iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 8 **Issue:** VII **Month of publication:** July 2020

DOI: https://doi.org/10.22214/ijraset.2020.30375

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 8 Issue VII July 2020- Available at www.ijraset.com

Mathematical Approach of the Tumor Growth Model - A Discussion on Glioma Tumor

Sathisha A B[1], Ashwini M Rao[2], Basavarajappa K S[3]
[1]*Department of Mathematics, Government Science College, Chitradurga. Karnataka, India.*
[2]*Department of Mathematics, Bapuji Institute of Engineering and Technology, Davanagere. Karnataka, India.*
[3]*Department of Mathematics, Bapuji Institute of Engineering and Technology, Davanagere. Karnataka, India.*

Abstract: ***In this paper mathematical study to compute the model parameters proliferation, lethal and survival of the brain tumor growth. The reference of glioma tumor is considered to model the growth of tumor and the spatial spread of tumor. The asymptotic form of spread of glioma tumor is the special case of the mathematical model. The tumor cell motility model consists of the differential equation in terms of spatial coordinates as time period and the position. The concentration term varies as small perturbations are on the rate of growth to the rate of birth. Numerical results give strong suggestion to the physiological response of detection of glioma tumor cells proliferation.***

I. INTRODUCTION

Human brain is highly developed mass and enlarged of nervous tissue which forms the end of the central nervous system. Brain lies within the rigid body of skull which is well protected and plays an important role in the central nervous system. The three parts of the brain are the paired cerebral hemisphere, the cerebellum and the brain stem. These parts are responsible for controlling the main functions of the human activities in reference to intelligence, speech and memory. Brain stem occurs into the top of the spinal cord and maintains the vital functions of the body. Generally brain weighs approximately 2% of the total body weight (1400gms in an adult human being). When any abnormal swelling occurs inside or on the part of the brain leads to the abnormal growth of tissue which may be benign or malignant.

This is the tumor which occurs on the brain and swelling of this inflamed area is with the leakage of small blood vessels of clear protein containing fluid which accumulates between the cells. The present study concerns mathematical analysis of the tumor on the brain. Glioma is the type of tumor the most malignant of brain tumor constituting the damage on grey matter and white matter of the brain. The malignancy of this tumor(brian tumor) produces the indication of the pressure on the surrounding structures which destroys normal brain cells. This becomes the causative factor for the progressive loss of the function and rises the intracranial pressure, causing head ache, omitting and drowsiness. Mathematical study has been carried out to understand the dynamics of tumor growth on quantitative measurements. The use of mathematical modeling employed in the analysis confirms the case of tumor invasion with respect to concentration of cells varying with respect to time and position.

Martin Braun [1] derived the survival time for patients with cancerous tumors. G. Philip M. Mooreet. al. [2] tested the null hypothesis that there were no differences between the effects. S. A. Maggelakis [3] used to measure the degree of nonuniformity of the inhibitor production rate. H. M. Byrne et. al. [4] presented a mathematical model describing the growth and development of capillary sprouts. Philippe Tracqui [5] simulated the bifurcation from a homogeneous distribution of cells at the tumor surface. Francis G.Blankenberg [6] demonstrated that VDt and DNA ploidy may be better prognosticators than histologic grade. N. Bellomoet. al. [7] provided a survey of mathematical models dealing with the analysis and simulation of behavior of tumor dynamics. XuemeiWeiet. al. [8] concerned the problem of tumor angiogenesis.

HeikoEnderling et al [9] discussed some ideas for combining tractable dynamical tumor growth models with radiation response models using biologically accessible parameters. Pamela R. Jackson et al [10] focuses on a series of clinical relevant results using patient- specific mathematical modeling. SubhasKhajanchi et al [11] describes a mathematical model with synergistic interaction between the malignant glioma cells and the immune system. Renee Brady et al [12] discussed the emerging trend in mathematical oncology publications to predict novel, optimal, sometimes even patient specific treatments and propose a model to prsdict the treatments.

The present study concerns mathematical analysis of the tumor on the brain. Glioma is the type of tumor the most malignant of brain tumor constituting the damage on grey matter and white matter of the brain. To analyze the dynamics of tumor invasion and employ the tumor cell density.

