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On Sum-Distance in Fuzzy Graphs

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***Abstract:** The distance and related concepts like eccentricity, radius, diameter, center, periphery, etc. were already defined and used in many applications of graph theory. In this we paper, we investigate the idea of Fuzzy distance, which is a metric, in fuzzy graphs. The concepts of eccentricity, radius, diameter, centre and self centered fuzzy graphs are studied using this metric and some properties of eccentric nodes, peripheral nodes and central nodes are obtained. We will show that any complete fuzzy graph can be embedded as an induced subgraph of a self-centered fuzzy graph.*
***Keywords:** Fuzzy graph, Fuzzy distance, Sum Distance, Centre, Self-Cantered graph,*
***AMS Subject Classification Number [2010]:** 05C72, 05C12, 05C38, 05C40, 90C35.*

I. INTRODUCTION

A mathematical frame work to describe the phenomena of uncertainty in the real life problems are suggested by Zadeh [5]. The advantage of replacing the classical sets by fuzzy sets is that it takes the practical diversity and the absence of an absolute precision into account and thus gives more accuracy in theory and more efficiency and system compatibility in applications.

Cohen [1] introduced the notion of competition graphs to model ecological problems. Sunitha and Vijayakumar [4] studied some metric aspects of fuzzy graphs Akram et al. [3] defined length, distance, eccentricity, radius and diameter of a bipolar fuzzy graph and has introduced the concept of self centered bipolar fuzzy graphs. Mini Tom and Sunitha [2] introduced the notion of Sum Distance in Fuzzy Graphs. In this paper, we study the Fuzzy distance, which is a metric, in fuzzy graphs. Section 2 contains the preliminaries and in section 3, the sum distance in fuzzy graphs, which is a metric, is studied. Based on this metric, eccentricity, radius, diameter, centre in fuzzy graphs are defined. In section 4, we have the complete embedding theorem which describes the construction of a complete self-centered fuzzy graph *G* from a given complete self-centered fuzzy graph *H* such that H is an induced subgraph of the self-centered fuzzy graph, G.

II. PRELIMINARIES

Definition 2.1. A fuzzy graph is denoted by $G:(V,\sigma,\mu)$ where V is a vertex set, σ is a fuzzy subset of V and μ is a fuzzy relation on σ such that $\mu(\mathrm{x},\mathrm{y}) \le \sigma(x)\wedge\sigma(y), \forall x,y\in V$.

It is assumed that V is finite and nonempty, μ is reflexive and symmetric. In all the examples σ is chosen suitably. We denote the underlying crisp graph by $G^*:(\sigma^*,\mu^*)$ where $\sigma^*=\{u\in V:\sigma(u)>0\}$ and $\mu^*=\{(u,v)\in V\times V:\mu(u,v)>0\}$. It is assumed that $\sigma^* = V$

Definition 2.2. A fuzzy graph $H:(V,\tau,\nu)$ is said to be partial fuzzy subgraph of $G:(V,\sigma,\mu)$ if $\tau(u)\le\sigma(u), \forall u\in\tau^*$ and $\nu(u,v)\le\mu(u,v), \forall(u,v)\in\nu^*$.

$H:(V,\tau,\nu)$ is called a fuzzy subgraph of $G:(V,\sigma,\mu)$ if $\tau(u)=\sigma(u), \forall u\in\tau^*$ and $\nu(u,v)=\mu(u,v), \forall(u,v)\in\nu^*$

If in addition $\tau^*=\sigma^*$,then H is called a spanning subgraph of G.

Definition 2.3. An arc in $G:(V,\sigma,\mu)$ is called weakest arc if it has least membership.

Definition 2.4. In a fuzzy graph $G:(V,\sigma,\mu)$, a path P of length n is a sequence of distinct nodes $u_0,u_1,u_2,......u_n$ such that $\mu(u_{i-1},u_i)>0, i=1,2,3,.....n$ and Strength of a path is the degree of membership of a weakest arc in a path.

If $u_0=u_n$ and $n\ge 3$ then P is called a cycle and P is called a fuzzy cycle if it contains more than one weakest arc.

Definition 2.5. A fuzzy graph, $G:(V,\sigma,\mu)$ is a complete fuzzy graph if $\mu(u,v)=\sigma(u)\wedge\sigma(v), \forall u,v\in\sigma^*$

Definition 2.6. μ -length of a $u-v$ path P is the sum of the reciprocals of the arc-weights in P and the distance between u and v is called the μ -distance and is denoted by $d_\mu(u,v)$, the smallest μ -length of P.

