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Boundary Nodes and Interior Nodes in Bipolar Fuzzy Graphs

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Abstract: A bipolar fuzzy graph is a fuzzy graph which admits positive and negative membership grades. In this paper, we introduce and study the concepts of boundary nodes, interior nodes, cut nodes and complete nodes in bipolar fuzzy graphs.
Keywords: Fuzzy graph, Bipolar fuzzy graph, Sum distance, boundary node, Interior node.

I. INTRODUCTION

Fuzzy set theory was first introduced by Zadeh [7] in 1965. In 1994, Zhang [8] introduced the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are extensions of fuzzy sets whose range of membership degree is $[-1, 1]$. In Bipolar fuzzy sets membership degree 0 of an element means that the element is irrelevant to the respective property, a positive membership degree of an element indicates that the element somewhat satisfies the property and a negative membership degree of an element indicates the element somewhat satisfies the implicit counter property. Several studies on bipolar fuzzy graphs [3, 4, 5, 6] were occurred. Sunil M.P. and J. Sureshkumar [1, 2] studied sum distance in fuzzy graphs. Mini Tom and M.S. Sunitha [3] introduced and studied the concept of boundary nodes and interior nodes in fuzzy graphs using the notion of sum distance. In this paper, we introduce and study the concepts of boundary nodes, interior nodes, cut nodes and complete nodes in bipolar fuzzy graphs.

II. MAIN RESULTS

A. *Bipolar Fuzzy Graphs*

In this section, we recall the definition of bipolar fuzzy graphs and the related terminology.

1) *Definition 2.1.1.* Let X be a non empty set. A bipolar fuzzy set M in X is an object having the form $M = \{(x, \mu^+(x), \mu^-(x))/ x \in X)\}$ where $\mu^+: X \to [0,1]$ and $\mu^-: X \to [-1,0]$ are mappings.
2) *Definition 2.1.2.* A bipolar fuzzy graph with an underlying set V is defined to be the pair (A, B), where $A = (\sigma^+, \sigma^-)$ is a bipolar fuzzy set on V and $B = (\mu^+, \mu^-)$ is a bipolar fuzzy set on E such that $\mu^+(x, y) \le min\{\sigma^+(x), \sigma^+(y)\}$ and $\mu^-(x, y) \ge max\{\sigma^-(x), \sigma^-(y)\}$ for all $(x, y) \in E$. The set A is called bipolar fuzzy vertex set on V and the sets B is called bipolar fuzzy edge set on E.
3) *Definition 2.1.3.* A bipolar fuzzy graph $G = (A, B)$ is connected if the underlying crisp graph is connected.
4) *Definition 2.1.4.* Let G be a bipolar fuzzy graph. A vertex v is called central vertex if radius of G is equal to the eccentricity of v. The set of all central vertices in G is denoted by $C(G)$.The graph, G is said to be self-centered if each vertex of G is a central vertex.
5) *Definition 2.1.5.* Let $G = (A, B)$ be a bipolar fuzzy graph. Then the fuzzy subgraph induced by $C(G)$ is called the center of G.
6) *Definition.2.1.6.* Let $G = (A, B)$ be a connected bipolar fuzzy graph and $P = (u_0, u_1, u_2 \ldots \ldots \ldots u_n)$ be any path in G. The positive length of P is defined as the sum of the positive weights of the edges in P and denoted by. $L^+(P) = \sum_{i=1}^{n} \mu^+(u_{i-1}, u_i)$. The negative length of P is defined as the sum of the negative weights of the edges in P and denoted by $L^-(P) = \sum_{i=1}^{n} \mu^-(u_{i-1}, u_i)$.
7) *Definition 2.1.7.* The positive sum distance between u and v is defined as $d_s^+(u, v) = Min\{L^+(P)\}$ and the negative sum distance between u and v is defined as $d_s^-(u, v) = Max\{L^-(P)\}$.
8) *Definition 2.1.8.* The positive eccentricity $e^+(u)$ is the positive sum distance to a vertex farthest from u. i.e, $e^+(u) = max\{d_s^+(u, v): v \in V\}$. The negative eccentricity $e^-(u)$ is the negative sum distance to a vertex farthest from u. ie, $e^-(u) = min\{d_s^-(u, v): v \in V\}$. The eccentricity of a vertex u is defined as $e(u) = (e^+(u), e^-(u))$.

