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Fuzzy Generalised Mean and Variance

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Abstract: A data consists of several values (called data points) and the aim of Statistical analysis is to explore or describe the data values. Suresh Kumar [1] introduced Generalised Arithmetic mean and Generalised Geometric mean. Suresh Kumar and Sarika M Nair [2] studied them in detail and extended the work to the Generalised Variance and the Generalised covariance and the Generalised Correlation coefficient. In this paper, we investigate the notion of the Fuzzy Generalised Mean, the Fuzzy Generalised Variance, the Fuzzy Generalised Covariance and Fuzzy Generalised Correlation by taking the fuzziness of the edges also into account and to explore the approach of Non-linear, Fuzzy and Combinatorial relationship among the data, through the Fuzzy graph models.
Keywords: Data, Fuzzy Graphs, Generalised Arithmetic Mean, Generalised Variance, Generalised Covariance,

I. INTRODUCTION

Statistical data consists of several values (called data points) and the aim of Statistical analysis is to explore or describe the data values and investigates the relationship of the data. The disadvantage of the mean is that it is sensitive to some extreme value, especially when the sample size is small. So, it is not an appropriate measure of central tendency for the skewed distribution. For example, in a company in which a few employees draw cadre wise salaries plus allowances and performance based incentives, the usual mean will not give any realistic measure of the employee's average monthly income.

In many situations, the Fuzziness also needs to be considered since the employees are subject to variation by firing or self-movement.

Sunil M.P. and J. Suresh Kumar [3, 4] studied the fuzzy graphs and showed how they can be used to accommodate the fuzziness in the distance concept in Graphs. In the modern new-generation business Environment, there are several layers of employees like Managers, Advisors, Sales Representatives etc. so that there is an inter-relationship (usually Hierarchical) among the employees as well so that the actual monthly income of an employee depends on the amount of business capital generated by that person,.

The Fuzzy Generalised Arithmetic mean (FGAM) can be used for the data where some data points are more important than some other values so that they shall contribute more to the final "average" and it also gives the inter-relationships among the data values and the degree of Fuzziness of the pints and their relationships as well. J. Suresh Kumar and Sunil M.P.[5] introduced and studied the Vertex-Edge Fuzzy generalised mean and variance taking both vertex and edge membership values into account.

In a University, various courses have to be awarded credit points depending on their relevance and applicability to the core discipline. Professors or teachers handling the curriculum design shall assign credits to the courses, The courses and their relationships are Fuzzy if rapid changes or advancements in the courses are accommodated in time. The Fuzzy Generalised Arithmetic mean can be effectively used when calculating a credit for a specific course by seeing the connections among the various courses.

The Fuzzy Generalised Arithmetic Mean is similar to an ordinary arithmetic mean with an exception that instead of each data point contribute equally to the final average, some data points are inter-related to many others and thus contribute more than others. This concept plays a vital role in descriptive statistics and also occurs in a more general form in several other areas of applied mathematics. When the weights of all data points are equal, then the Fuzzy Generalised Arithmetic Mean is the same as the usual arithmetic mean.

Covariance is a statistical measure of dispersion used in the Correlation analysis to explore the "Linear" relationship among the data. We propose the notion of the Fuzzy Generalised Covariance to give an approach of Non-linear "Combinatorial" relationship among the data, through Fuzzy graph models. For statistical terms and notations not explicitly mentioned, reader may refer VK Rohatgi [6]. For graph terms and notations not explicitly mentioned, reader may refer Harary [7].

II. MAIN RESULTS

A. Fuzzy Generalized Arithmetic Mean (FGAM)

Arithmetic mean is an average and represents a measure of the central tendency of the data values. For a given set of n elements from a_1 to a_n, the arithmetic mean of these numbers is defined as $(a_1 + a_2 + \cdots + a_n)/n$ and is referred to as AM.

