_{11}]=\{v_9, v_{10}, v_{11}, v_{12}, v_1\}$ $Max\{nbd[v_{11}]\}=v_{12}$

$nbd[v_{12}]=\{v_{10}, v_{11}, v_{12}, v_1, v_2\}$ $Max\{nbd[v_{12}]\}=v_{12}$

f) Algorithm verification for Total Domination number of $G=P_{15}^2$

i) Step 1 start

ii) Step 2 $D=\phi$

iii) Step 3 $v_3 \in D$

iv) Step 4 $max\{nbd[v_3]\}=v_5 \in D$

v) Step 5 $max\{nbd[v_5]\}=v_7 \in D$

vi) Step 6 $max\{nbd[v_7]\}=v_9 \in D$

vii) Step 7 $max\{nbd[v_9]\}=v_{11} \in D$

viii) Step 8 $max\{nbd[v_{11}]\}=v_{12}$

ix) Step 10 $D= \{v_3, v_5, v_7, v_9, v_{11}\}$

x) Step 11 stop.

Total Dominating set of Graph G is $D= \{v_3, v_5, v_7, v_9, v_{11}\}$

Total Domination number of G is $\gamma_t(G)=5$

Our algorithm is verified true the Total domination number of $G=C_{12}^2$

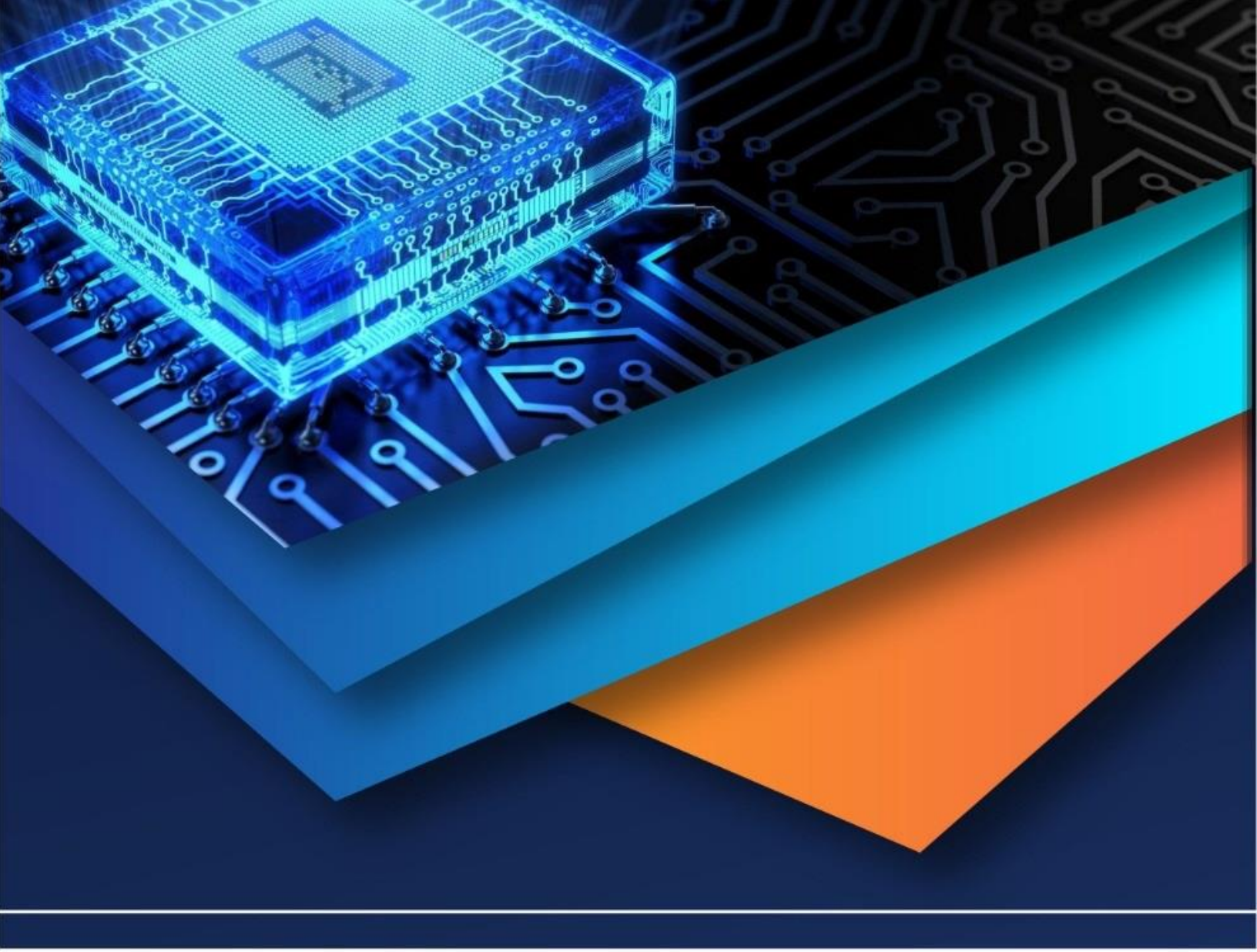
IV. CONCLUSION

In this paper we find the formulas to find domination number, chromatic number and total domination number of all powers of cycles using a circular-arc graph G. And we find algorithms to find domination number and total domination number of all powers of cycles using a circular-arc graph G and verified them with experimental problems.

REFERENCES

[1] F. Harary, Graph Theory, Narosa Publishing House, New Delhi, (1988).
[2] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, Inc, New York, 1998.
[3] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, Domination in Graphs Advanced Topics, Marcel Dekker, Inc, New York, 1998.
[4] D. B. West, Introduction to Graph Theory, 2nd ed., Prentice Hall, USA, 2001.
[5] T. Jensen and B. Toft. Graph Coloring Problems. John Wiley & Sons, New York. 1995.
[6] O. Ore. Theory of graphs, Amer, math. Sec. colloq. Publ.38, providence (1962). P. 206.
[7] C. Berge. Graphs and Hypergraphs, North Hollond.Amseterdram in graphs, Networks. Vol. 10(1980), 211-215.
[8] M. A. Henning and A. Yeo, Total domination in graphs (Springer Monographs in Mathematics). (2013) ISBN 978-1-4614-6524-9 (print) 978-1-4614-6525-6 (online).
[9] P Erdos and A. Hajnd. On chromatic number of graphs and set-systems.Acta.Math. Acad. Sci. Hungar..17:61 99, 1966.

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