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 8 Issue VII July 2020- Available at www.ijraset.com

II. FORMULATION

With reference to the cells proliferation and the diffusion for malignancy of this tumor, we modeled for the concentration term as, Let$\bar{c}(\bar{x},\bar{t})$ be the number of cells at a position $\bar{x}$ and at time $\bar{t}$. We take the basic model, in dimension form, as a conservation equation,

$$\frac{\partial \bar{c}}{\partial \bar{t}} = \bar{\nabla}_{(x,t,z)}.J + (b-d)\bar{c} \quad (1)$$

Where $\alpha = (b-d)$

Then equation (1) becomes,

$$\frac{\partial \bar{c}}{\partial \bar{t}} = \bar{\nabla}_{(x,t,z)}.J + \alpha\bar{c} \quad (2)$$

Where $\alpha = (b-d)$ gives the net growth of cells among rate of growth(b) to the decay(d). The diffusion flux of cells, J, is proportional to the gradient of the cell density:

$$J = \overline{(x_1 - x_f)}\,.\,\bar{\nabla}_{(x,t,z)}\bar{c} \quad (3)$$

where$\overline{(x_1 - x_f)}$gives the net behavior of malignancy into the white matter (x_i) and grey matter (x_f). With constant diffusion, the governing equation (2) with (3) is then,

$$\frac{\partial \bar{c}}{\partial \bar{t}} = \overline{(x_1 - x_f)}\,.\,\bar{\nabla}_{(x,t,z)}.\bar{\nabla}_{(x,t,z)}\bar{c} + \alpha.\bar{c} \quad (4)$$

Referring to the computations of diffusional concentration of grey and white matter

This gives, in place of (4),

$$\frac{\partial \bar{c}}{\partial \bar{t}} = \bar{\nabla}_{(x,t,z)}.\left[\overline{(x_1 - x_f)}(\bar{x}).\bar{\nabla}_{(x,t,z)}.\bar{c}\right] + \alpha.\bar{c} \quad (5)$$

Defining $\overline{(x_1 - x_f)} = \bar{x}_{WG}$

$$\frac{\partial \bar{c}}{\partial \bar{t}} = \bar{x}_{WG}.\left(\bar{\nabla}_{(x,t,z)}\right)^2 + \alpha.\bar{c} \quad (6)$$

Then equation (6) becomes,

$$\frac{\partial \bar{c}}{\partial \bar{t}} = \bar{x}_{WG}.\bar{\nabla}_{(x,t,z)}^{\ 2} + \alpha.\bar{c} \quad (7)$$

Again defining $\bar{x}_{WG} = \bar{x}_{WG}(\bar{x})$

The flux is assumed to be zero at the boundary. Then the required condition is given by,

$$n.\bar{x}_{WG}(\bar{x}).\bar{\nabla}_{(x,t,z)}\bar{c} = 0 \text{for} x \quad (8)$$

Numerical simulation and analytic results of this simpler case will be compared with experimental data. We first nondimensionalise the spatially heterogeneous model, which as usual, also decreases the number of effective parameters in the system and get some idea of the relative importance of various terms (without regard to units).

With the boundary condition for the analytical comparison with numerical predictions,

$$\frac{\partial \bar{c}}{\partial \bar{t}} = \bar{\nabla}_{(x,t,z)}.\left[\bar{x}_{WG}(\bar{x}).\bar{\nabla}_{(x,t,z)}\bar{c}\right] + \alpha.\bar{c} \quad (9)$$

$$\frac{\partial c}{\partial t} = \bar{\nabla}_{(x,t,z)}.\left[x_{WG}\bar{\nabla}_{(x,t,z)}c\right] + c \quad (10)$$

where

$$\bar{x}_{WG}(\bar{x}) = \begin{cases} 1 \text{ for x in white matter} \\ \gamma = \frac{x_w}{x_G} \text{ for x in grey matter} \end{cases} \quad (11)$$

With$c(x,0) = f(x) = \frac{x_w}{\alpha.N_0}\bar{f}\left(\sqrt{\frac{\alpha}{x_w}}\bar{x}\right)$ and $n.x_{WG}(x).\bar{\nabla}_{(x,t,z)}.c = 0$ for x on ∂B.