The Fuzzy Generalized Arithmetic mean (FGAM) can be used for the data where some data points or the relationships among them are fuzzy so that vertices contribute differently to the final "average". In this paper, while calculating it, we consider only the fuzziness of the relationships among the data values. The accommodation of the vertex fuzziness (in terms of vertex memberships) is reserved for the future work.

1) *Definition.* Let $a_1, a_2 \ldots\ldots a_n$ be a given set of numbers. Consider a fuzzy graph $G = (V, \sigma, \mu)$ with n vertices and assign these numbers as its vertex labels. For any edge $\{v_i, v_j\}$ of the graph G, assign the label $[\mu(v_i, v_j)\,(a_i + a_j)]\,/n$. Then Fuzzy Generalized Arithmetic Mean (FGAM) is defined as the sum of all the edge labels of G and is denoted by FGAM(G). We recall that the degree of a vertex, v, in a Fuzzy graph G is the sum of the membership values of the edges incident at v.
2) *Theorem.* For a Fuzzy graph $G = (V, \sigma, \mu)$ with vertex degrees, $d_1, d_2 \ldots\ldots d_n$, Fuzzy Generalised Arithmetic mean is given by $FGAM = \sum_{i=1}^{n} a_i d_i / n$.

a) *Proof:* For any edge $\{v_i, v_j\}$ of G, we are assigning the label $[\mu(v_i, v_j)\,(a_i + a_j)]\,/n$. Thus the contribution of the vertex v_i is $\mu(v_i, v_j)\,a_i/n$ for this edge. Hence, if we count the sum of all the edge labels of G, then each vertex v_i with degree $d_i = \sum_{v_j} \mu(v_i, v_j)$ contribute $d_i a_i / n$ to the sum of the edge labels of all edges of the graph. Hence Fuzzy Generalised Arithmetic mean is $\sum_{i=1}^{n} a_i d_i / n$.

The following corollaries are immediate from the above theorem.

3) *Corollary.* For 1-Regular graph, FGAM is same as the usual AM of a set of numbers
4) *Corollary.* For 2-Regular graph, FGAM will be twice that of the usual AM of a set of numbers.
5) *Corollary.* For a k-regular graph, FGAM of a set of numbers is k times that of usual AM.

The following is a bound for Fuzzy Generalised Arithmetic Mean of a given set of numbers.

6) *Corollary.* For a Fuzzy graph G with n vertices, $FGAM(G) \leq (n-1)AM$

a) *Proof.* Since for a graph G with vertex degrees, $d_1, d_2 \ldots\ldots d_n$, Fuzzy Generalised Arithmetic mean is given by $\sum_{i=1}^{n} a_i d_i / n$. Also the FGAM is the maximum when $d_1, d_2 \ldots\ldots d_n$ have the maximum value, which is (n-1). Hence, the inequality follows. The complement of the Fuzzy graph $G = (V, \sigma, \mu)$ is defined as the Fuzzy graph, $G' = (V, \sigma', \mu')$, where $\sigma'(v) = 1 - \sigma(v), \mu'(\{u, v\}) = 1 - \mu(\{u, v\})$ Now we proceed to investigate the Nordhaus-Gaddum type result for the graph parameter, the Fuzzy Generalised Arithmetic Mean. Let G be a graph with vertex degrees, $d_1, d_2 \ldots\ldots d_n$ so that $\bar{G}$ is the graph with vertex degrees, $n-1-d_1, n-1-d_2 \ldots\ldots n-1-d_n$. Then the Fuzzy Generalised Arithmetic mean of the numbers $a_1, a_2 \ldots\ldots a_n$ with respect to the graph G satisfies the same lower and upper Nordhaus-Gaddum bounds as below.