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9) *Definition 2.1.9.* For a vertex u, each vertex at sum distance $e^+(u)$ from u is positive eccentric vertex of u, denoted by $u^{(*,+)}$ and each vertex at sum distance $e^-(u)$ from u is negative eccentric vertex of u, denoted by $u^{(*,-)}$. The eccentric vertex of u is denoted by $u^* = \left(u^{(*,+)}, u^{(*,-)}\right)$.
10) *Definition 2.1.10.* The radius of a bipolar fuzzy graph G is defined as the pair $r(G) = \left(r^+(G),\ r^-(G)\right)$, where $r^+(G) = min\{e^+(u): u \in G\}$ and $r^-(G) = max\{e^-(u): u \in G\}$.
11) *Definition 2.1.11.* The diameter of a bipolar fuzzy graph G is defined as the pair $d(G) = \left(d^+(G), d^-(G)\right)$, where $d^+(G) = max\{e^+(u): u \in V\}$ and $d^-(G) = min\{e^-(u): u \in V\}$.
12) *Definition 2.1.12.* A vertex u is called a central vertex if $e^+(u) = r^+(G)$ and $e^-(u) = r^-(G)$.
13) *Definition 2.1.13.* A vertex u is called peripheral vertex if $e^+(u) = d^+(G)$ and $e^-(u) = d^-(G)$.
14) *Definition 2.1.14.* A bipolar fuzzy graph G is self centered if each vertex in G is a central vertex.

B. *Boundary Nodes of Bipolar Fuzzy Graph*

In this section, we introduce and discuss the concepts of boundary nodes and boundary of a bipolar fuzzy graph based on sum distance.

1) *Definition 2.2.1.* A node v in a connected bipolar fuzzy graph $G(V, E)$ is a boundary node of a node u if for each neighbour w of v in G, $d_s^+(u, v) \geq d_s^+(u, w)$ for the edges of positive weight and $d_s^-(u, v) \leq d_s^-(u, w)$ for the edges of negative weight.
2) *Definition 2.2.2.* A node v is called a boundary node of a bipolar fuzzy graph G if v is a boundary node of some node of G. The fuzzy subgraph induced by the boundary nodes of G is called the boundary of G, denoted by $\partial(G)$.

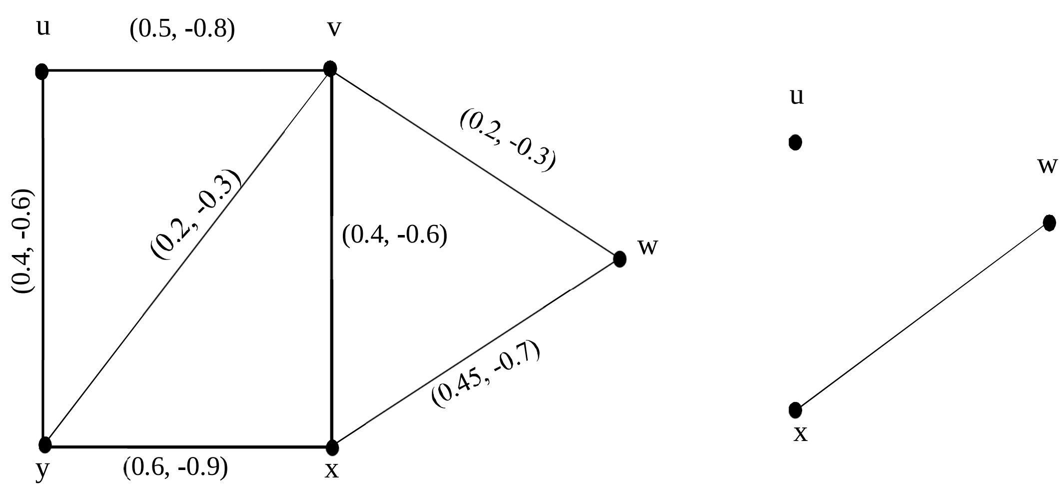

Figure.1. Boundary nodes and Boundary $\partial(G)$ of a Bipolar Fuzzy Graph

3) *Example 2.2.3.* Consider the bipolar fuzzy graph in figure.1.

