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Some Improved Classes of Estimators using Auxiliary Information

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Abstract: This paper addresses the problem of estimating the population mean using auxiliary information. A class of linear combination of estimators have been proposed including Srivastava and Walsh type estimators for estimating the population mean. The properties of the suggested family have been discussed. Expressions for the bias and mean square error (MSE) of the suggested family have been derived. It has been shown that the proposed class of estimators has minimum mean square of error as compared to various estimators available in the literature of sampling. An empirical study has been also included at the end to support the fact.
Keywords: Multiple auxiliary variable, bias, mean square error, efficiency.

I. INTRODUCTION

In sampling, the use of auxiliary information has been permeated the important role to improve the efficiency of the estimators. It is well known that the use of auxiliary information results in substantial gain in efficiency over the estimators obtained from those which do not use such information. Out of many, ratio, product and regression methods of estimation are good examples in this context. When the correlation between the study variate y and the auxiliary variate x is positive (high), the ratio method of estimation is quite effective. On the other hand if this correlation is negative (high), the product method of estimation envisaged by Robson (1957) and rediscovered by Murthy (1964), can be employed. Estimators using information of the known population mean of an auxiliary variable have generalized to the cases when such information is available for more than one auxiliary variables by several authors like Olkin (1958), Raj (1965), Rao and Mudholkar (1967), Singh (1967), Srivastava (1965) and Shukla (1966) and Agrawal and Panda (1993) etc. This paper deals with the problem of estimating the population mean of the study variable using single auxiliary information and thereafter, the proposed class of estimators has been extended to the use of multiple auxiliary information. Many authors have made use of linear combination of various estimators available in literature, Singh and Solanki (2011) is one example from the list. In this paper, we have suggested an alternative class of estimators using a linear combination of Srivastava and Walsh estimators in section 2. Section 3 deals with the extension of the proposed class of estimators using two auxiliary variables, related bias and mean square error are obtained up to the first order of approximation. Furthermore, section 4 gives the ultimate extension of the proposed class of estimators using multiple auxiliary information along with the bias and minimum mean square error of the proposed one. Theoretical comparisons with some known estimators of the literature like, mean per unit, ratio, product and some special cases of the proposed class of estimators are given under section 5 and 6 respectively. An illustration, to support the theoretical comparisons, is given as an empirical study in section 7.

II. THE SUGGESTED CLASS OF ESTIMATORS T

Consider a finite population $U=(U_1,U_2,...,U_N)$ of size N from which a sample of size n is drawn in accordance with SRSWOR. Let y_i and x_i denotes the study variable and the auxiliary variable for the i^{th} unit respectively. We define a class of estimators for the population mean as

$$T = w_1\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^g + w_2\bar{y}\left(\frac{\bar{X}}{\bar{X}+\delta(\bar{x}-\bar{X})}\right) \tag{2.1}$$

It is also to be mention that

(i). For $(w_1,w_2)=(1,0)$, the class of estimators T reduces to the class of estimators due to Srivastava

$$t_s = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^g \qquad (2.2)$$

(ii). For $(w_1, w_2) = (w_1, 0)$, the class of estimators T turns out to be

$$\eta_s = w_1\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^g \qquad (2.3)$$

(iii). For $(w_1, w_2) = (0,1)$, the class of estimators T reduces to the class of estimators due to Walsh

$$t_w = \bar{y}\left(\frac{\bar{X}}{\bar{X}+\delta(\bar{x}-\bar{X})}\right) \qquad (2.4)$$

(iv). For $(w_1, w_2) = (0, w_2)$,the class of estimators T transforms to

$$\eta_w = w_2\bar{y}\left(\frac{\bar{X}}{\bar{X}+\delta(\bar{x}-\bar{X})}\right) \qquad (2.5)$$

(v). For $(w_1, w_2, g) = (1,0,-1)$ and $(w_1, w_2, \delta) = (0,1,1)$, the class of estimators T reduces to the usual ratio estimator.

$$t_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \qquad (2.6)$$

(vi). For $(w_1, w_2, g) = (1,0,1)$, the class of estimators T transforms to the usual product estimator

$$t_P = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right) \qquad (2.7)$$