7) *Theorem.* For a graph G with n vertices, $FGAM(G) + FGAM(G') = (n-1)\,AM$

a) *Proof:* Let G be a graph with n vertices. $a_1, a_2 \ldots\ldots a_n$. Any edge $\{v_i, v_j\}$ of G gets the label $[\mu(v_i, v_j)\,(a_i + a_j)]\,/n$. Thus when we calculate the Fuzzy Generalised Arithmetic mean of G, v_i contributes $\mu(v_i, v_j)\,a_i/n$. Thus v_i contribute $\sum_{v_i} \sigma(v_i)\,a_i/n = d_i a_i/n$ to the sum, $FGAM(G)$. Similarly, when we calculate the Fuzzy Generalised Arithmetic mean with respect to G', an edge $\{v_i, v_j\}$ of G' gets the label $[1 - \mu(v_i, v_j)][\,(a_i + a_j)]\,/n$. Thus when we calculate the Fuzzy Generalised Arithmetic mean of G', v_i contributes $[1 - \mu(v_i, v_j)]\,a_i/n$. Thus, v_i contributes $\sum_{v_j}[1 - \mu(v_i, v_j)]\,a_i/n = (n-1-d_i)a_i/n$ to the sum, $FGAM(G')$. Thus, v_i contributes $(n-1)\,a_i/n$ to the sum, $FGAM(G) + FGAM(G')$. Hence the Theorem follows.

B. Fuzzy Generalised Geometric Mean

In this section, we introduce the Fuzzy Generalised Geometric mean (FGGM) of a graph.

1) *Definition.* Let $a_1, a_2, \ldots\ldots\ldots a_n$ be a given set of numbers. Let $G = (V, \sigma, \mu)$ be a Fuzzy graph with n vertices and assign these numbers as its vertex labels. For any edge $\{v_i, v_j\}$, assign the label $(\mu(v_i, v_j)\,a_i a_j)^{1/n}$. Fuzzy Generalised Geometric mean is defined as the product of edge labels of G.
2) *Theorem.* For a graph G with vertex degrees, $d_1, d_2 \ldots\ldots d_n$, Fuzzy Generalised Geometric mean is given by $(a_1{}^{d_1} a_2{}^{d_2} \ldots\ldots a_n{}^{d_n})^{1/n} \prod_{\{v_i, v_j\}} \left(\mu(v_i, v_j)\right)^{1/n}$

a) *Proof:* For any edge $\{v_i, v_j\}$, we assign the label: $\left(\mu(v_i, v_j)a_i a_j\right)^{1/n} = \left(\mu(v_i, v_j)\right)^{1/n} {a_i}^{1/n} {a_j}^{1/n}$. Hence, if we compute the product of all the edge labels of G, then each vertex v_i with degree d_i contributes $\left({a_i}^{1/n}\right)^{d_i}$ to the product of the edge labels of all edges of the graph. Also, each edge $\{v_i, v_j\}$ contributes $\left(\mu(v_i, v_j)\right)^{1/n}$ to the product of the edge labels of all edges of the graph. Hence the Fuzzy Generalised Geometric mean is given by $({a_1}^{d_1}{a_2}^{d_2} \dots \dots .. {a_n}^{d_n})^{1/n} \prod_{\{v_i, v_j\}} \left(\mu(v_i, v_j)\right)^{1/n}$

The following corollaries are immediate from the above theorem.

3) *Corollary*. For a 1-Regular graph, G with $\mu(v_i, v_j) = 1$ for all edges, $\{v_i, v_j\}$ of G, FGGM(G) is the same as the usual GM of given set of numbers and

4) *Corollary*. For a 2-Regular graph G with $\mu(v_i, v_j) = 1$ for all edges, $\{v_i, v_j\}$ of G, FGGM(G) is square of the usual GM of given set of numbers.

5) *Corollary*. For a k-regular graph G with $\mu(v_i, v_j) = 1$ for all edges, $\{v_i, v_j\}$ of G, $FGGM(G) = (GM)^k$

The following is a useful bound for the Generalised Geometric Mean of a set of numbers.

