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A Polynomial Time Algorithm for Minimal Roman Coloring of a Graph

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Abstract: The Roman coloring of a graph, G which is an assignment of three colors {0, 1, 2} to the vertices of G such that any vertex with color, 0 must be adjacent to a vertex with color, 2. It was introduced Suresh Kumar [1] motivated from ancient Roman military strategy. The weight of a Roman coloring is defined as the sum of all the vertex colors. A Roman coloring of G with the minimal weight is called a minimal Roman coloring of G. In this Paper, we present a polynomial time algorithm for finding a minimal Roman coloring for a graph.
Keywords: Graph, Roman coloring, minimal Roman coloring, algorithm, algorithmic complexity.

I. INTRODUCTION

Recent decades witnessed Graph theory emerging as a highly potential research area due to the rapid evolution and growth of Computer Science and Information Technology. Graph algorithms were studied by Mathematicians as well as Computer scientists and Engineers. Graph algorithms form an integral part of the design and analysis of algorithms. Also, graphs and trees are some prominent data structures. Apart from this, Graph theory finds applications in real time problems like Vehicle routing problem [2] as well as in the analysis of Number Theory topics [3].

Motivated from the ancient Roman military strategy, Suresh Kumar [1] introduced the Roman coloring of G as an assignment of three colors {0, 1, 2} to the vertices of G such that any vertex with color, 0 must be adjacent to a vertex with color, 2. The weight of a Roman coloring is defined as the sum of all vertex colors. The Roman Chromatic number of a graph G is defined as the minimum weight of a Roman coloring of G. A Roman coloring of G with the minimal weight is called a minimal Roman coloring of G.

In this Paper, we present a polynomial time algorithm for finding a minimal Roman coloring for a graph. We consider only connected undirected graphs with no multiple edges or loops. For the Graph theoretic terms and definitions not defined explicitly here, reader may refer Harary [4]. For the Algorithmic terms and definitions not defined explicitly here, reader may refer Gibbons [5].

II. MAIN RESULTS

In this section, we present an algorithm to find a minimal Roman coloring of a graph.

1) *Problem:* Find a minimal Roman Coloring for a given graph G

Algorithm

Step-1: Arrange the vertices of G into a list in descending order of their degrees.
Step-2: Choose a vertex of the largest degree (the first entry in the list) and color it by the color 2 and color all its neighbours by the color 0. Remove that vertex and all its neighbours from the list.
Step-3: Choose a vertex with the maximum degree in the remaining list. If its degree is at least 2, then assign the color 2 to it. and color all its neighbours by the color 0. Otherwise color that vertex by the color 1. Then remove that vertex of the largest degree and all its neighbours from the list.
Step-4: If there are vertices remaining in the list, then go to step-3.
Stpe-5: If there is no vertex in the remaining list, then STOP.

Each time a vertex is given a color, if we print that vertex and its color to output, we will get a Roman coloring of the given graph as the output at the end. Now we are going to prove that the algorithm will produce the desired result in all cases.

2) *Theorem.* The above algorithm works and finds a minimal Roman coloring for all graphs.

a) *Proof:* We use Mathematical induction on the largest degree, say d, among the vertices in G. If d=1, then all the vertices of G has degree, 1 and the Graph must be a K_2. The Roman Chromatic number of K_2 is two [4]. The algorithm gives a coloring with the color, 1 to each vertex of K_2 so that the Roman Chromatic number of K_2 is 2. Thus, it is a minimal Roman coloring. Assume that the algorithm gives a minimal Roman coloring for all graphs with the maximum degree, n. Consider a graph G with the maximum degree, $n+1$. Let v be a vertex of G with the degree, $n+1$. Choose any vertex u of G that is adjacent to v. Spit the vertex, v into two vertices v_1, v_2 such that is v_2 is a vertex of degree 1 and v_2 is adjacent only to u and v_1 is adjacent to all other neighbours of v other than u. This may be done for all vertices with degree, $n+1$. Call the resulting graph as H. Then H is a graph with the maximum degree, n, where v_1 is a vertex of H with the degree, n. So be induction assumption, the algorithm produces a minimal Roman coloring for H where the vertex v_2 has the color, 1 and v_1 has the color 2. So by identifying the vertices, v_1, v_2, we can get back to the graph G and the minimal Roman coloring of H is also a minimal Roman coloring for G by giving the color 2 to the vertex, v. Hence, by induction, the algorithm works for all graphs and produces the desired result.