1) Theorem 2.1

The bias and MSE of the proposed estimator is given by

$$Bias(T) = \bar{Y}\left\{w_1\left(1+\lambda g\rho_{yx}C_yC_x+\frac{g(g-1)}{2}\lambda C_x^2\right)+w_2\left(1-\lambda\delta\rho_{yx}C_yC_x+\lambda\delta^2C_x^2\right)-1\right\} \qquad (2.8)$$

$$MSE(T) = \bar{Y}^2\left[\begin{array}{l} w_1^2\left(1+\lambda C_y^2+g^2\lambda C_x^2+4g\lambda\rho_{yx}C_yC_x+g(g-1)\lambda C_x^2\right) \\ +w_2^2\left(1+\lambda C_y^2+\delta^2\lambda C_x^2-4\delta\lambda\rho_{yx}C_yC_x+2\delta^2\lambda C_x^2\right) \\ -2w_1\left(1+g\lambda\rho_{yx}C_yC_x+\dfrac{g(g-1)\lambda C_x^2}{2}\right) \\ -2w_2\left(1-\delta\lambda\rho_{yx}C_yC_x+\delta^2\lambda C_x^2\right) \\ 2w_1w_2\left(\begin{array}{l}1+2g\lambda\rho_{yx}C_yC_x-2\delta\lambda\rho_{yx}C_yC_x \\ +\dfrac{g(g-1)\lambda C_x^2}{2}+\delta\lambda C_x^2-\delta g\lambda C_x^2+\lambda C_y^2\end{array}\right)+1 \end{array}\right]$$

$$= \bar{Y}^2\left[1+w_1^2A+w_2^2C+2w_1w_2D-2w_1B-2w_2E\right] \qquad (2.9)$$

where

$$A=\left(1+\lambda C_y^2+g^2\lambda C_x^2+4g\lambda\rho_{yx}C_yC_x+g\left(g-1\right)\lambda C_x^2\right)$$

$$B=\left(1+g\lambda\rho_{yx}C_yC_x+\frac{g\left(g-1\right)\lambda C_x^2}{2}\right)$$

$$C=\left(1+\lambda C_y^2+3\delta^2\lambda C_x^2-4\delta\lambda\rho_{yx}C_yC_x\right)$$

$$D=\left(1+2g\lambda\rho_{yx}C_yC_x-2\delta\lambda\rho_{yx}C_yC_x+\frac{g\left(g-1\right)\lambda C_x^2}{2}+\delta^2\lambda C_x^2-\delta g\lambda C_x^2+\lambda C_y^2\right)$$

$$E=\left(1-\delta\lambda\rho_{yx}C_yC_x+\delta^2\lambda C_x^2\right)$$

2) *Corollary 2.2*

The MSE of the class of estimators is minimized for

$$w_1=\frac{\left(BC-DE\right)}{\left(AC-D^2\right)}=w_{1(opt)} \qquad (2.10)$$

$$w_2=\frac{\left(AE-BD\right)}{\left(AC-D^2\right)}=w_{2(opt)} \qquad (2.11)$$

Substituting (2.10) and (2.11) we get the minimum MSE of the class of estimators as

$$MSE_{\min}\left(T\right)=\overline{Y}^2\left[1-\frac{\left(B^2C-2BDE+AE^2\right)}{\left(AC-D^2\right)}\right] \qquad (2.12)$$

3) *Theorem 2.3*

To the first order of approximation

$$MSE_{\min}\left(T\right)\geq\overline{Y}^2\left[1-\frac{\left(B^2C-2BDE+AE^2\right)}{\left(AC-D^2\right)}\right]$$

with equality holding if

$$w_1=w_{1(opt)}$$

$$w_2=w_{2(opt)}$$

Putting $\left(w_1,w_2\right)=\left(1,0\right),\left(w_1,0\right),\left(0,1\right),\left(0,w_2\right)$, $\left(w_1,w_2,g\right)=\left(1,0,-1\right),\left(w_1,w_2,\delta\right)=\left(0,1,1\right)$ and $\left(w_1,w_2,g\right)=\left(1,0,1\right)$ in (2.9), we get the MSEs of the estimators $t_s,\eta_s,t_w,\eta_w\ t_R$ and t_P respectively, to the first order of approximation as

$$MSE\left(t_s\right)=\overline{Y}^2\left[1+A-2B\right]=\lambda\overline{Y}^2\left[C_y^2+g^2C_x^2+2g\rho_{yx}C_yC_x\right] \qquad (2.13)$$

$$MSE\left(\eta_s\right)=\overline{Y}^2\left[1+w_1^2A-2w_1B\right] \qquad (2.14)$$

$$MSE\left(t_w\right)=\overline{Y}^2\left[1+C-2E\right]=\lambda\overline{Y}^2\left[C_y^2+\delta^2C_x^2-2\delta\rho_{yx}C_yC_x\right] \qquad (2.15)$$

$$MSE\left(\eta_w\right)=\overline{Y}^2\left[1+w_2^2C-2w_2E\right] \qquad (2.16)$$

$$MSE\left(t_R\right)=\lambda\overline{Y}^2\left[C_y^2+C_x^2-2\rho_{yx}C_yC_x\right] \qquad (2.17)$$

$$MSE\left(t_P\right)=\lambda\overline{Y}^2\left[C_y^2+C_x^2+2\rho_{yx}C_yC_x\right] \qquad (2.18)$$