6) *Corollary*. For a Fuzzy graph G with n vertices, $FGGM(G) \leq GM$

a) *Proof.* Since for a graph G with vertex degrees, $d_1, d_2 \dots \dots .. d_n$, Fuzzy Generalised Geometric mean is given by $\prod_{i=1}^{n} \left({a_i}^{1/n}\right)^{d_i} \prod_{\{v_i, v_j\}} \left(\mu(v_i, v_j)\right)^{1/n}$. Now, the required inequality follows at once since the usual Geometric mean is $\prod_{i=1}^{n} {a_i}^{1/n}$ and we have: $0 \leq \mu(v_i, v_j) \leq 1$. Now we proceed to investigate the Nordhaus-Gaddum type result for the graph parameter, the Fuzzy Generalised Geometric Mean. Let G be a graph with vertex degrees, $d_1, d_2 \dots \dots .. d_n$ so that $\bar{G}$ is the graph with vertex degrees, $n - 1 - d_1, n - 1 - d_2 \dots \dots .. n - 1 - d_n$. Then the Fuzzy Generalised Geometric mean of the numbers $a_1, a_2 \dots \dots . a_n$ with respect to the graph G satisfies the the same lower and upper Nordhaus-Gaddum bounds as below.

7) *Theorem.* $FGGM(G).FGGM(G') = (GM)^{n-1} \prod_{\{v_i, v_j\}} \left(\mu(v_i, v_j)[1 - \mu(v_i, v_j)]\right)^{1/n}$

a) *Proof:* Let G be a graph with n vertices. $a_1, a_2 \dots \dots . a_n$ be its vertex labels. Any edge $\{v_i, v_j\}$ of G gets the label, $\left(\mu(v_i, v_j)\right)^{1/n} {a_i}^{1/n} {a_j}^{1/n}$. So when we calculate the Fuzzy Generalised Geometric mean with respect to G, each vertex v_i contribute $\left(\mu(v_i, v_j)\right)^{1/n} {a_i}^{1/n}$. Similarly, when we calculate the Fuzzy Generalised Geometric mean with respect to G', each vertex v_i contribute $\left([1 - \mu(v_i, v_j)]\right)^{1/n} {a_i}^{1/n}$ Thus when we calculate the product $GGM(G).GGM(G')$ each vertex v_i contributes $\left(\mu(v_i, v_j)\right)^{1/n} {a_i}^{1/n} . \left([1 - \mu(v_i, v_j)]\right)^{1/n} {a_i}^{1/n}$. Hence the Theorem follows.

C. *Fuzzy Generalised Variance*

Variance is defined as the arithmetic mean of the squares of the deviations of the observations from their arithmetic mean. The Square root of the variance is called the standard deviation.

1) *Definition.* Let $a_1, a_2, \dots \dots \dots a_n$ be a given set of numbers with mean μ. Let G be any Fuzzy graph with n vertices $v_1, v_2 \dots \dots v_n$ and assign the vertex label $\bar{v}_i = (a_i - \mu)^2$ to each vertex v_i of G. For any edge $\{v_i, v_j\}$ of G assign the label $\mu(v_i, v_j)\left(\bar{v}_i + \bar{v}_j\right)/n$. Then Fuzzy Generalised Variance (FGV) is defined as the sum of all edge labels of G. Square root of the Fuzzy Generalised variance is called the Fuzzy Generalised Standard Deviation (FGSD).

2) *Theorem.* For a graph G with vertex degrees, $d_1, d_2 \dots \dots .. d_n$, Fuzzy Generalised variance is given by $\frac{1}{n}\sum_{i=1}^{n} d_i \bar{v}_i$, where $\bar{v}_i = (a_i - \mu)^2$

a) *Proof:* We assign the label: $\mu(v_i, v_j)\left(\bar{v}_i + \bar{v}_j\right)/n$ to any edge, $\{a_i, a_j\}$. So, if we count the sum of the edge labels of G, then each vertex v_i with degree d_i contribute $d_i\left(\bar{v}_i + \bar{v}_j\right)/n$ to the sum of the edge labels of all edges of the graph. Hence, Fuzzy Generalised variance is $\frac{1}{n}\sum_{i=1}^{n} d_i \bar{v}_i$, where $\bar{v}_i = (a_i - \mu)^2$.