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Definition2.7. Let $G:(V,\sigma,\mu)$ be a fuzzy graph. The strength of connectedness between two nodes u and v is defined as the maximum of the strength of all paths between u and v and is denoted by $CONN_G(u,v)$. A fuzzy graph $G:(V,\sigma,\mu)$ is connected if for every $u,v\in\sigma^*$, $CONN_G(u,v)>0$.

Definition 2.8. An arc of a fuzzy graph $G:(V,\sigma,\mu)$ is called strong if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted.

Definition 2.9. A strong path from u to v is a $u-v$ geodesic if there is no shorter strong path from u to v and the length of a $u-v$ geodesic is the geodesic distance from u to v denoted by $d_g(u,v)$.

Definition 2.10. An isomorphism between two fuzzy graphs $G_1(V_1,\sigma_1,\mu_1)$ and $G_2(V_2,\sigma_2,\mu_2)$ is a bijective map $h:V_1\to V_2$ that satisfies $\sigma_1(u)=\sigma_2(\mathrm{h}(u))$, $\forall u\in V_1$ and $\mu_1(u,v)=\mu_2(\mathrm{h}(u),\mathrm{h}(v))$, $\forall u,v\in V_1$.

III. SUM DISTANCE IN FUZZY GRAPHS

Definition 3.1. Let $G:(V,\sigma,\mu)$ be a connected fuzzy graph. For any path P: $(u_0,u_1,\dots u_n)$ length P is defined as the sum of the membership grades of the arcs in P. $L(P)=\sum_{i=1}^{n}\mu(u_{i-1},u_i)$ If n=0, define $L(P)$=0 and for $n\geq 1$, $L(P)>0$.

For any two nodes u, v in G, let $P=\{P_i : P_i$ is a $u-v$ path, $i=1,2,3,\dots\dots\}$. The sum distance between u and v is defined as $d_s(u,v)=Min\{L(P_i): P_i\in P, i=1,2,3,\dots\dots\}$

Remark. 3.2. If $\mu(u,v)=1,\forall(u,v)\in\mu^*$, then $d_s(u,v)$ is the length of the shortest path as in the crisp graph.

Theorem 3.3. In a fuzzy graph $G:(V,\sigma,\mu)$, $d_s: V\times V\to[0,1]$ is a metric on V.

a) $d_s(u,v)\geq 0, \forall u,v\in V$ and $d_s(u,v)=0$ if and only if $u=v$ $\forall u,v\in V$ (Positivity)
b) $d_s(u,v)=d_s(u,v)$ $\forall u,v\in V$ (Symmetry)
c) $d_s(u,v)\leq d_s(u,w)+d_s(w,v)$ $\forall u,v,w\in V$ (Transitivity)

Definition 3.4. Let $G:(V,\sigma,\mu)$ be a connected fuzzy graph and let u be a node of G. The eccentricity $e((u)$ of u is the sum distance to a node farthest from u. ie, $e((u)=\max\{d_s(u,v): v\in V\}$. For a node u, each node at sum distance $e(u)$ from u is an eccentric node for u denoted by u^*

Definition 3.5. A fuzzy graph G is a Unique Eccentric Node Fuzzy Graph if each node in G has a unique eccentric node.

Definition 3.6. The radius $r(G)$ of a fuzzy graph G is the minimum eccentricity of the nodes and the diameter $d(G)$ is the maximum eccentricity.

Definition 3.7. A node u is a central node if $e((u)=r(G)$. If $C(G)$ is the set of all central nodes, fuzzy subgraph induced by $C(G)$, denoted by $\langle C(G)\rangle = H:(V,\tau,\upsilon)$ is the centre of G.

Definition 3.8. A connected fuzzy graph is self-centred if each node is a central node. i. e, $G\approx H$. A node u is a peripheral node if $e((u)=d(G)$

Theorem 3.9. For any connected graph $G:(V,\sigma,\mu)$, $r(G)\leq d(G)\leq 2r(G)$

Theorem 3.10. For any connected fuzzy graph $G:(V,\sigma,\mu)$, $|e(u)-e(v)|\leq d_s(u,v)$.

Theorem 3.11. If $G:(V,\sigma,\mu)$ is a self-centred fuzzy graph, then each node of G is eccentric.