Now, $MSE(t_s)$, $MSE(\eta_s)$, $MSE(t_w)$ and $MSE(\eta_w)$ are, respectively minimised for

$$g_{(opt)} = -\rho_{yx}\frac{C_y}{C_x}$$

$$w^*_{1(opt)} = \frac{B}{A}$$

$$\delta_{(opt)} = \rho_{yx}\frac{C_y}{C_x}$$

$$w^*_{2(opt)} = \frac{E}{C}$$

The resultant minimum mean square error of t_s, η_s, t_w, η_w are respectively, given by

$$MSE_{\min}(t_s) = MSE_{\min}(t_w) = \lambda\bar{Y}^2 C_y^2\left(1-\rho_{yx}^2\right) \tag{2.19}$$

$$MSE_{\min}(\eta_s) = \bar{Y}^2\left(1-\frac{B^2}{A}\right) \tag{2.20}$$

$$MSE_{\min}(\eta_w) = \bar{Y}^2\left(1-\frac{E^2}{C}\right) \tag{2.21}$$

The $MSE_{\min}(t_s), MSE_{\min}(t_w)$ given by (3.32) is same as the MSE of usual regression estimator $\bar{y}_{lr}$.

III. EXTENSION OF THE PROPOSED ESTIMATOR USING TWO AUXILIARY VARIABLES x_1 and x_2

Consider a finite population of size N from which a sample of size n is drawn with the help of simple random sampling without replacement. Let y denotes the variable under study whose mean is to be estimated making the use of two auxiliary variables x_1 and x_2. It is to be assumed that the population mean is known. The suggested class of estimators under the scheme of two auxiliary variables transforms to

$$T^* = w_1\bar{y}\left(\frac{\bar{x}_1}{\bar{X}_1}\right)^{g_1}\left(\frac{\bar{x}_2}{\bar{X}_2}\right)^{g_2} + w_2\bar{y}\left(\frac{\bar{X}_1}{\bar{X}_1+\delta_1\left(\bar{x}_1-\bar{X}_1\right)}\right)\left(\frac{\bar{X}_2}{\bar{X}_2+\delta_2\left(\bar{x}_2-\bar{X}_2\right)}\right) \tag{3.1}$$

where g_i and δ_i are the characterising scalars whose values can be determined from the non-Searls form of the above proposed class of estimators, $\bar{y}$ and $\bar{x}_i$ denotes the sample mean of the study variable y and sample means of the two auxiliary variables x_i respectively. $(i=1,2)$

It is important tom note that this class of proposed estimator extend the use of Olkin (1958), Srivastava (1965) and Walsh using the Searls approach.

1) Theorem 3.1

Bias and MSE of the above suggested class of estimators using two auxiliary variables is given by

$$Bias\left(T^*\right) = \bar{Y}\left\{\begin{array}{l} w_1\left(1+g_1\lambda\rho_{yx_1}C_yC_{x_1}+g_2\lambda\rho_{yx_2}C_yC_{x_2}+g_1g_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}+\frac{g_1(g_1-1)}{2}\lambda C_{x_1}^2+\frac{g_2(g_2-1)}{2}\lambda C_{x_2}^2\right) \\ +w_2\left(1-\delta_1\lambda\rho_{yx_1}C_yC_{x_1}-\delta_2\lambda\rho_{yx_2}C_yC_{x_2}+\delta_1\delta_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}+\delta_1^2\lambda C_{x_1}^2+\delta_2^2\lambda C_{x_2}^2\right)-1 \end{array}\right\} \tag{3.2}$$