The following corollaries are immediate from the above theorem.

3) *Corollary*. For a 1-Regular graph with $\mu(v_i, v_j) = 1$ for all edges, $\{v_i, v_j\}$ of G, Fuzzy Generalised variance is same as the usual Variance of given set of numbers and

4) *Corollary*. For a 2-Regular graph with $\mu(v_i, v_j) = 1$ for all edges, $\{v_i, v_j\}$ of G, Fuzzy Generalised variance will be twice of the usual variance of given set of numbers.
5) *Corollary*. For a k-regular graph with $\mu(v_i, v_j) = 1$ for all edges, $\{v_i, v_j\}$ of G,, $FGV = k(Variance)$ and $FGSD = \sqrt{k}\, SD$
6) *Corollary*. For a 1-Regular graph Fuzzy Generalised Standard deviation (FGSD) is same as the usual Standard deviation of given set of numbers. For a 2- regular graph FGSD is same as $\sqrt{2}\, SD$.

D. Fuzzy Generalised Covariance

Covariance is a statistical measure used in the Correlation analysis to explore the "Linear" relationship among the data. Fuzzy Generalised Covariance (FGCOV) gives an approach to the Non-linear Combinatorial relationship among the data, through graph models. In this section, we introduce the notion of the Fuzzy Generalised Covariance to give an approach of Non-linear, Fuzzy, Combinatorial relationship among the data, through Fuzzy graph models.

1) *Definition.* Let $\{(x_i, y_i)\}_{i=1,2,\dots n}$ be a given data. Let G be a Fuzzy graph with n vertices. Assign (x_i, y_i) to each vertex v_i of G. Let a be the mean of x_i's and b be the mean of y_i's. To each edge $\{v_i, v_j\}$ of G, assign the label $\mu(v_i, v_j)(\overline{v_i} + \overline{v_j})/n$ where $\overline{v_i} = (x_i - a)(y_i - b)$ and $\overline{v_j} = (x_j - a)(y_j - b)$. Then Fuzzy Generalised Covariance is the sum of all the edge labels of G.
2) *Theorem.* For a Fuzzy graph G with vertex degrees, $d_1, d_2 \dots\dots d_n$, Fuzzy Generalised Covariance is given by $\frac{1}{n}\sum_{i=1}^{n} d_i \overline{v_i}$, where $\overline{v_i} = (x_i - a)(y_i - b)$

a) *Proof:* For any edge $\{a_i, a_j\}$ assign the label: $\mu(v_i, v_j)(\overline{v_i} + \overline{v_j})/n$ where $\overline{v_i} = (x_i - a)(y_i - b)$ and $\overline{v_j} = (x_j - a)(y_j - b)$.Hence, if we count the sum of all the edge labels of G, then each vertex v_i with degree d_i contribute $d_i \overline{v_i}/n$ to the sum of the edge labels of all edges of the graph. Hence the Fuzzy Generalised Covariance is given by $\sum_{i=1}^{n} \overline{v_i} d_i/n$.

The following corollaries are immediate from the above theorem.

3) *Corollary*. For 1-regular graph, Fuzzy Generalised Covariance is same as the usual covariance
4) *Corollary* For 2- regular graph, generalised covariance is twice of the usual covariance.
5) *Corollary*. For k-regular graph, $FGCOV$ is k times the usual covariance.

The following is a useful bound for Fuzzy Generalised Covariance of a given set of numbers.