Diffusion on spatial coordinate in reference to white matter is considered to be with respect to time coordinates on the growth of the tumor.

III. ANALYSIS

At the positive initial condition, the tumor is considered with finite number of cancerous cells.

$$c(x,0) = a\exp\left(\frac{-|x-x_0|^2}{b}\right) \quad (12)$$

Using exponential series we can write equation (15) becomes

$$c(x,0) = a\left\{1 + \frac{(-1)|x-x_0|^2}{b} + \frac{(-1)^2(|x-x_0|^2)^2}{2b^2} + \frac{(-1)^3(|x-x_0|^2)^3}{6b^3} + \frac{(-1)^4(|x-x_0|^2)^4}{24b^4} + \cdots\right\} \quad (13)$$

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 8 Issue VII July 2020- Available at www.ijraset.com

IV. RESULTS AND DISCUSSION

The present study analyzes the dynamics of tumor growth with respect to glioma tumor. Spatial coordinates 'x' and't' in diffusional concentration are varied with respect to time for the quantification of tumor cell density. Results give the analytical evidence that the proliferation rate of tumor cells effects on the growth of normal cells which is growth loss of such cells. The mathematical comparison shown in the graphs gives the evidence that decrease of normal cells due to the malignancy is caused by proliferation of tumor cells.