$$MSE\left(T^{*}\right)=\bar{Y}^{2}\left[\begin{array}{l} w_1^2\begin{pmatrix}1+\lambda C_y^2+g_1^{\,2}\lambda C_{x_1}^2+g_2^{\,2}\lambda C_{x_2}^2+4g_1\lambda\rho_{yx_1}C_yC_{x_1}+4g_2\lambda\rho_{yx_1}C_yC_{x_2}\\ +4g_1g_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}+g_1\left(g_1-1\right)\lambda C_{x_1}^2+g_2\left(g_2-1\right)\lambda C_{x_2}^2\end{pmatrix}\\ +w_2^2\begin{pmatrix}1+\lambda C_y^2+3\delta_1^{\,2}\lambda C_{x_1}^2+3\delta_2^{\,2}\lambda C_{x_2}^2-4\delta_1\lambda\rho_{yx_1}C_yC_{x_1}\\ -4\delta_2\lambda\rho_{yx_2}C_yC_{x_2}+4\delta_1\delta_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\end{pmatrix}\\ -2w_1\begin{pmatrix}1+g_1\lambda\rho_{yx_1}C_yC_{x_1}+g_2\lambda\rho_{yx_2}C_yC_{x_2}+g_1g_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\\ +\dfrac{g_1\left(g_1-1\right)}{2}\lambda C_{x_1}^2+\dfrac{g_2\left(g_2-1\right)}{2}\lambda C_{x_2}^2\end{pmatrix}\\ -2w_2\left(1+\delta_1^{\,2}\lambda C_{x_1}^2+\delta_2^{\,2}\lambda C_{x_2}^2-\delta_1\lambda\rho_{yx_1}C_yC_{x_1}-\delta_2\lambda\rho_{yx_2}C_yC_{x_2}+\delta_1\delta_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\right)\\ 2w_1w_2\begin{pmatrix}1+\lambda C_y^2+\delta_1^2\lambda C_x^2+\delta_2^2\lambda C_x^2-\delta_1g_1\lambda C_{x_1}^2-\delta_2g_2\lambda C_{x_2}^2+2g_1\lambda\rho_{yx_1}C_yC_{x_1}\\ +2g_2\lambda\rho_{yx_2}C_yC_{x_2}-2\delta_1\lambda\rho_{yx_1}C_yC_{x_1}-2\delta_2\lambda\rho_{yx_2}C_yC_{x_2}+\delta_1\delta_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\\ +g_1g_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}-g_1\delta_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\\ -\delta_1g_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}+\dfrac{g_1\left(g_1-1\right)\lambda C_{x_1}^2}{2}+\dfrac{g_2\left(g_2-1\right)\lambda C_{x_2}^2}{2}\end{pmatrix}+1 \end{array}\right]$$

$$=\bar{Y}^{2}\left[1+w_1^2A^*+w_2^2C^*+2w_1w_2D^*-2w_1B^*-2w_2E^*\right] \tag{3.3}$$

where

$$A^*=\begin{pmatrix}1+\lambda C_y^2+g_1^{\,2}\lambda C_{x_1}^2+g_2^{\,2}\lambda C_{x_2}^2+4g_1\lambda\rho_{yx_1}C_yC_{x_1}+4g_2\lambda\rho_{yx_1}C_yC_{x_2}\\ +4g_1g_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}+g_1\left(g_1-1\right)\lambda C_{x_1}^2+g_2\left(g_2-1\right)\lambda C_{x_2}^2\end{pmatrix}$$

$$B^*=\begin{pmatrix}1+g_1\lambda\rho_{yx_1}C_yC_{x_1}+g_2\lambda\rho_{yx_2}C_yC_{x_2}+g_1g_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\\ +\dfrac{g_1\left(g_1-1\right)}{2}\lambda C_{x_1}^2+\dfrac{g_2\left(g_2-1\right)}{2}\lambda C_{x_2}^2\end{pmatrix}$$

$$C^*=\begin{pmatrix}1+\lambda C_y^2+3\delta_1^{\,2}\lambda C_{x_1}^2+3\delta_2^{\,2}\lambda C_{x_2}^2-4\delta_1\lambda\rho_{yx_1}C_yC_{x_1}\\ -4\delta_2\lambda\rho_{yx_2}C_yC_{x_2}+4\delta_1\delta_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\end{pmatrix}$$

$$D^*=\begin{pmatrix}1+\lambda C_y^2+\delta_1^2\lambda C_{x_1}^2+\delta_2^2\lambda C_{x_2}^2-\delta_1g_1\lambda C_{x_1}^2-\delta_2g_2\lambda C_{x_2}^2+2g_1\lambda\rho_{yx_1}C_yC_{x_1}\\ +2g_2\lambda\rho_{yx_2}C_yC_{x_2}-2\delta_1\lambda\rho_{yx_1}C_yC_{x_1}-2\delta_2\lambda\rho_{yx_2}C_yC_{x_2}+\delta_1\delta_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\\ +g_1g_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}-g_1\delta_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\\ -\delta_1g_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}+\dfrac{g_1\left(g_1-1\right)\lambda C_{x_1}^2}{2}+\dfrac{g_2\left(g_2-1\right)\lambda C_{x_2}^2}{2}\end{pmatrix}$$

$$E^*=\left(1+\delta_1^{\,2}\lambda C_{x_1}^2+\delta_2^{\,2}\lambda C_{x_2}^2-\delta_1\lambda\rho_{yx_1}C_yC_{x_1}-\delta_2\lambda\rho_{yx_2}C_yC_{x_2}+\delta_1\delta_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\right)$$