$d_s^+(u,v) = 0.5$, $d_s^+(u,w) = 0.7$, $d_s^+(u,x) = 0.90$, $d_s^+(u,y) = 0.4$, $d_s^+(v,w) = 0.2$,
$d_s^+(v,x) = 0.4$, $d_s^+(v,y) = 0.2$, $d_s^+(w,x) = 0.45$, $d_s^+(w,y) = 0.4$, $d_s^+(x,y) = 0.6$
$d_s^-(u,v) = -0.8$, $d_s^-(u,w) = -1.1$, $d_s^-(u,x) = -1.4$, $d_s^-(u,y) = -0.6$, $d_s^-(v,w) = -0.3$ $d_s^-(v,x) = -0.6$, $d_s^-(v,y) = -0.3$,
$d_s^-(w,x) = -0.7$, $d_s^-(w,y) = -0.6$, $d_s^-(x,y) = -0.9$.
Thus, $u^b = \{x\}$, $v^b = \{u,x\}$, $w^b = \{u,x\}$, $x^b = \{u,w\}$, $y^b = \{u,x\}$
Also the boundary nodes are u, w and x.

4) *Remark 2.2.4.* In a connected bipolar fuzzy graph $G(V,E)$ all the peripheral nodes are eccentric nodes but the converse need not be true. Also every eccentric node is a boundary node, but every boundary node need not be an eccentric node. In Figure.1, $e^+(u) = d_s^+(u,x)$ and $e^-(u) = d_s^-(u,x) \Rightarrow u^* = x$, $e^+(v) = d_s^+(v,u)$ and $e^-(v) = d_s^-(v,u) \Rightarrow v^* = u$, $e^+(w) = d_s^+(w,u)$ and $e^-(w) = d_s^-(w,u) \Rightarrow w^* = u$, $e^+(x) = d_s^+(x,u)$ and $e^-(x) = d_s^-(x,u) \Rightarrow x^* = u$, $e^+(y) = d_s^+(y,x)$ and $e^-(y) = d_s^-(y,x) \Rightarrow y^* = x$. The eccentric nodes u and x are boundary nodes of G but the boundary node w is not an eccentric node of G.

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5) *Remark 2.2.5.* Self centered bipolar fuzzy graphs have the same set of nodes as eccentric nodes, peripheral nodes and boundary nodes. But a bipolar fuzzy graph having the same set of nodes as eccentric nodes, peripheral nodes and boundary nodes need not be self centered.

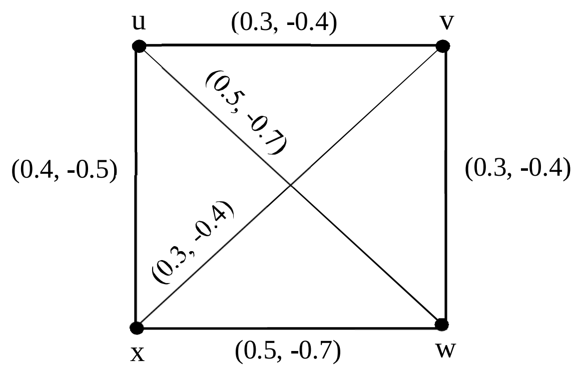

Figure.2. A Bipolar Fuzzy Graph, which is not self centered

6) *Example 2.2.6.* Consider the Figure. 2. $e^+(u) = 0.5$, $e^+(v) = 0.3$, $e^+(w) = 0.5$, $e^+(x) = 0.5$, $e^-(u) = -0.7$ $e^-(v) = -0.4$, $e^-(w) = -0.7$, $e^-(x) = -0.7$, $d^+(G) = 0.5$, $d^-(G) = -0.7$.
Since $e^+(u) = e^+(w) = e^+(x) = d^+(G)$ and $e^-(u) = e^-(w) = e^-(x) = d^-(G)$. Thus, the nodes u, w, x are peripheral nodes. Further, $e^+(u) = d_s^+(u,w)$ and $e^-(u) = d_s^-(u,w) \Rightarrow u^* = w$,
$e^+(v) = d_s^+(v,u) = d_s^+(v,w) = d_s^+(v,x)$,$e^-(v) = d_s^-(v,u) = d_s^-(v,w) = d_s^-(v,x) \Rightarrow v^* = u, w, x$
$e^+(w) = d_s^+(w,u) = d_s^+(w,x)$ and $e^-(w) = d_s^-(w,u) = d_s^-(w,x) \Rightarrow w^* = u, x$.
$e^+(x) = d_s^+(x,w)$ and $e^-(x) = d_s^-(x,w) \Rightarrow$ $x^* = w$. i.e, the nodes u, w and x are eccentric nodes.
$u^b = \{w\}$, $v^b = \{u, w, x\}$, $w^b = \{u, x\}$, $x^b = \{w\}$. i.e, the nodes u, w and x are boundary nodes. Here the bipolar fuzzy graph G have the same set of nodes as eccentric nodes, peripheral nodes and boundary nodes is not self centered.

