iJRASET
International Journal For Research in Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 8 **Issue:** VII **Month of publication:** July 2020

DOI: https://doi.org/10.22214/ijraset.2020.30376

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 8 Issue VII July 2020- Available at www.ijraset.com

Mathematical Study on Spread of Infectious Diseases: S-I-R Model with a Reference to Industrial Pollution

Ashwini M Rao[1], Sathisha A B[2], Basavarajappa K S[3]
[1, 3]*Department of Mathematics, Bapuji Institute of Engineering and Technology, Davanagere. Karnataka, India.*
[2]*Department of Mathematics, Government Science College, Chitradurga. Karnataka, India.*

Abstract: Mathematical analysis for the quantification of susceptible (S(t)), Infectives (I(t)) and Removal (R(t)) populations in SIR model due to industrial pollution has been studied in the paper. The study concerns the analysis of SIR model to quantify the varying populations of S(t), I(t) and R(t) with three different sizes in the populations. A system of non- linear ordinary differential equations are solved using Runge - Kutta fourth order method to investigate the long term effect of industrial pollution on the human population in terms of spread of diseases.
Keywords: Pollution, Epidemics, susceptible, populations, industrial.

I. INTRODUCTION

The study refers to quantification of susceptible(S(t)), Infectives(I(t)) and Removal(R(t)) populations in S-I-R model due to industrial pollution. The spread of infectious diseases arise among human population due to discharge of untreated industrial discharges as sewage water, sludge water. The undissolved heavy metals appear in these discharges and the untreated water is being utilized not only for various vegetables growth and the growth of other crops but also the same water being used for various other purposes. Then the untreated water and other discharges are continuously utilized, they cause various types of health hazards. The one another important industrial pollution is air pollution due to untreated various toxic gases released into the atmosphere which include lead, carbon-dioxide, sulpher dioxide etc. All these will cause the health effects like branchiatic asthma, tuberculosis, infection of lungs etc. Also equally important the one another pollution is due to noise. The sound from industries, vehicles generated exceeds 87 dB (as per I.B.S, Indian Beauro of Standards) leads to noise and the longer period exposure to this type of noise causes various effects on human health such as hearing loss, leading to permanent effect) hardening of arteries of flow of blood, disturbance on the microcirculation, fatigue etc. Vincenzo Capasso [1] examined a general stochastic epidemic model with immigration in a large population. David Greenhalgh [2] studied a mathematical model for the control of an infectious disease with a reference to stochastic and geographical factors influencing spread of disease. W Y Tan [3] developed a stochastic model for HIV epidemic in a homosexual population under general conditions. M Y Kim et. al. [4] described two models for the dynamics of a SIR type by randomly screening the active population. L Stone et. al. [5] analysed the rationale of the pulse vaccination strategy in the simple SIR epidemic model and showed that repeatedly vaccinating the susceptible population in a series of pulses, it is possible to eradicate the measles infection from the entire model population. Timothy B Hallettet. al. [6] developed a mathematical model of HIV transmission by the incorporation of potential for natural changes in the epidemic as it matures. Huiyan Kang et. al. [7] studied a S-I-S model with delay on scale-free networks to describe some epidemics which are not only transmitted by a vector but also spread between individuals by direct contacts. Thomas Wetere Tulu et. al. [8] developed a mathematical model to study and analyze the dynamics of Ebola epidemics and the results showed that quarantine and vaccination are very effective ways to control Ebola epidemic. Meghendra Singh et al [9] optimized a cost vector associated with the various behaviors to match the behavior distributions observed in a detailed survey of human behaviors during influenza outbreaks. Jürgen Hackl et al [10] presented is a prototype that can be used to analyze multiple scenarios in the case of a disease spread at an urban scale, considering variations of different model parameters settings. The present study focuses on the mathematical analysis of SIR model to quantify the varying populations of S(t), I(t), and R(t) with three different sizes in populations. The sensitivity constants are introduced to fulfill the conditions of the assumptions proposed for formulations and the variations of population sizes S_1, S_2 and S_3 for S(t) , I_1, I_2 and I_3 for I(t) and R_1, R_2 and R_3 for R(t). Total populations of these three sizes of S, I and R give the insight for proper quantification of populations directly affected due to industrial pollution. Mathematical models consist of non-linear systems of ordinary differential equations. These ordinary differential equations are solved using numerical method (R.K fourth order) to investigate the long term effect of pollution (Industrial) on the human population in terms of spread of diseases. The population under consideration is proposed to be divided in compartments based with the specific variations on S(t) with S_1, S_2 and S_3 , I(t) with I_1, I_2 and I_3 and R(t) with R_1, R_2 and R_3.