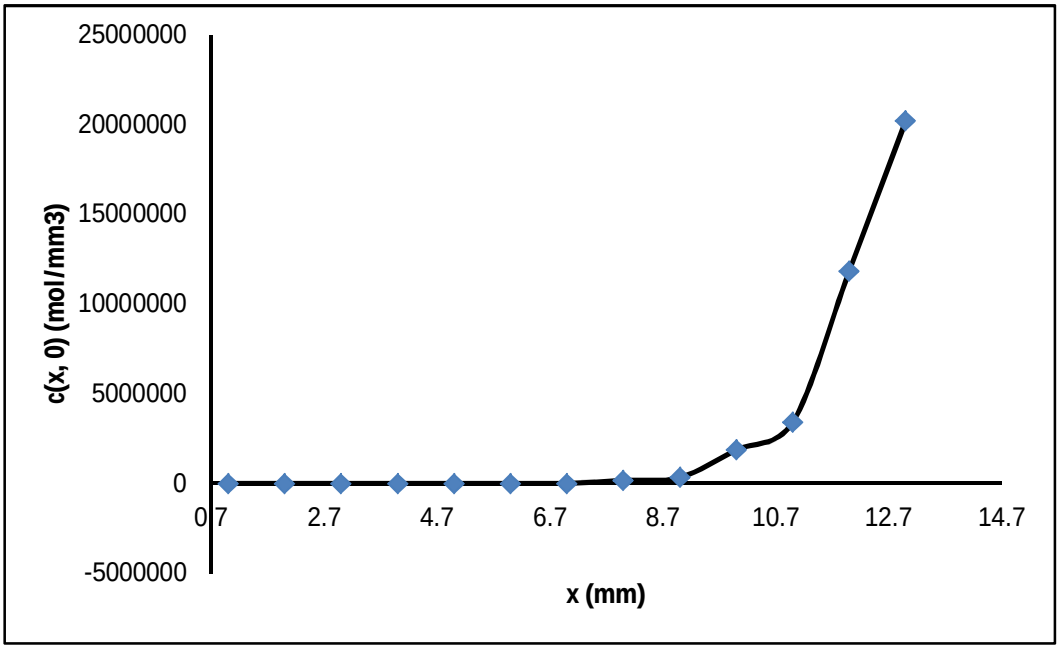


Figure 1: concentration of tumor cells against time at cell density a=10.6

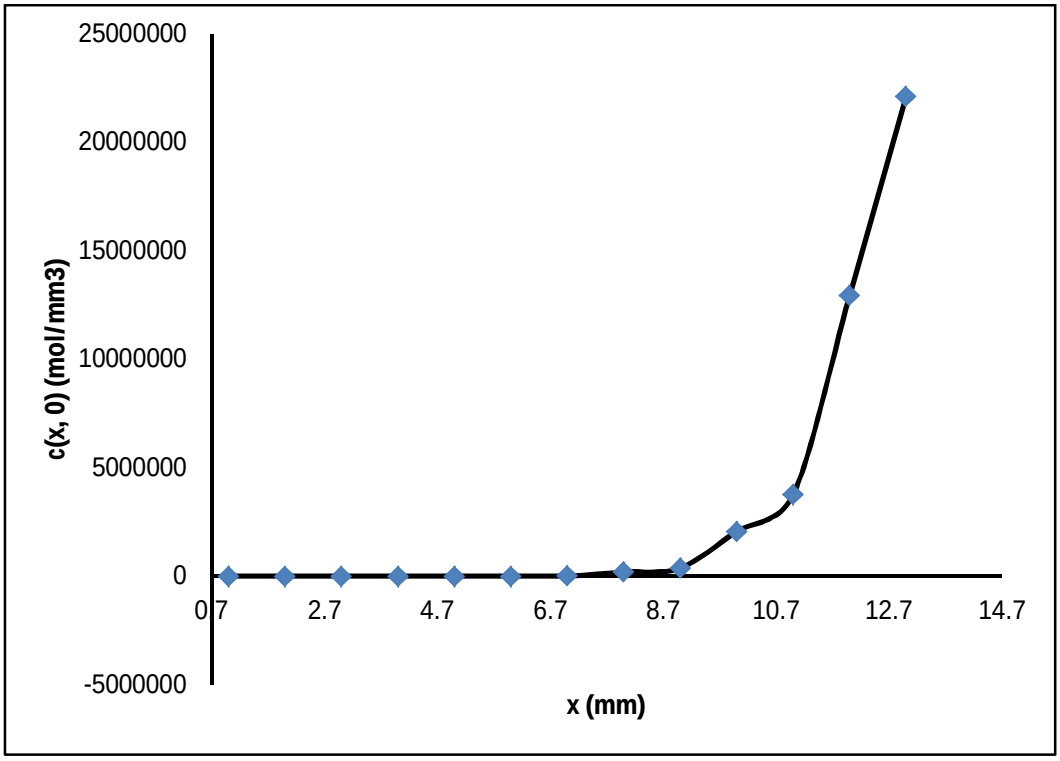


Figure 2: concentration of tumor cells against time at cell density a=11.6