7) *Remark 2.2.7.* In crisp graphs, no cut node of a connected graph G is a boundary node of G, but in bipolar fuzzy graphs, a fuzzy cut node can be a boundary node. In Figure.1, node x is a fuzzy cut node and also a boundary node.
8) *Theorem 2.2.8.* In a bipolar fuzzy graph $G = (V, E)$, a node u is a cut node implies u is not a boundary node of G.

a) *Proof.* Let G be a bipolar fuzzy graph. Assume, to the contrary that there exists a cut node u of G such that u is a boundary node of some node v of G. let G_1 be the component of $G - u$ containing v and G_2 be any other component of $G - u$. If node w is a neighbour of u belonging to G_2, then $d_s^+(v,w) = d_s^+(v,w) + l$, $0 < l \leq 1$, for the edges of positive weight and $d_s^-(v,w) = d_s^-(v,w) + l$, $-1 \leq l < 0$, for the edges of negative weight ,which contradicts our assumption that u is a boundary node of v.

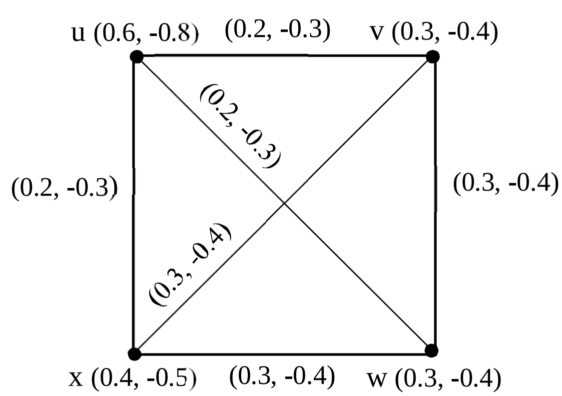

Figure.3. Complete node in a Bipolar Fuzzy Graph

9) *Remark 2.2.9.* In a crisp graphs, a vertex v is a boundary node of every vertex distinct from v if and only if v is a complete node [8]. A complete node in a bipolar fuzzy graph does not need to be boundary node. A node which is a boundary node of all the others, need not be complete.

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10) *Example 2.2.10.* In Figure.3, $u^b = \{v, w, x\}$, $v^b = \{w, x\}$, $w^b = \{v, x\}$, $x^b = \{v, w\}$. The node u is a complete node since its strong neighbours x, v, w form a complete bipolar fuzzy graph but u is not a boundary node. Also node w is a boundary node of all the other nodes but w is not complete since its strong neighbours u, v, x do not form a complete graph.

11) *Remark 2.2.11.* A vertex is an end node of a tree if and only if it is a boundary vertex. But a boundary node of a bipolar fuzzy tree need not be a fuzzy end node. Also a node is a fuzzy end node of a bipolar fuzzy tree need not imply that it is a boundary node.

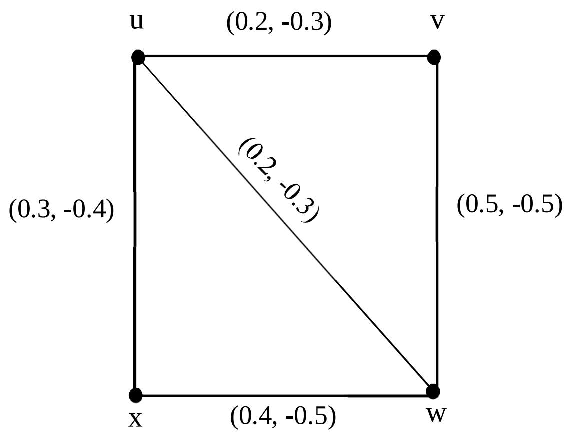

Figure.4. Bipolar Fuzzy Tree

12) *Example 2.2.12.* In Figure.4, u and v are fuzzy end nodes since they have exactly one strong neighbour in G. Boundary nodes are v and x. But x is not a fuzzy end node since there are two strong arcs incident on x and x is a boundary node. The fuzzy end node u is the central node of G.