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Theorem 3.12. In a fuzzy graph $G:(V,\sigma,\mu)$ all peripheral nodes are eccentric nodes.

Remark. 3.13. Converse of the above theorem is not true.

IV. EMBEDDING THEOREM FOR SELF-CENTRED FUZZY GRAPHS

In this section, we show that any complete self-centred fuzzy graph can be embedded as an induced subgraph of a complete self-centred fuzzy graph.

Theorem.4.2. (Complete Embedding Theorem) Let $H:(V,\sigma',\mu')$ be a complete self-centred fuzzy graph. Then there exists a connected complete self-centred fuzzy graph $G:(V,\sigma,\mu)$ such that H is contained in G as an induced subgraph.

Proof: Let $H:(V,\sigma',\mu')$ be a complete self-centred fuzzy graph with n nodes. Let $c = \wedge\sigma'(u)$, $\forall u \in H$, with $e(u) = 2t, \forall u \in H$, $(0 < t \leq c)$. Let $G:(V,\sigma,\mu)$ be a fuzzy graph obtained by adding four new nodes $\{u_1,u_2,u_3,u_4\}$ to H and let $\sigma^* = \sigma'^* \cup \{u_1,u_2,u_3,u_4\}$ where $\sigma = \sigma'$ for all nodes in H and $\mu = \mu'$ for all arcs in H. Put $\sigma(u_i) = t, i = 1,2,3,4$ and $\mu(u_1,u_2) = \mu(u_3,u_4) = t$. Also let $\mu(u_i,w) = t, \forall w \in H$. Then $G:(V,\sigma,\mu)$ is a complete fuzzy graph with $e(x) = 2t, \forall x \in G$ so that $r(G) = d(G) = 2t$. Thus, G is self-centred fuzzy graph and clearly H is contained in G as an induced subgraph.

It is illustrated for a complete self-centred fuzzy graph $H:(V,\sigma',\mu')$. In H, $\sigma(u)=\sigma(v) = \sigma(w) = \sigma(x)=0.2$ and $\mu(u,v) = \mu(u,w) = \mu(u,x) = \mu(v,w) = \mu(v,x) = \mu(w,x) = 0.2$. In the complete self-centered fuzzy graph, G, Let $\sigma(u_i) = 0.1$, $i = 1,2,3,4$ and $\mu(u_1,u_2) = \mu(u_3,u_4) = 0.1$, $\mu(u_i,w) = 0.1, \forall w \in H$