©IJRASET: All Rights are Reserved

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 8 Issue VII July 2020- Available at www.ijraset.com

II. FORMULATION

By considering the assumptions defined for epidemic model as with S(t):I(t):R(t) as S-I- R no control work mechanism, the coupled non linear ordinary differential equations for susceptible(S(t)) and infective(I(t)) where R(t) is the case of removals.

S-I-R Epidemic model in Industrial population is given by,

$$\frac{dS_1}{dt} = -\beta_1 S_1 I_1 - \beta_2 S_1 I_2 - \beta_3 S_1 I_3 + \frac{1}{L_1}(K - S)$$

$$\frac{dI_1}{dt} = \beta_1 S_1 I_1 + \beta_2 S_1 I_2 + \beta_3 S_1 I_3 - \gamma_1 I_1 - \frac{1}{L_1} I$$

$$\frac{dR_1}{dt} = \gamma_1 I_1 - \frac{1}{L_1} R \qquad (1)$$

$$\frac{dS_2}{dt} = -\beta_4 S_2 I_2 - \beta_5 S_2 I_1 + \frac{1}{L_2}(K - S)$$

$$\frac{dI_2}{dt} = \beta_4 S_2 I_2 + \beta_5 S_2 I_1 - \gamma_2 I_2 - \frac{1}{L_2} I_2$$

$$\frac{dR_2}{dt} = \gamma_2 I_2 - \frac{1}{L_2} R \qquad (2)$$

$$\frac{dS_3}{dt} = -\beta_6 S_3 I_3 - \beta_7 S_3 I_1 - \frac{1}{L_3}(K - S)$$

$$\frac{dI_3}{dt} = \beta_6 S_3 I_3 + \beta_7 S_3 I_1 - \gamma_3 I_3 - \frac{1}{L_3} I_3$$

$$\frac{dR_3}{dt} = \gamma_3 I_3 - \frac{1}{L_3} R_3 \qquad (3)$$

This is a compartmental model for the spread of disease in three communities by taking into account the birth rate in each community and an intra-interaction of members of community C_1 and C_2 and C_2 and C_3 where $C_i (i = 1,2,3)$ are the three communities of human population the total population $K_i (i = 1,2,3)$ is taken to be constant in each community.

III. ANALYSIS

To obtain the set of tabulated values for $S_i(t)$, $I_i(t)$ and $R_i(t)$ (with i = 1, 2, 3), we employ numerical method with three stages of approximations and sufficient number of decimal places in rounding off the numerical result.

For the first group,

$$\frac{dS_1}{dt} = -\beta_1 S_1 I_1 - \beta_2 S_1 I_2 - \beta_3 S_1 I_3 + \frac{1}{L_1}(K - S)$$

$$\frac{dI_1}{dt} = \beta_1 S_1 I_1 + \beta_2 S_1 I_2 + \beta_3 S_1 I_3 - \gamma_1 I_1 - \frac{1}{L_1} I$$

$$\frac{dR_1}{dt} = \gamma_1 I_1 - \frac{1}{L_1} R$$

The number of susceptible, infective and removal for the first group is,

$$S_1 = S_{1,0} + \frac{1}{6}(e_1 + 2e_2 + 2e_3 + e_4) = 7.9 \times 10^{18} I_1 = I_{1,0} + \frac{1}{6}(g_1 + 2g_2 + 2g_3 + g_4) = -7.9 \times 10^{18}$$

$$R_1 = R_{1,0} + \frac{1}{6}(j_1 + 2j_2 + 2j_3 + j_4) = -7.17 \times 10^{8} \qquad (4)$$

For the second group,

$$\frac{dS_2}{dt} = -\beta_4 S_2 I_2 - \beta_5 S_2 I_1 + \frac{1}{L_2}(K - S)$$

$$\frac{dI_2}{dt} = \beta_4 S_2 I_2 + \beta_5 S_2 I_1 - \gamma_2 I_2 - \frac{1}{L_2} I_2$$

$$\frac{dR_2}{dt} = \gamma_2 I_2 - \frac{1}{L_2} R$$

©IJRASET: All Rights are Reserved

The number of susceptible, infective and removal for the second group is,

$$S_2 = S_{2,0} + \frac{1}{6}(e_1 + 2e_2 + 2e_3 + e_4) = 1.41 \times 10^{20}$$

$$I_2 = I_{2,0} + \frac{1}{6}(g_1 + 2g_2 + 2g_3 + g_4) = -1.42 \times 10^{20}$$

$$R_2 = R_{2,0} + \tfrac{1}{6}(j_1 + 2j_2 + 2j_3 + j_4) = -3.33 \times 10^{9} \quad (5)$$