13) *Theorem 2.2.13.* Let $G = (V, E)$ be a complete bipolar fuzzy graph on n nodes $n \geq 3$ and let u_0 be a node of G. Every node distinct from u_0 is a boundary node of u_0 if and only if u_0 is a weakest node of G.

a) *Proof.* Let $G = (V, E)$ be a complete bipolar fuzzy graph on n nodes, $n \geq 3$. Let u_0 be a node of G such that every node distinct from u_0 is a boundary node of u_0. i.e, u_0^b={$u_i, i = 1,2,3, \dots . n-1$}. We have to prove that u_0 is a weakest node of G. Suppose u_0 is not a weakest node of G. Let u_k be a node of G such that $\sigma(u_k)$ has the least value for the edges of positive weight and maximum value for the edges of negative weight. But for complete bipolar fuzzy graph, $\mu^+(u_{0\,,}u_k)$=$\sigma^+(u_k)$, $\mu^-(u_{0\,,}u_k)$=$\sigma^-(u_k)$ and $\mu^+(u_{0\,,}u_i)$=$\sigma^+(u_0) \wedge \sigma^+(u_i)$ for $i \neq k$, $\mu^-(u_{0\,,}u_i)$=$\sigma^-(u_0) \vee \sigma^-(u_i)$ for $i \neq k$. Thus $\mu^+(u_{0\,,}u_k) < \mu^+(u_{0\,,}u_i)$ and $\mu^-(u_{0\,,}u_k) > \mu^-(u_{0\,,}u_i)$. Also we have $d_s^+(u_0, u_k) = \mu^+(u_{0\,,}u_k)$, $d_s^-(u_0, u_k) = \mu^-(u_{0\,,}u_k)$ and $d_s^+(u_0, u_i) = min\{\mu^+(u_{0\,,}u_i), 2\sigma^+(u_k)\}, i \neq k$ and $d_s^-(u_0, u_i) = max\{\mu^-(u_{0\,,}u_i), 2\sigma^-(u_k)\}, i \neq k$ [5]. Thus $d_s^+(u_0, u_k) < d_s^+(u_0, u_i)$ and $d_s^-(u_0, u_k) > d_s^-(u_0, u_i)$. Hence by the definition of boundary node, u_k is not a boundary node of $u_{0\,,}$ which is a contradiction to the assumption that every node distinct from u_0 is a boundary node of $u_{0\,.}$ Hence u_0 is a weakest node of G. Conversely assume that u_0 is a weakest node of G. By the definition of complete bipolar fuzzy graph, $\mu^+(u_{0\,,}u_i) = \sigma^+(u_0)$, $\mu^-(u_{0\,,}u_i) = \sigma^-(u_0)$, $i = 1,2,3, \dots . n-1$. For any node u_i of G, $d_s^+(u_0, u_i) = \mu^+(u_{0\,,}u_i)$, $d_s^-(u_0, u_i) = \mu^-(u_{0\,,}u_i)$. Hence by the definition of boundary node, every node distinct from u_0 is a boundary node of $u_{0.}$

14) *Corollary 2.2.14.* In a complete bipolar fuzzy graph $G = (V, E)$, if u_0 is the unique weakest node, then every node distinct from u_0 are boundary nodes of G, whereas if the weakest node is not unique, then all the nodes of G are boundary nodes of G.

C. Interior Nodes of a Bipolar fuzzy graph

Interior node of a bipolar fuzzy graph is defined based on sum distance. In crisp graph, the interior nodes are precisely those nodes that are not boundary nodes. But in bipolar fuzzy graphs, there are nodes which are neither boundary nodes nor interior nodes.

1) *Definition 2.3.1.* A node w in a connected bipolar fuzzy graph G is said to lie between two other nodes u and v, which are different from w, with respect to the sum distance if $d_s^+(u, v) = d_s^+(u, w) + d_s^+(w, v)$ for the edges of positive weight and $d_s^-(u, v) = d_s^-(u, w) + d_s^-(w, v)$ for the edges of negative weight.