3) *Theorem.* The above algorithm is a polynomial time algorithm.

a) *Proof:* Let the number of vertices of G be n. The step-2 and step-3 together assigns a color each to each vertex of G so that only n assignments operations are needed. In step-1, the degrees of vertices of G can be sorted in descending order using a bubble sort and it takes only $\frac{n(n-1)}{2}$ operations. Hence the complexity our algorithm is $n + \frac{n(n-1)}{2} = O(n^2)$. Hence our algorithm is a polynomial time algorithm for finding a minimal Roman coloring for a graph.

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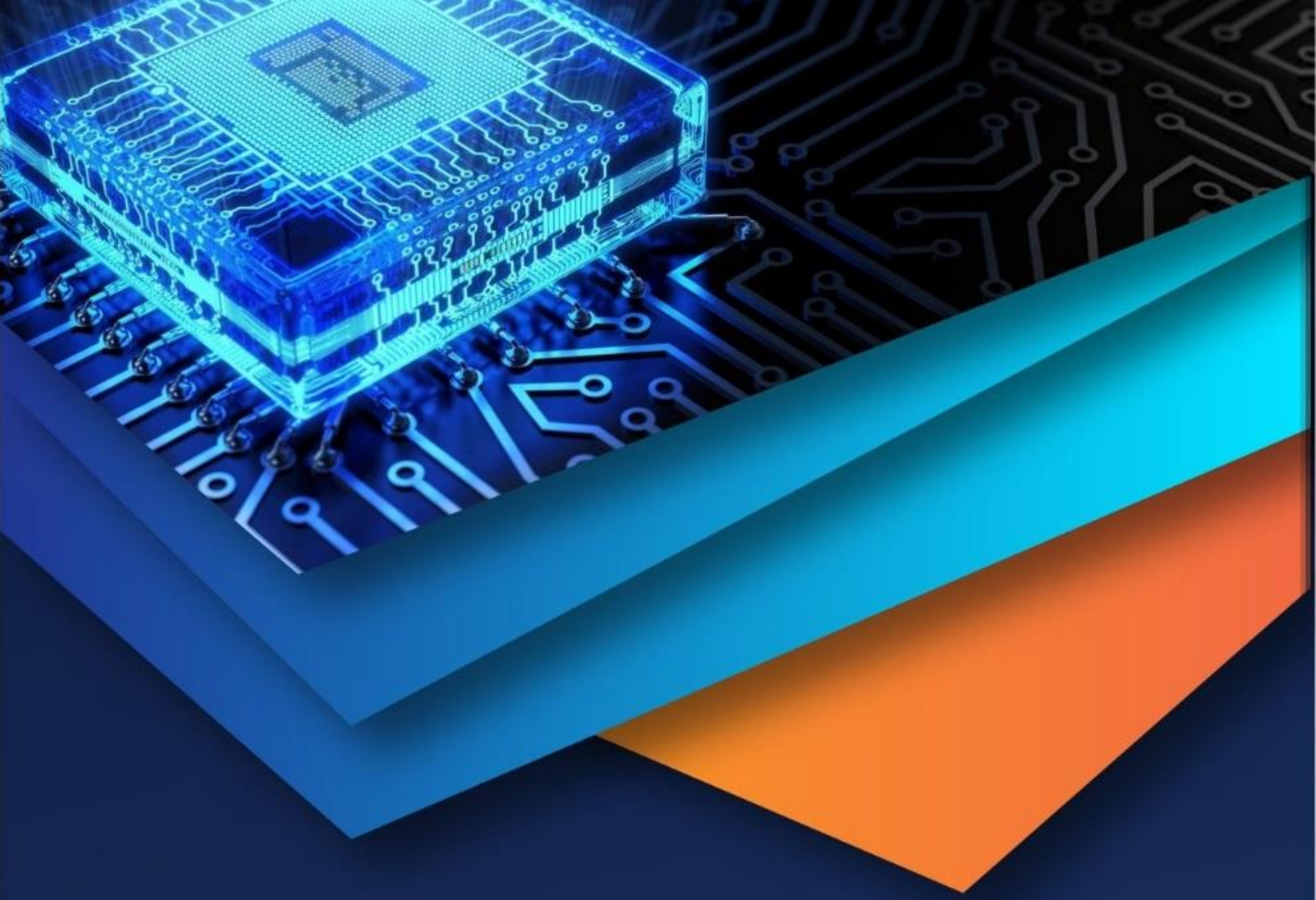

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