2) Corollary 3.2

The MSE of the class of estimators is minimized for

$$w_1=\frac{\left(B^*C^*-D^*E^*\right)}{\left(A^*C^*-D^{*2}\right)}=w_{1(opt)} \tag{3.4}$$

$$w_2=\frac{\left(A^*E^*-B^*D^*\right)}{\left(A^*C^*-D^{*2}\right)}=w_{2(opt)} \tag{3.5}$$

Substituting (3.4) and (3.5) we get the minimum MSE of the class of estimators as

$$MSE_{\min}\left(T^*\right)=\overline{Y}^2\left[1-\frac{\left(B^{*2}C^*-2B^*D^*E^*+A^*E^{*2}\right)}{\left(A^*C^*-D^{*2}\right)}\right] \tag{3.6}$$

IV. MULTIVARIATE EXTENSION OF THE PROPOSED CLASS OF ESTIMATORS USING MULTIPLE AUXILIARY INFORMATION

Let there are k auxiliary variables then we can use the variables by taking a linear combination of these k estimators of the form (). Then the estimator for population mean will be defined as

$$T_m=w_1\overline{y}\prod_{i=1}^{k}\left(\frac{\overline{x}_i}{\overline{X}_i}\right)^{g_i}+w_2\overline{y}\prod_{i=1}^{k}\left(\frac{\overline{X}_i}{\overline{X}_i+\delta_i\left(\overline{x}_i-\overline{X}_i\right)}\right) \tag{4.1}$$

where g_i and δ_i are the characterising scalars $(i=1,2,...,k)$

1) *Theorem 2.6*

Bias and MSE of the above estimator can be obtained as

$$Bias\left(T_m\right)=\overline{Y}\left\{\begin{array}{l} w_1\left(1+\sum_{i=1}^{k}g_i\lambda\rho_{yx_i}C_yC_{x_i}+\sum_{i\neq j=1}^{k}g_ig_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}\frac{g_i\left(g_i-1\right)}{2}\lambda C_{x_i}^2\right) \\ +w_2\left(1-\sum_{i=1}^{k}\delta_i\lambda\rho_{yx_i}C_yC_{x_i}+\sum_{i\neq j=1}^{k}\delta_i\delta_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}\delta_i^2\lambda C_{x_i}^2\right)-1 \end{array}\right\} \tag{4.2}$$

$$MSE\left(T^*\right)=\overline{Y}^2\left[\begin{array}{l} w_1^2\left(\begin{array}{l}1+\lambda C_y^2+\sum_{i=1}^{k}g_i^2\lambda C_{x_i}^2+4\sum_{i=1}^{k}g_i\lambda\rho_{yx_i}C_yC_{x_i}+4g_2\lambda\rho_{yx_1}C_yC_{x_2} \\ +4\sum_{i\neq j=1}^{k}g_ig_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}g_i\left(g_i-1\right)\lambda C_{x_i}^2\end{array}\right) \\ +w_2^2\left(1+\lambda C_y^2+3\sum_{i=1}^{k}\delta_i^2\lambda C_{x_i}^2-4\sum_{i=1}^{k}\delta_i\lambda\rho_{yx_i}C_yC_{x_i}+4\sum_{i\neq j=1}^{k}\delta_i\delta_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}\right) \\ -2w_1\left(1+\sum_{i=1}^{k}g_i\lambda\rho_{yx_i}C_yC_{x_i}+\sum_{i\neq j=1}^{k}g_ig_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}\frac{g_i\left(g_i-1\right)}{2}\lambda C_{x_i}^2\right) \\ -2w_2\left(1-\sum_{i=1}^{k}\delta_i\lambda\rho_{yx_i}C_yC_{x_i}+\sum_{i\neq j=1}^{k}\delta_i\delta_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}\delta_i^2\lambda C_{x_i}^2\right) \\ 2w_1w_2\left(\begin{array}{l}1+\lambda C_y^2+\sum_{i=1}^{k}\delta_i^2\lambda C_{x_i}^2-\sum_{i=1}^{k}\delta_ig_i\lambda C_{x_i}^2+2\sum_{i=1}^{k}g_i\lambda\rho_{yx_i}C_yC_{x_i} \\ -2\sum_{i=1}^{k}\delta_i\lambda\rho_{yx_i}C_yC_{x_i}+\sum_{i\neq j=1}^{k}\delta_i\delta_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i\neq j=1}^{k}g_ig_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j} \\ -\sum_{i\neq j=1}^{k}g_i\delta_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}\frac{g_i\left(g_i-1\right)}{2}\lambda C_{x_i}^2\end{array}\right)+1 \end{array}\right]$$