6) *Corollary*. For a Fuzzy graph G with n vertices, $COV \le FGCOV(G) \le (n-1)COV$Now we proceed to investigate the Nordhaus-Gaddum type result for the graph parameter, the Fuzzy Generalised Covariance. Let G be a graph with vertex degrees, $d_1, d_2 \dots\dots d_n$ so that G' is the graph with vertex degrees, $n-1-d_1, n-1-d_2 \dots\dots n-1-d_n$. Then the Fuzzy Generalised Covariance of the numbers $a_1, a_2 \dots\dots a_n$with respect to the graph G satisfies the the same lower and upper Nordhaus-Gaddum bounds as below.
7) *Theorem.* For a Fuzzy graph G with n vertices, $FGCOV(G) + FGCOV(G') = (n-1)COV$

a) *Proof:* Let G be a Fuzzy graph with n vertices and $a_1, a_2 \dots\dots a_n$ be its vertex labels. Any edge $\{v_i, v_j\}$ of G gets the label $(\overline{v_i} + \overline{v_j})/n$ where $\overline{v_i} = (x_i - a)(y_i - b)$ and $\overline{v_j} = (x_j - a)(y_j - b)$. Thus when we calculate the Generalised Covariance with respect to G, each vertex v_i contribute $\overline{v_i}/n$ to each edge to which it is incident with. Thus each vertex v_i contributes $(n-1)\,\overline{v_i}/n$ to the sum $FGAM(G) + FGAM(G')$. Hence the Theorem follows.

E. Generalised Correlation Coefficient

The Correlation Coefficient quantifies the strength of the linear relationship between two variables in the correlation analysis. The Correlation Coefficient is defined by

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}}$$

1) *Definition.* Let $\{(x_i, y_i)\}_{i=1,2,\dots n}$ be the given data. Let $G = (V, \sigma, \mu)$, be a Fuzzy graph with n vertices. Assign (x_i, y_i) to each vertex v_i of G. Let $X = \{x_i\}_{i=1,2,\dots n}$ and $Y = \{y_i\}_{i=1,2,\dots n}$. Then

$$\text{Fuzzy Generalised Correlation coefficiant} = \frac{\text{Fuzzy } Generalsed\ Covariance\ of\ G}{(FGSD\ of\ X)(FGSD\ of\ Y)}$$

2) *Theorem.* For all regular Fuzzy graphs with n vertices, the Fuzzy Generalised Correlation coefficient is same as the usual Correlation coefficient.