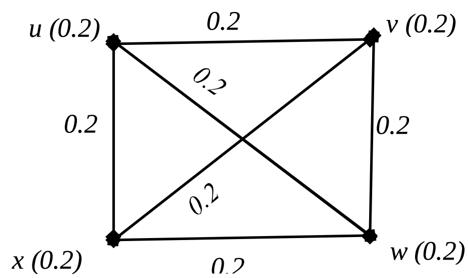

Figure.1. A complete fuzzy graph, H

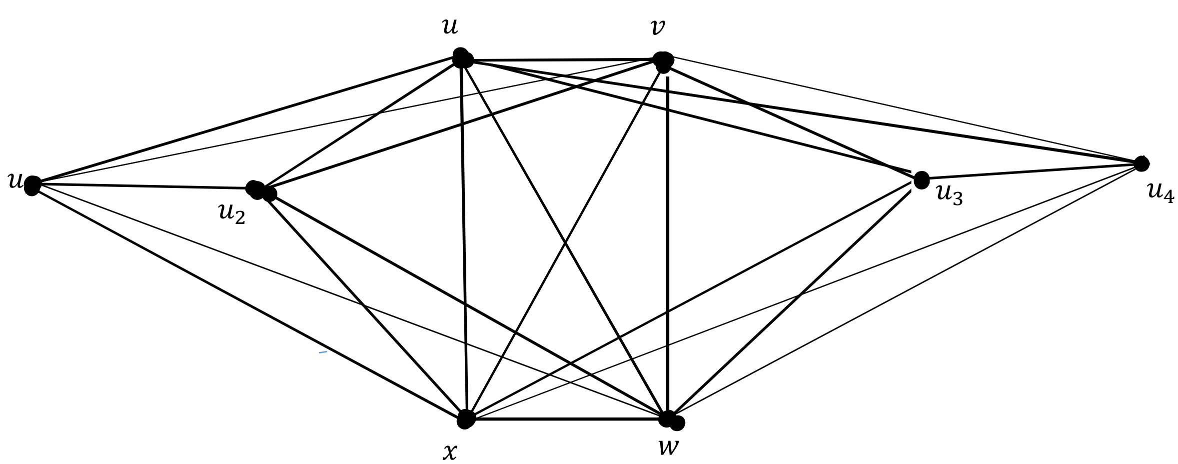

Figure.2. A complete self-centered fuzzy graph, G

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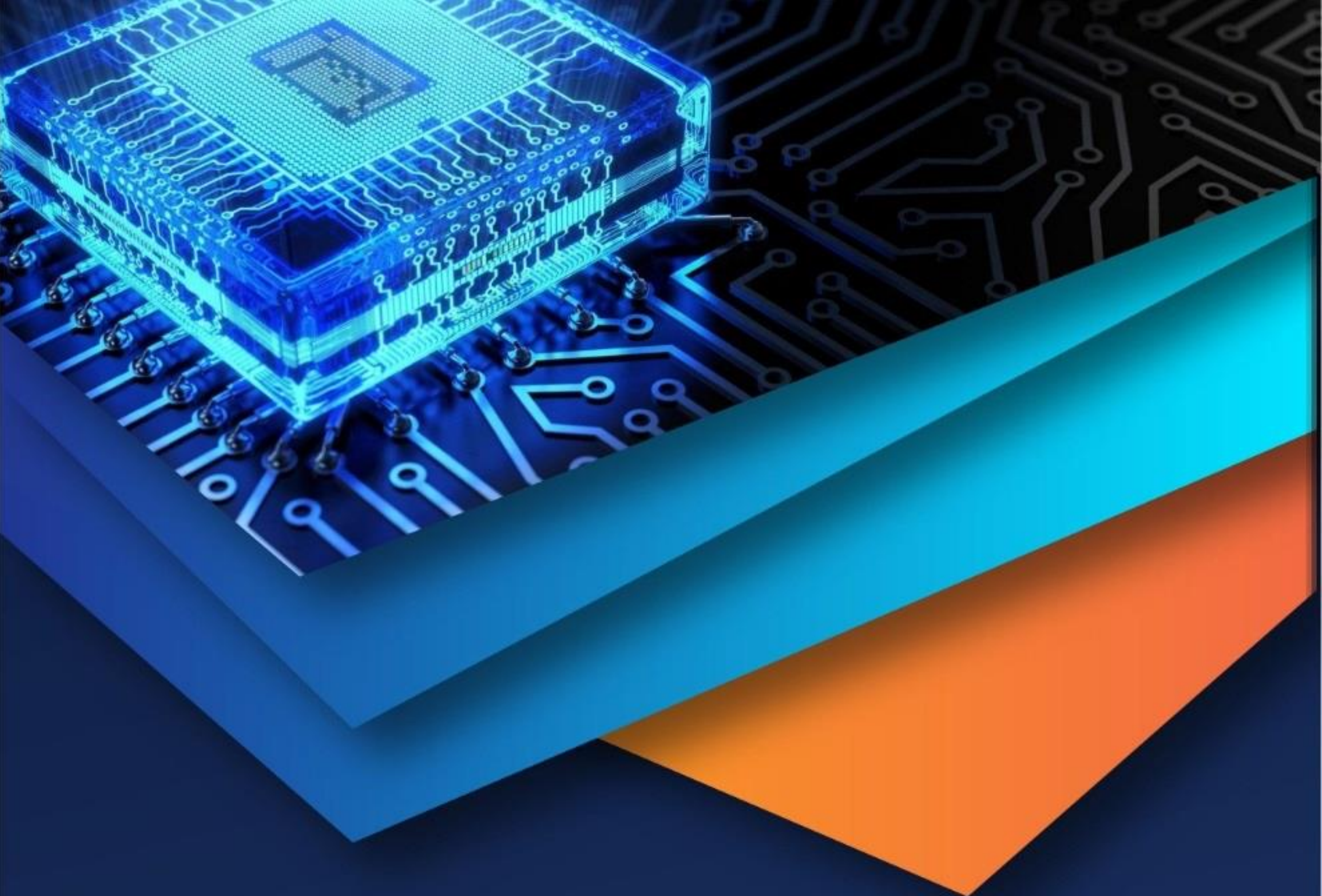

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