$$=\bar{Y}^2\left[1+w_1^2A_m+w_2^2C_m+2w_1w_2D_m-2w_1B_m-2w_2E_m\right] \tag{4.3}$$

where

$$A_m=\left(\begin{array}{l}1+\lambda C_y^2+\sum_{i=1}^{k}g_i^2\lambda C_{x_i}^2+4\sum_{i=1}^{k}g_i\lambda\rho_{yx_i}C_yC_{x_i}+4g_2\lambda\rho_{yx_1}C_yC_{x_2}\\+4\sum_{i\neq j=1}^{k}g_ig_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}g_i\left(g_i-1\right)\lambda C_{x_i}^2\end{array}\right)$$

$$B_m=\left(1+\sum_{i=1}^{k}g_i\lambda\rho_{yx_i}C_yC_{x_i}+\sum_{i\neq j=1}^{k}g_ig_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}\frac{g_i\left(g_i-1\right)}{2}\lambda C_{x_i}^2\right)$$

$$C_m=\left(1+\lambda C_y^2+3\sum_{i=1}^{k}\delta_i^2\lambda C_{x_i}^2-4\sum_{i=1}^{k}\delta_i\lambda\rho_{yx_i}C_yC_{x_i}+4\sum_{i\neq j=1}^{k}\delta_i\delta_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}\right)$$

$$D_m=\left(\begin{array}{l}1+\lambda C_y^2+\sum_{i=1}^{k}\delta_i^2\lambda C_{x_i}^2-\sum_{i=1}^{k}\delta_ig_i\lambda C_{x_i}^2+2\sum_{i=1}^{k}g_i\lambda\rho_{yx_i}C_yC_{x_i}\\-2\sum_{i=1}^{k}\delta_i\lambda\rho_{yx_i}C_yC_{x_i}+\sum_{i\neq j=1}^{k}\delta_i\delta_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i\neq j=1}^{k}g_ig_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}\\-\sum_{i\neq j=1}^{k}g_i\delta_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}\frac{g_i\left(g_i-1\right)}{2}\lambda C_{x_i}^2\end{array}\right)$$

$$E_m=\left(1-\sum_{i=1}^{k}\delta_i\lambda\rho_{yx_i}C_yC_{x_i}+\sum_{i\neq j=1}^{k}\delta_i\delta_j\lambda\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}\delta_i^2\lambda C_{x_i}^2\right)$$

2) Corollary 2.7

The MSE of the class of estimators is minimized for

$$w_1=\frac{\left(B_mC_m-D_mE_m\right)}{\left(A_mC_m-D_m^2\right)}=w_{1(opt)} \tag{4.4}$$

$$w_2=\frac{\left(A_mE_m-B_mD_m\right)}{\left(A_mC_m-D_m^2\right)}=w_{2(opt)} \tag{4.5}$$

Substituting (4.4) and (4.5) we get the minimum MSE of the class of estimators as

$$MSE_{\min}\left(T_m\right)=\bar{Y}^2\left[1-\frac{\left(B_m^2C_m-2B_mD_mE_m+A_mE_m^2\right)}{\left(A_mC_m-D_m^2\right)}\right] \tag{4.6}$$

V. COMPARISON OF THE ESTIMATORS

A comparison of the proposed classes of estimators with some of the known estimators available in the literature viz., the usual mean per unit estimator, usual ratio, product estimators in terms of biases and mean square error up to order has been shown under this section. Also a comparison with some special cases of the proposed class of estimators has been given thereafter.