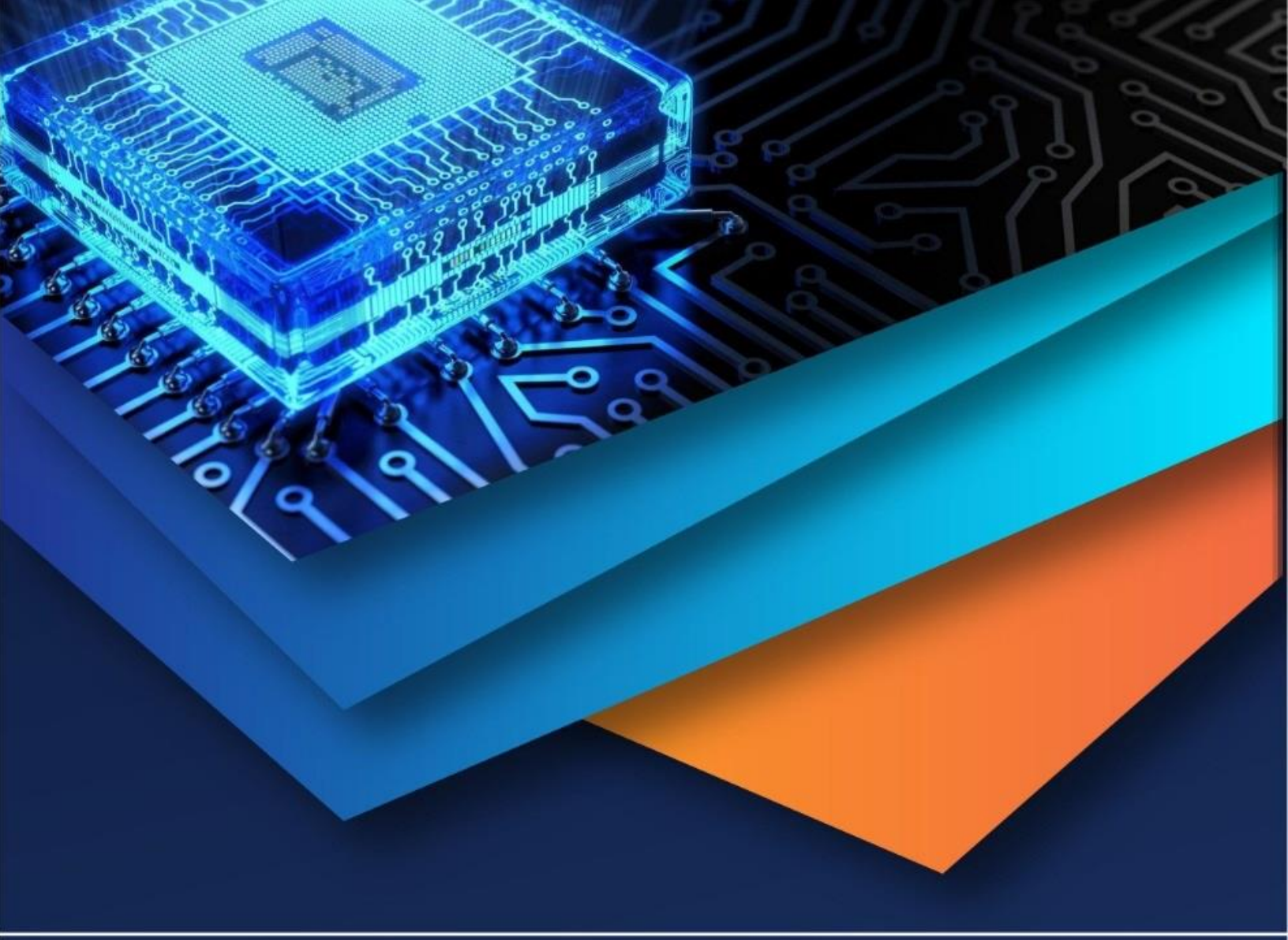
b) *Proof:* Let G be a k-regular Fuzzy graph with n vertices and assign (x_i, y_i) to each vertex v_i of G. Since G is k −regular, $GCOV(G) = k(COV)$, $FGSD(X) = \sqrt{k}\, SD(X)$ and $FGSD(Y) = \sqrt{k}\, SD(Y)$. Hence, the Fuzzy Generalised Correlation coefficient is:

$$\frac{\text{Fuzzy } Generalsed\ Covariance\ of\ G}{(FGSD\ of\ X)(FGSD\ of\ Y)} = \frac{k.Covariance\ of\ G}{(\sqrt{k}.\ SD\ of\ X)(\sqrt{k}\ SD\ of\ Y)}$$

Which is the usual Correlation coefficient and the theorem follows.

REFERENCES

[1] J. Suresh Kumar, Graph Algorithms and Abstraction for Number Theoretic Concepts, International Journal of Mathematics and its Applications (IJMAA), 7(2) (2019), 39-41.
[2] J. Suresh Kumar and Sarika M Nair, Generalized Mean and Variance, Submitted to International Journal of Research and Technology in Engineering (IJRTE), May, 2020 (Pre Print)
[3] J. Suresh Kumar and Sunil M.P., On Fuzzy Distance in Fuzzy Graphs, International Journal of Mathematics and its Applications (IJMAA), 8(1)(2020), 89-93.
[4] J. Suresh Kumar and Sunil M.P., On Sum-Distance in Fuzzy Graphs, International Journal for Research in Applied Science & Engineering Technology (IJRASET), Volume 8 Issue I, Jan 2020.
[5] J. Suresh Kumar and Sunil M.P., VE-Fuzzy Generalized Mean and Variance, International Journal for Research in Applied Science & Engineering Technology (IJRASET), Volume 8 Issue III Mar 2020.
[6] V. K. Rohatgi, AK Md. Ehsanes Saleh, An Introduction to Probability and Statistics, John Wiley and Sons, 2011.
[7] Harary Frank, Graph Theory, Reading Mass, 1969.



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