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2) *Definition 2.3.2.* A node w is an interior node of a connected bipolar fuzzy graph G if for every node u distinct from w, there exist a node v such that w lies between u and v.

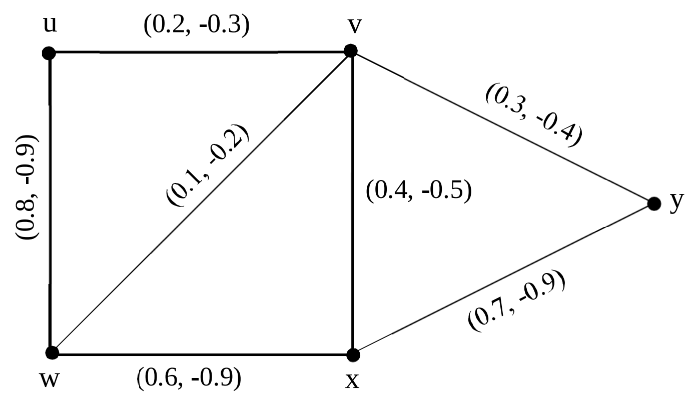

Figure.5. Interior node of a Bipolar Fuzzy Graph

3) *Remark 2.3.3.* In fuzzy graphs, nodes which are not boundary nodes need not be interior nodes. In Figure.5, the boundary nodes are x , u and y. The only interior node is v. The node w is neither a boundary node nor an interior node.

4) *Theorem 2.3.4.* Let G be a bipolar fuzzy graph. A boundary node of G is not an interior node of G.

a) *Proof.* Let v be a boundary node of a connected bipolar fuzzy graph $G = (V, E)$. Let v be the boundary node of the node u. Assume that v is an interior node of G. i.e, there exists a node w which is distinct from u and v such that v lies between u and w. Let P be a $u - w$ path in which v lies. i.e, $P = u, v_1, v_2, v_3, \dots \dots \dots, v = v_j, v_{j+1}, \dots \dots \dots, v_k = w$, $1 < j < k$. Now v_{j+1} is a neighbour of u and $d_s^+(u, v_{j+1}) = d_s^+(u, v) + k$, $0 < k \leq 1$, for edges of positive weight and $d_s^-(u, v_{j+1}) = d_s^-(u, v) + k$,$-1 \leq k < 0$, for edges of negative weight, which is a contradiction to the assumption that v is a boundary node of u.

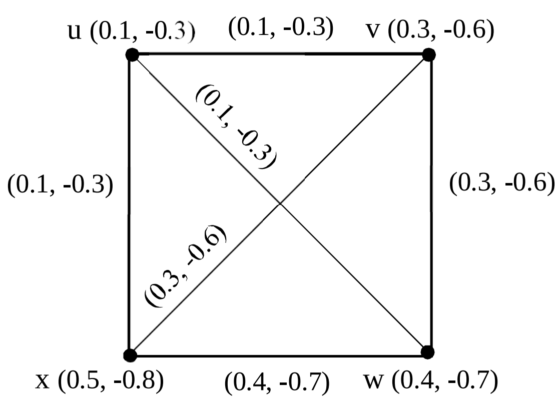

Figure.6. A Complete Bipolar Fuzzy Graph with an interior node

5) *Remark 2.3.5.* Let $G = (V, E)$ be a complete bipolar fuzzy graph. Then by Theorem 2.2.13 and Corollary 2.2.14, there exists at most one node which is not a boundary node. Thus there exists at most one node which is an interior node. Interior node if it exists is the unique weakest node of G.

6) *Example 2.3.6.*

In Figure.6, $d_s^+(u, v) = d_s^+(u, w) = d_s^+(u, x) = 0.1$, $d_s^+(v, w) = d_s^+(v, x) = d_s^+(w, x) = 0.2$
$d_s^-(u, v) = d_s^-(u, w) = d_s^-(u, x) = -0.3$, $d_s^-(v, w) = d_s^-(v, x) = d_s^-(w, x) = -0.6$,
$d_s^+(v, w) = d_s^+(v, u) + d_s^+(u, w)$,$d_s^+(v, x) = d_s^+(v, u) + d_s^+(u, x)$,$d_s^+(w, x) = d_s^+(w, u) + d_s^+(u, x)$, $d_s^-(v, w) = d_s^-(v, u) + d_s^-(u, w)$,$d_s^-(v, x) = d_s^-(v, u) + d_s^-(u, x)$,$d_s^-(w, x) = d_s^-(w, u) + d_s^-(u, x)$. Hence u is the only interior node.