1) Mean per unit estimator

It is an unbiased estimator of population mean and its variance is given by

$$Var\left(\bar{y}\right)=\lambda\bar{Y}^2C_y^2 \tag{5.1}$$

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2) Ratio estimator

$$\bar{y}_R = \bar{y}\frac{\bar{X}}{\bar{x}}$$

where x may be chosen as x_1 or x_2

$$Bias\left(\bar{y}_R\right) = \lambda\bar{Y}C_x\left(C_x - \rho_{yx}C_y\right) \quad (5.2)$$

$$MSE\left(\bar{y}_R\right) = \lambda\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right) \quad (5.3)$$

3) Product estimator

$$\bar{y}_P = \bar{y}\frac{\bar{x}}{\bar{X}}$$

$$Bias\left(\bar{y}_P\right) = \lambda\bar{Y}\rho_{yx}C_yC_x \quad (5.4)$$

$$MSE\left(\bar{y}_P\right) = \lambda\bar{Y}^2\left(C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x\right) \quad (5.5)$$

4) If $(w_1, w_2) = (1,0), (w_1, w_2) = (w_1, 0), (w_1, w_2) = (0,1)$ and $(w_1, w_2) = (0, w_2)$ then the proposed classes of estimators become:

(i). $t_s = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^g$

(ii). $\eta_s = w_1\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^g$

(iii). $t_w = \bar{y}\left(\frac{\bar{X}}{\bar{X} + \delta\left(\bar{x} - \bar{X}\right)}\right)$

(iv). $\eta_w = w_2\bar{y}\left(\frac{\bar{X}}{\bar{X} + \delta\left(\bar{x} - \bar{X}\right)}\right)$

The biases and the mean square errors of the above estimators are given below:

(i). $Bias(t_s) = \bar{Y}\left(g\lambda\rho_{yx}C_yC_x + \frac{g(g-1)}{2}\lambda C_x^2\right)$ (5.6)

$MSE_{\min}(t_s) = \lambda\bar{Y}^2C_y^2\left(1 - \rho_{yx}^2\right)$ (5.7)

(ii). $Bias(\eta_s) = \bar{Y}\left\{w_1\left(1 + \lambda g\rho_{yx}C_yC_x + \frac{g(g-1)}{2}\lambda C_x^2\right) - 1\right\}$ (5.8)

$MSE_{\min}(\eta_s) = \bar{Y}^2\left(1 - \frac{B^2}{A}\right)$ (5.9)

(iii). $Bias(t_w) = \bar{Y}\left(\lambda\delta^2C_x^2 - \lambda\delta\rho_{yx}C_yC_x\right)$ (5.10)

$MSE_{\min}(t_s) = \lambda\bar{Y}^2C_y^2\left(1 - \rho_{yx}^2\right)$ (5.11)

(iv). $Bias(t_w) = \bar{Y}\left\{w_2\left(1 - \lambda\delta\rho_{yx}C_yC_x + \lambda\delta^2C_x^2\right) - 1\right\}$ (5.12)

$$MSE_{\min}(\eta_w)=\overline{Y}^2\left(1-\frac{E^2}{C}\right) \qquad (5.13)$$

5) The proposed class of estimators using two auxiliary variables

$$T^* = w_1\overline{y}\left(\frac{\overline{x}_1}{\overline{X}_1}\right)^{g_1}\left(\frac{\overline{x}_2}{\overline{X}_2}\right)^{g_2} + w_2\overline{y}\left(\frac{\overline{X}_1}{\overline{X}_1+\delta_1\left(\overline{x}_1-\overline{X}_1\right)}\right)\left(\frac{\overline{X}_2}{\overline{X}_2+\delta_2\left(\overline{x}_2-\overline{X}_2\right)}\right)$$

Bias and minimum MSE can be seen from equations (3.2) and (3.6) respectively.

6) Multivariate extension of the proposed estimator

$$T_m = w_1\overline{y}\prod_{i=1}^{k}\left(\frac{\overline{x}_i}{\overline{X}_i}\right)^{g_i} + w_2\overline{y}\prod_{i=1}^{k}\left(\frac{\overline{X}_i}{\overline{X}_i+\delta_i\left(\overline{x}_i-\overline{X}_i\right)}\right)$$

where g_i and δ_i are the characterising scalars $(i=1,2,...,k)$

The respective biases and minimum mean square errors of the above proposed class of estimators can be seen from (4.2) and (4.6).