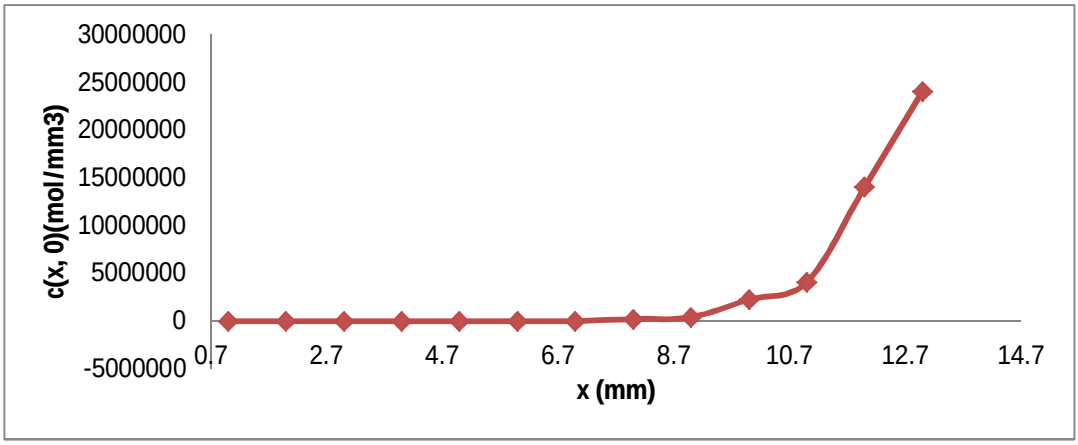


Figure 3: concentration of tumor cells against time at cell density a=12.6

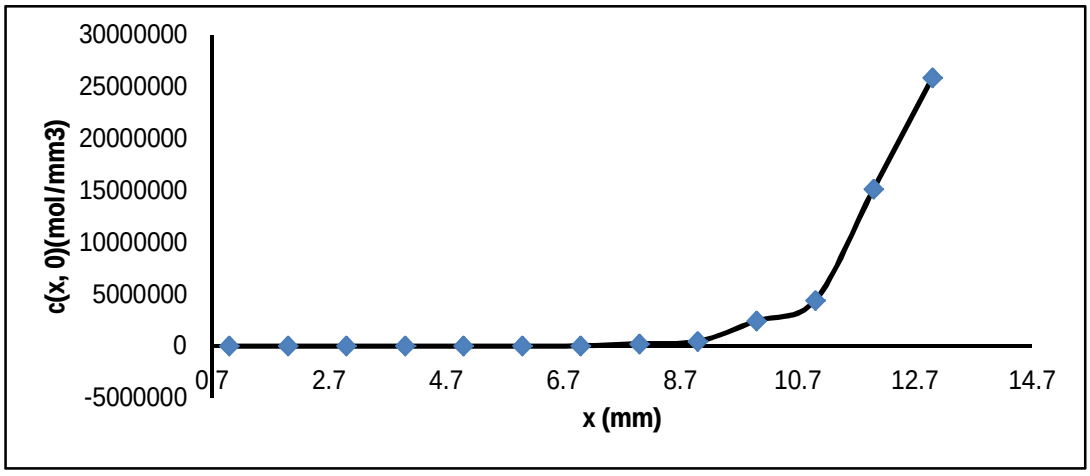
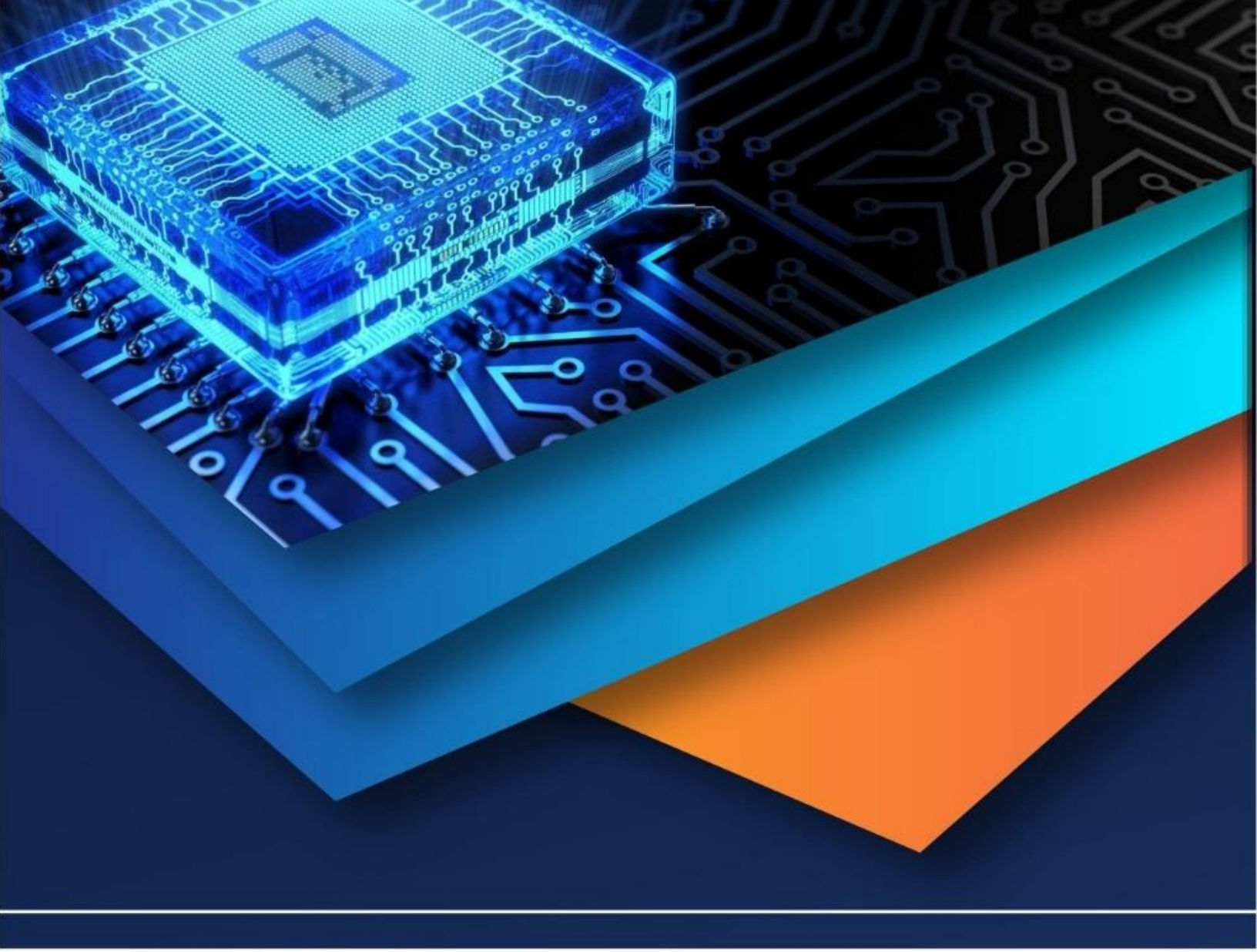