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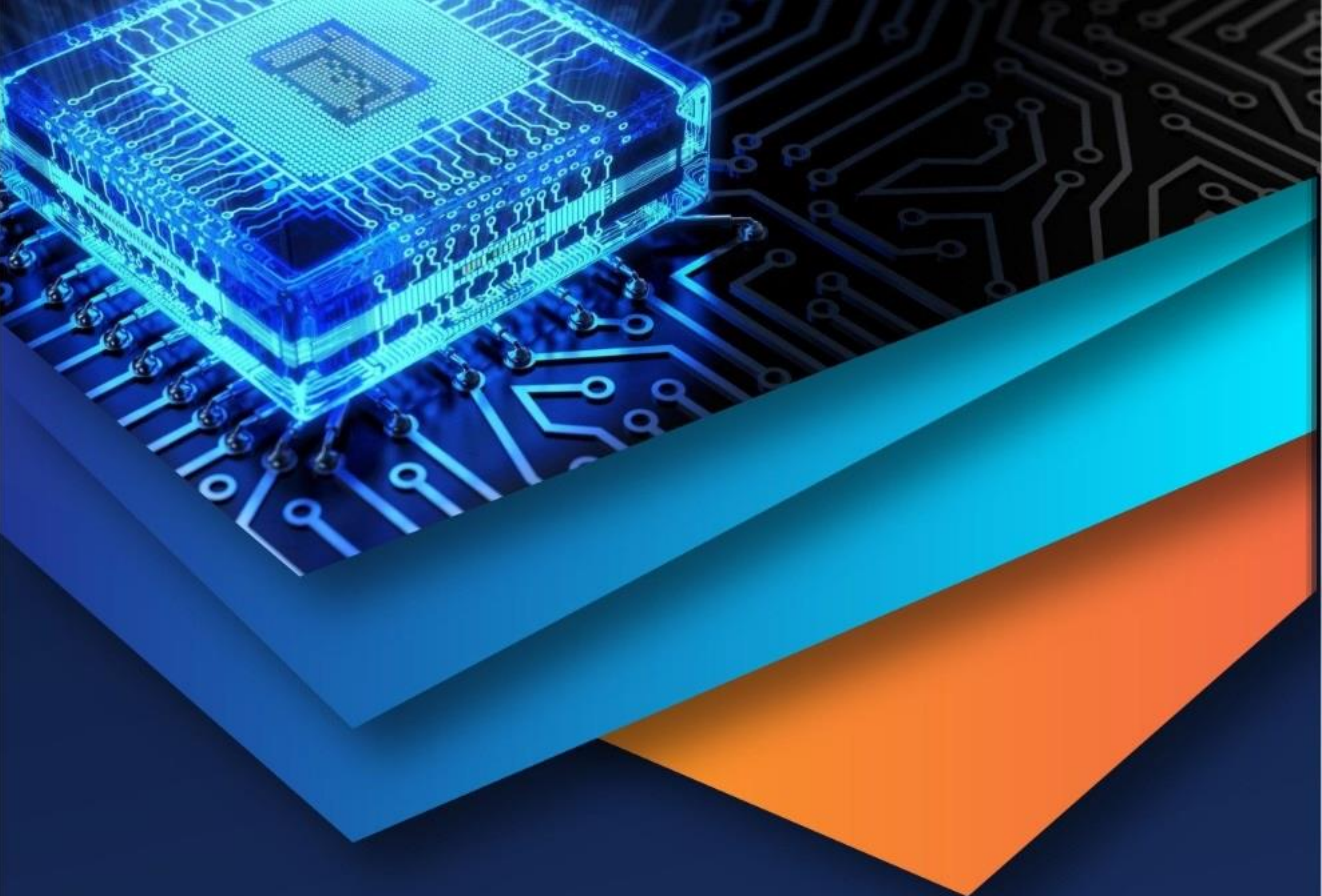

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