VI. EFFICIENCY COMPARISON

In this section, we have compared the efficiency of the proposed class of estimators T with usual $\overline{y}$ (usual unbiased estimator), $\overline{y}_R\,(=t_R)$ (ratio estimator) and $\overline{y}_P\,(=t_P)$ (product estimator) to the first order of approximation,

$$Var(\overline{y}) = MSE(\overline{y}) = \lambda\overline{Y}^2C_y^2 \qquad (6.1)$$

$$MSE(\overline{y}_R) = MSE(t_R) = \lambda\overline{Y}^2\left[C_y^2+C_x^2-2\rho_{yx}C_yC_x\right] \qquad (6.2)$$

$$MSE(\overline{y}_P) = MSE(t_P) = \lambda\overline{Y}^2\left[C_y^2+C_x^2+2\rho_{yx}C_yC_x\right] \qquad (6.3)$$

From (2.12), (2.13), (2.15), (2.17), (2.18), (2.19), (2.20), (2.21), (6.1), (6.2) and (6.3), we get

$$Var(\overline{y})-\left[MSE_{\min}(t_s)=MSE_{\min}(t_w)=MSE(\overline{y}_{lr})\right]=\lambda\overline{Y}^2C_y^2\rho_{yx}^2\geq 0 \qquad (6.4)$$

$$\left[MSE(\overline{y}_R)=MSE(t_R)\right]-MSE(\overline{y}_{lr})=\lambda\overline{Y}^2\left(C_x-\rho_{yx}C_y\right)^2\geq 0 \qquad (6.5)$$

$$\left[MSE(\overline{y}_P)=MSE(t_P)\right]-MSE(\overline{y}_{lr})=\lambda\overline{Y}^2\left(C_x+\rho_{yx}C_y\right)^2\geq 0 \qquad (6.6)$$

$$MSE(t_s)-\left[MSE_{\min}(t_s)=MSE(\overline{y}_{lr})\right]=\lambda\overline{Y}^2\left(gC_x+\rho_{yx}C_y\right)^2\geq 0 \qquad (6.7)$$

$$MSE(t_w)-\left[MSE_{\min}(t_w)=MSE(\overline{y}_{lr})\right]=\lambda\overline{Y}^2\left(\delta C_x-\rho_{yx}C_y\right)^2\geq 0 \qquad (6.8)$$

$$MSE(t_s)-MSE_{\min}(\eta_s)=\overline{Y}^2\frac{(A-B)^2}{A}\geq 0 \qquad (6.9)$$

$$MSE(t_w)-MSE_{\min}(\eta_w)=\overline{Y}^2\frac{(C-E)^2}{E}\geq 0 \qquad (6.10)$$

$$MSE_{\min}(\eta_s)-MSE_{\min}(T)=\overline{Y}^2\frac{(AE-BD)^2}{A\left(AC-D^2\right)}\geq 0 \qquad (6.11)$$

$$MSE_{\min}(\eta_w)-MSE_{\min}(T)=\overline{Y}^2\frac{(BC-DE)^2}{C\left(AC-D^2\right)}\geq 0 \qquad (6.12)$$

VII. EMPIRICAL STUDY

The comparison among these estimators is given in this section using a real data set. The data for this study is taken from [1], District Handbook of Aligarh, India. The population contains 332 villages. A simple random sample 80 villages is taken for the study. We consider the variables Y , X_1 and X_2 as the number of cultivators, area of the village and number of household in the village respectively. We compute the bias and the MSE for all estimators. The following values were obtained using the whole data sets:

$\overline{Y} = 1093.1$, $\overline{X_1} = 181.57$, $\overline{X_2} = 143.31$

$C_y = 0.7626$, $C_{x_1} = 0.7684$, $C_{x_2} = 0.7616$

$\rho_{yx_1} = 0.973$, $\rho_{yx_2} = 0.862$, $\rho_{x_1x_2} = 0.842$

Using the above results we have calculated the MSE and PRE for all the estimators in section 5. The PRE for each estimator with respect to the sample mean of a SRS is defined as follows:

$$e\left(y'\right) = \left[\frac{MSE\left(\overline{y}\right)}{MSE\left(y'\right)}\right]*100$$

where $MSE\left(y'\right)$ is the mean square error for each estimator suggested in Section 5 and $MSE\left(\overline{y}\right) = Var\left(\overline{y}\right)$ for a sample of size 80.

Estimators	Auxiliary variables	MSE	Percent Relative Efficiency (PRE)
$\overline{y}$	None	6593.04	100
$\overline{y}_R$	x_1	359.11	1835.92
$\overline{y}_R$	x_2	1817.31	362.79
$\overline{y}_P$	x_1	26214.39	25.15
$\overline{y}_P$	x_2	24520.31	26.89
t_s	x_1	351.22	1877.19
t_s	x_2	1694.12	389.17
η_s	x_1	351.04	1878.15
η_s	x_2	1690.50	390.01
t_w	x_1	351.22	1877.19
t_w	x_2	1694.12	389.18
η_w	x_1	351.12	1877.75
η_w	x_2	1691.72	389.72
T	x_1	351.20	1877.31
T	x_2	1694.12	389.17
T^*	x_1 , x_2	4764.41	138.38

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