Figure 4: concentration of tumor cells against time at cell density a=13.6

REFERENCES

[1] Martin Braun [1980], 'Probability Survival Distributions For Cancerous Tumors'. Mathematical Modeling. Vol. I, pp. 27-31.
[2] G. Philip M. Moore, Basil A. Panaretto, and Nancy B. Carter [1985], 'Epidermal Hyperplasia and Wool Follicle Regression in Sheep. Infused with Epidermal Growth Factor'. The Journal of Investigative Dermatology, 84:172- 175.
[3] S. A. Maggelakis and J. A, Adam [1990], 'Mathematical Model of Prevascular Growth of a Spherical Carcinoma'. Mathematical and Computational Modeling, Vol. 13, No. 5, pp. 23-38.
[4] M. Byrne and M. A. J. Chaplin [1995], 'Explicit Solutions of a Simplified Model of Capillary Sprout Growth During Tumor Angiogenesis'. Applied Mathematical Letter, Vol. 8, No. 5, pp. 71-76.
[5] Philippe Tracqui [1995], 'From passive Diffusion to active Cellular Migration in Mathematical Models of Tumor Invasion'. ActaBiotheoretica 43, pp. 443-464.
[6] Francis G. Blankenberg, Raymond L. Teplitz, William Ellis, M. ShahriarSalamat, ByungHee Min, Lisa Hall, Derek B. Boothroyd, Iain M. Johnstone, and Dieter R. Enzmann [1995], 'The Influence of Volumetric Tumor Doubling Time, DNA Ploidy, and Histologic Grade on the Survival of Patients with IntracranialAstrocytomas'. AJNR Am J Neuroradio l16:1001–1012.
[7] N. Bellomo and L. Preziosi [2000], 'Modelling and Mathematical Problems Related to Tumor Evolution and Its Interaction with the Immune System'. Mathematical and Computer Modelling 32, 413-452.
[8] Xuemei Wei and Shangbin Cui [2008], 'Existence and uniqueness of global solutions for a mathematical model of antiangiogenesis in tumor growth'. Nonlinear Analysis: Real World Applications 9, pp. 1827 – 1836.
[9] HeikoEnderling, Mark A. J. Chaplain and Philip Hahnfeldt [2010], 'Quantitative Modeling of Tumor Dynamics and Radiotherapy'. ActaBiotheor, Vol. 58, pp. 341-353.
[10] Pamela R. Jackson, Joseph Juliano, Andrea Hawkins – Daarud, Russell C. Rockne and Kristin R. Swanson [2015], 'Patien – Specific Mathematical Neuro-Oncology: Using a simple Proliferation and Invasion Tumor Model to Inform Clinical Practice'.
[11] SubhasKhajanchi and Sandeep Banerjee [2017], 'Quantifying the Rolr of immunotherapeutic drug T11 target structure in progression of malignant gliomas: Mathematical Modeling and dynamical perspective'. Mathematical Biosciences, Vol. 289, pp. 69-77.
[12] Renee Brady and HeikoEnderling [2019], 'Mathematical Models of Cancer: When toPredictNovelTherapies and When Not To'. Bulletin of Mathematical Biology, Vol. 81, pp. 3722-3731.



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY