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Analysis of Different Method of Solving Quadratic Equations

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Abstract: In this paper we have studied how to calculate approximate value of roots of the polynomial equation by different ways. We can solve the equation by Bisection, Regular-Falsi and Newton-Raphson methods and how these methods differ from each other. There are several option to solve the same problem but the question arises that which method to be preferred and is less time consuming and more accurate. These methods are day by day improving by working on it. By changing variable and by some small change in calculation the time can be reduced and the easiness can be increased. At the end we will illustrate about the difference in solving a particular polynomial equation by different method the difference in the methods of solving and toughness of them. You will get to know how these method differ from each other

I. INTRODUCTION

The fact that the roots of the polynomial can be obtained immediately using computer program as MATLAB, does not diminish the importance of gaining the new ways for solving the polynomial equation, simpler than that of current ones. In this paper the author gave u the information about the different methods such as Bisection, Secant and Newton-Raphson methods and the difference between the ways of solving and the difference between each of them.

The root finding problem is one of the most relevant computational problem. It arises in a wide variety of practical application in Physics, Chemistry, Biosciences, etc. Different methods converges the roots at different rates.[2] That is

Some methods are faster in converging to the roots than others. The rate of convergence could be linear, quadratic or otherwise. The higher the order, the faster the method converges. The whole study is comparing the rate of performance of Bisection, and Newton-Raphson methods of finding roots.

It can be seen that Newton-Raphson may converge faster than any other method but when we compare the performance, it is needful to consider both cost and speed of convergence. An algorithm that converges quickly but takes a few second per iteration may take more time overall than an algorithm that converges more slowly, but takes a few milliseconds per iteration. As Secant methodand Newton-Raphson method are almost same, from geometric perspective. The difference is that Newton-Raphson method uses a line that is tangent to one point, while the Scent method uses a line that is secant to two points. In Newton-Raphson, the derivation of a function at a point is used to create the tangent line, whereas in Secant method, a numerical approximation of the derivative based on two points is used to create the secant line.

In comparing the rate of convergence if Bisection, Secant and Newton-Raphson methods used C++ program language to calculate cube root of number from 1 to 25, using the three methods. They observed that rate of convergence is in the following order: Bisection Methods < Newton-Raphson < Secant Method. They conclude that Newton-Raphson method is 7.6786 times better than the bisection method while Secant method is 1.3894 times better than Newton method.[5]

II. METHODS TO OBTAIN ROOTS OF EQUATION

A. Bisection Method

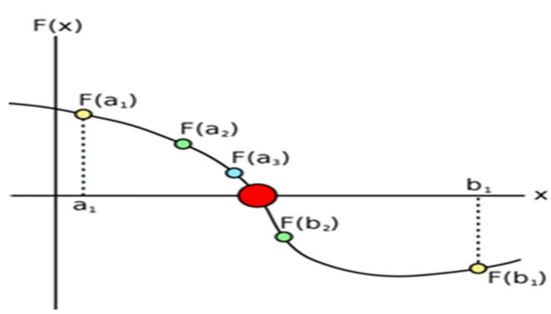

Fig 2.1: Graphical representation of Bisection method

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Given a function f(x)=0, continuous on a closed interval [a1,b1], such that f(a1).f(b1)< 0

The essential condition for bisection method is **f(a).f(b)< 0.** we find two values in the given interval such that the multiplication of them should be less than zero. After when we have find the values of "a" and "b" we take the out the value of c=a+b/2 and we put this value of c in the original equation.[1] Let us take an example to understand the concept-

F(x) =$x^3 - 2x - 5$ (This is the given polynomial equation and we have to find the roots of the equation)

F(1) = 1-2-5=-6 , f(2) = 8-4-5=-1,f(3)=27-6-5=16. Hence we can see that the f(1).f(3)<0 and the condition get satisfied. So the roots will lie between [2,3]. Therefore

1stiteration

$c = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$ then we have to find the value of f(2.5)=5.625 > 0

F(2).f(2.5)< 0 this condition again get full filled again we have to find the value of

2nd iteration

c=$\frac{2+2.5}{2}$=2.25 and again we find the value of f(2.25)=1.890>0 this process continues this until we find the value of c which is same after 3 place of decimal

Table 2.1: Result of Bisection Method

Number of iteration	Value of a	Value of b	C= $\frac{a+b}{2}$	F(c)=
1st	2	3	2.5	5.625
2nd	2.5	2	2.25	1.890
3rd	2.25	2	2.125	0.34
4th	2.125	2	2.0625	-0.3513
5th	2.0625	2.125	2.09375	-0.0089
6th	2.09375	2.125	2.109375	0.166835
7th	2.09375	2.109	2.101562	0.078562
8th	2.10156	2.093	2.09765	0.03464
9th	2.09765	2.093	2.0957	0.012827
10th	2.0957	2.093	2.0957	0.001936

Hence we find the value till we don't get the value of c till 3 place of decimal and hence we get it and the 10th iteration.

B. Regula-falsi Method

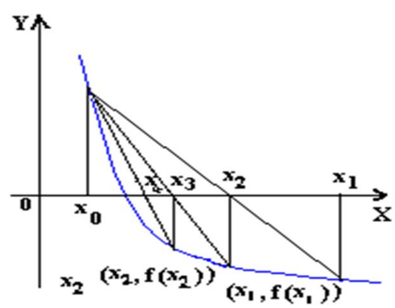

Fig 2.2: Graphical Representation of Regula-Falsi Method

Regula-Falsi method is quite same as secant method. In secant method the approximate function f(x) by a straight line. The point at which the line crosses x(ax1) is called approximate value of that function.[3]

If x(0) & x(1) are the initial approximation then approximate line to the function f(x) passes through point [X0,f(X0)]&[X1,f(X1)]

If approximation are select in such a way that if f(x) <0 then secant method is called as Regula-Falsi method.

f(X).f(Xn+1) < 0 crosses zero at certain point hence method converges fast.

Xn+1 = Xn - $\frac{Xn-Xn-1}{f(Xn)-f(Xn-1)} * f(Xn)$

let us understand this method by an example-

F(x) $=x^3-2x-5$ (This is the given polynomial equation and we have to find the roots of the equation)
F(0)= -5, f(1)= -6, f(2)= -1, f(3)= 16
Hence f(2).f(3)<0, X0=2 & X1=3 and f(X0)=-1& f(X1)=16
1st iteration
For n=1
X2 = X2 – $\frac{X1-X0}{f(X1)-f(X0)} * f(X1)$
= 2.058823
F(X2)= -0.390805
2nd iteration
For n=2
X3 = X2–$\frac{X2-X1}{f(X2)-f(X1)} * f(X2)$
=2.081263423
F(X3)=-0.147244

Table 2.2: Result of Regula-Falsi Method

Number of iteration	Value of X0	Value of X1	Xn+1 = Xn - $\frac{Xn-Xn-1}{f(Xn)-f(Xn-1)} * f(Xn)$	F(Xn+1
1st	2	3	2.058823	-0.390805
2nd	2.058823	3	2.081263423	-0.2055
3rd	2.081263423	2.058823	2.09483784	0.00277
4th	2.09483784	2.081263423	2.094579431	2.06*10-4

As we can see that value of 4th iteration and the value of 5th iteration are matching after 4 place of decimal point.

C. Newton-Raphson Method

Let x0 be the initial root of the equation. X
X1=X0+h--------(1)
Using Taylor series for equation
F(x1)= f(x0)+h.f'(x0)+h^2/2.f''(X0)+..............
Therefore, h is small so neglect higher order terms.
F(X1)=f(X0)+h.f'(x0)=0
h=-f(X0)/f'(X0)
substitute value of h in equation (1)
X1=X0--$\frac{f(X0)}{f'(X0)}$
In general, the formula is:
Xm+1=Xm--$\frac{f(Xm)}{f'(Xm)}$
(*) To check approximate is correct or not—
(1) If f'(X0)=0 change initial value
(2) Use the condition f(X).f'(X)> 0
Let us understand this with an example...
F(x) $=x^3-2x-5$
F'(x)$=3x^2-2$
F(1)=-6,F'(1)=1
F(2)=-1,F(2)=10
F(3)=16,F'(3)=25
H=-16/25=0.64
1st iteration

X0=3

X1=3+0.64=2.36

$X2 = 2.36 - \frac{3.4242}{14.7088} = 2.127200$

Table 2.3: Result of Newton-Raphson Method

Number of iteration	Values of X	Value came out from formula
1st	X1	2.36
2nd	X2	2.127200
3rd	X3	2.095136
4th	X4	2.09455167
5th	X5	2.094551482

As we can see that value of 4th iteration and the value of 5th iteration are matching after 4 place of decimal point.

III. CONCLUSION

The analysis carried out through this paper is that Newton Raphson method and Regula-Falsi method are easy to use and the implement to find roots of any equation as compared to Bisection method. In Bisection method we had to carry the calculations up to 10th iteration while in Newton Raphson method and Regula-Falsi method we got the desired result in the 5th iteration. Hence, we can conclude that for Bisection method, there is more probability for error since the convergence is slow and number of iterations is more. In Newton – Raphson, as we calculate the derivative of the given function, the calculation is converged at faster rate with less number of iterations. With help of Regula–Falsi method the result obtained are same like Newton – Raphson method, which may change with the function used. Depending on the function used the number of iterations and convergence rate changes. We also conclude that the Newton – Raphson method is having fast convergence rate to obtain the roots of any equations, hence it is most widely used.

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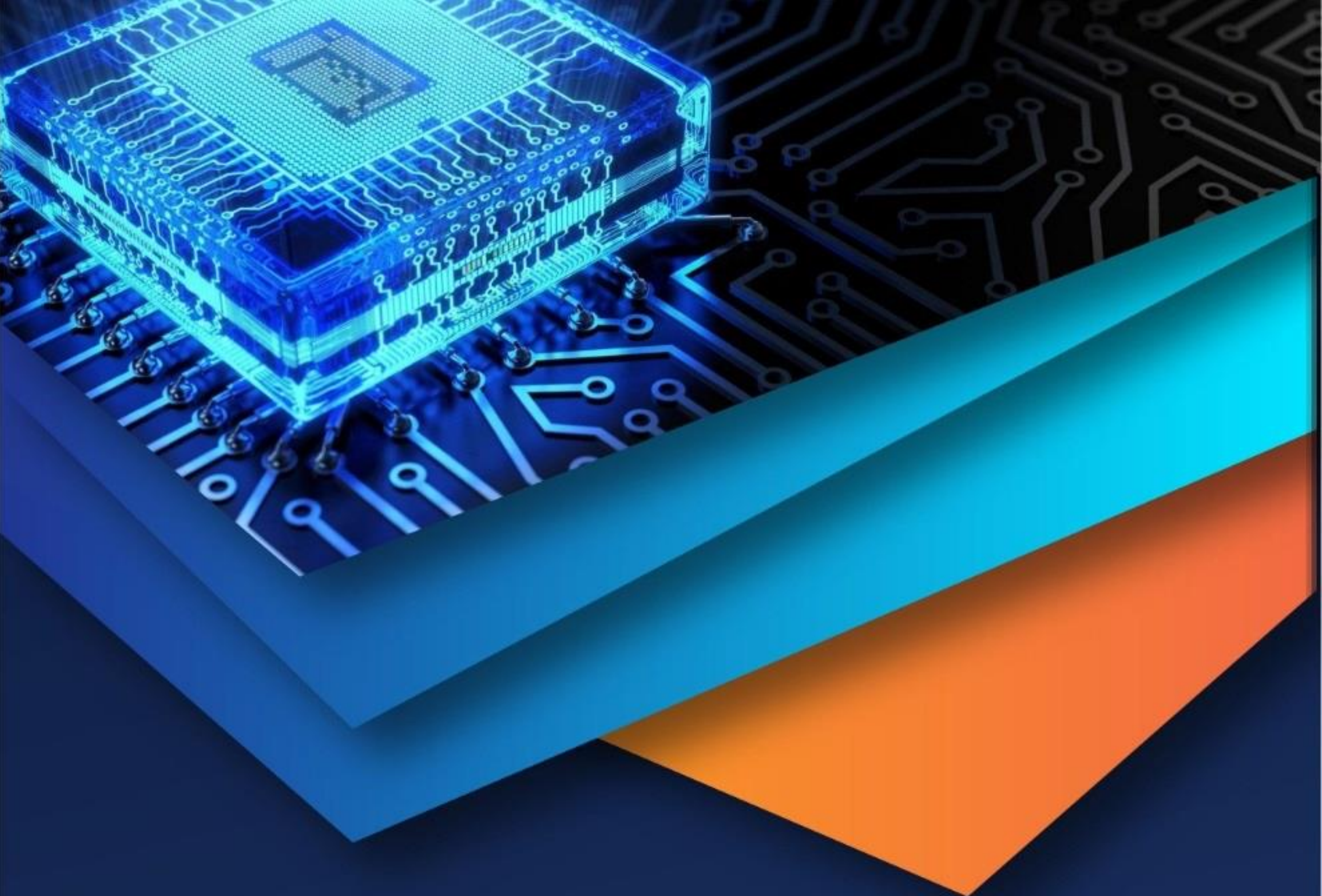

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