For the third group,

$$\frac{dS_3}{dt} = -\beta_6 S_3 I_3 - \beta_7 S_3 I_1 - \frac{1}{L_3}(K - S)$$

$$\frac{dI_3}{dt} = \beta_6 S_3 I_3 + \beta_7 S_3 I_1 - \gamma_3 I_3 - \frac{1}{L_3} I_3$$

$$\frac{dR_3}{dt} = \gamma_3 I_3 - \frac{1}{L_3} R_3$$

The number of susceptible, infective and removal for the third group is,

$$S_3 = S_{3,0} + \frac{1}{6}(e_1 + 2e_2 + 2e_3 + e_4) = 4.1 \times 10^{20}$$

$$I_3 = I_{3,0} + \frac{1}{6}(g_1 + 2g_2 + 2g_3 + g_4) = -4.1 \times 10^{20}$$

$$R_3 = R_{3,0} + \tfrac{1}{6}(j_1 + 2j_2 + 2j_3 + j_4) = -1.07 \times 10^{10} \quad (6)$$

It appears that the number of infectives at equilibrium level in community C_1 would be more as compared to the number of infectives in community C_2 and C_3 according to the distribution of total population in three communities. Since there is no interaction of the individuals of the community C_2 and C_3 , even though the infectives in community C_2would exceed than that in C_3. It is possibly due to interaction of the limit of two communities within themselves.

The effect of industrial pollution on the surrounding vegetation and life can always be computed by estimating the extent of pollution in a given industrial domain or land area. Pollution may be in terms of chemical or biological gases, fumes, particulate matter and polluted land or water resources. In addition to these many other industrial pollutants in industrial habitat area which also cause health hazards, the spread of infectious diseases is not uniform in the entire industrial population. Due to lack of awareness and other problems, then such locations are themselves are the means of pollution and cause the spread of infection. The sexually transmitted diseases in industrial towns are also very common. The infection of these diseases spread from one person to other either due to use of unhygienic equipments or other means.

Mathematical formulations are necessary to understand the infectious diseases for specific quantification with the assumption (I) population is closed with diseases without births and deaths, (II) migration is ignored, (III) the rate at which members become infective is proportional to the product of number of susceptible and the number of infectives, (IV) the rate of passage from infective to removed class through recovery or death is proportional to the number of infectives.

IV. RESULTS AND DISCUSSION

The study concerns the analysis of spread of epidemics with a reference to industrial pollution. We have computed number of S(t), I(t) and R(t) population with no control mechanism using numerical method. Due to sufficient number of approximations we can consider the number of I(t) will be increasing otherwise by employing computational method we can form the set of equations as coupled ordinary differential equations. Then, analytical method or numerical method can be employed for further prediction.

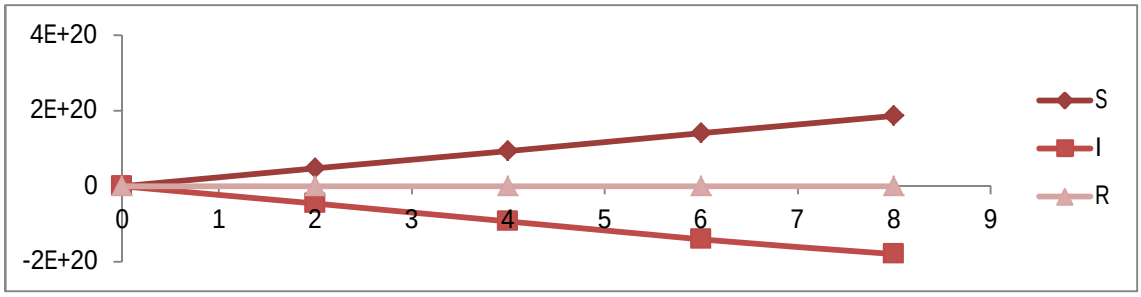


Fig 1: Time v/s population (S-I-R)

©IJRASET: All Rights are Reserved

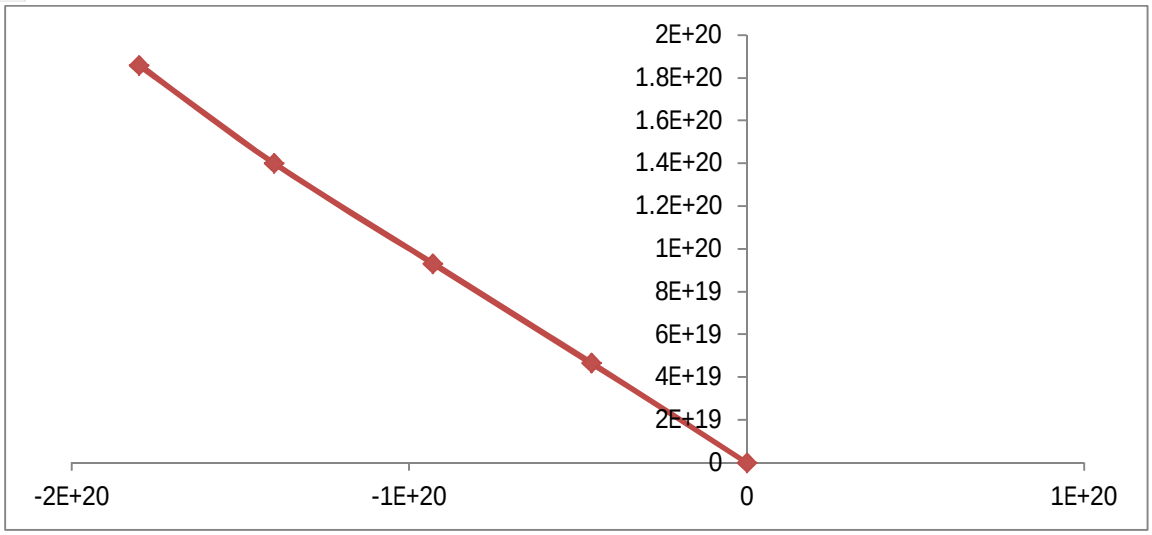


Fig 2: Susceptible Populationv/s Infective population

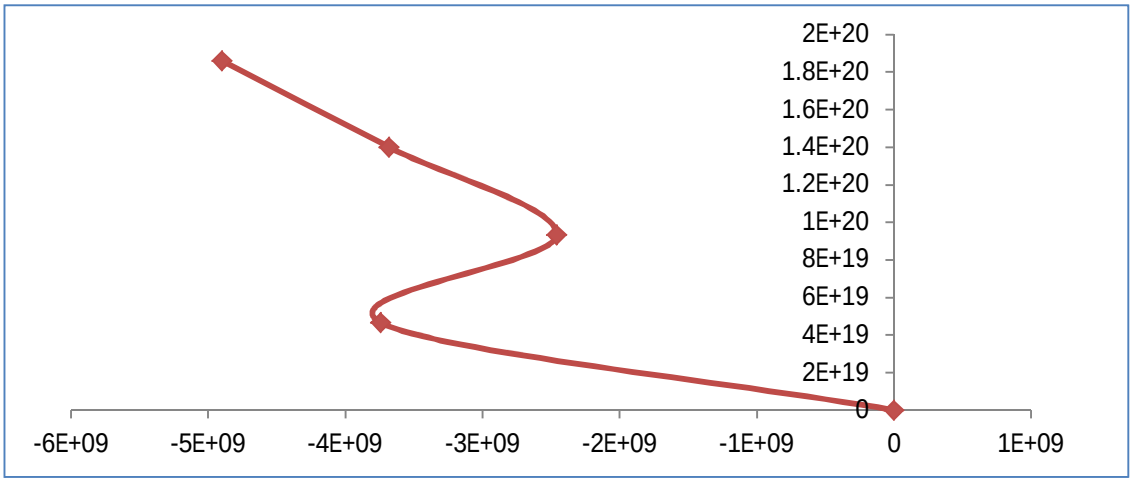
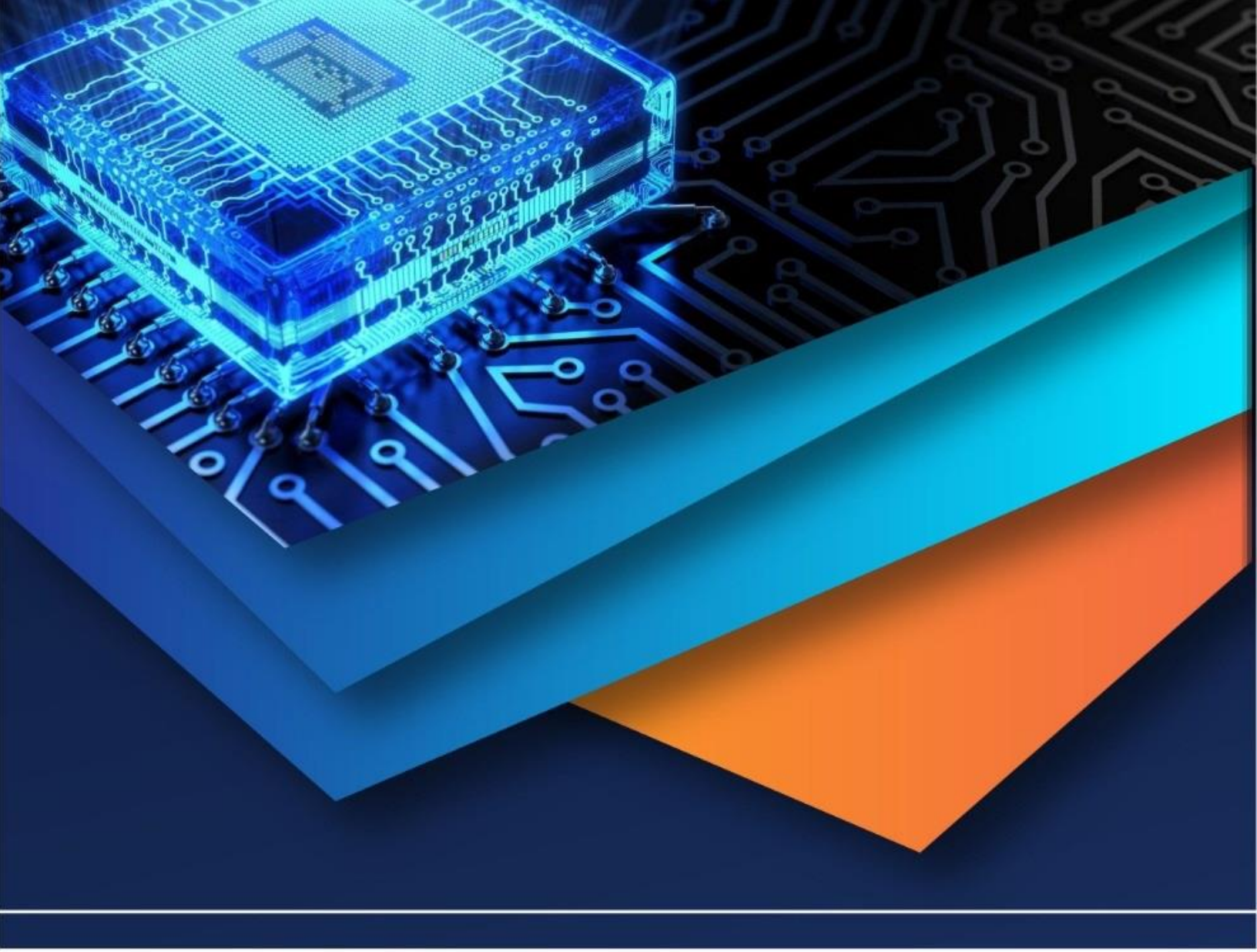


Fig 3: Susceptible Population v/s Removal population

REFERENCES

[1] Vincenzo Capasso [1977], 'A Semigroup Approach To The Study Of A General Stochastic Epidemic', Stochastic Processes and their Applications 5 PP 285-296.
[2] David Greenhalgh [1986], 'Simple Models for control of Epidemics', Mathematical Modeling, Vol 7, PP 753-763.
[3] W Y Tan [1991], 'Stochastic Models Of Hiv Epidemic In Homosexual Populations-The Effects Of Mixing Patterns', Mathematical Computer. Modelling, Vol 15, No.12, PP 37-65.
[4] M. Y. Kim and F. A. Milner [1995], 'A Mathematical Model of Epidemics with Screening and Variable Infectivity',Mathematical Computer Modelling, Vol. 21, No. 7, pp 29-42.
[5] L. stone, B. Shulgin, Z. Agur [2000], 'Theoretical Examination of the Pulse Vaccination Policy in the SIR Epidemic Model', Mathematical and Computer Modelling 31, PP 207-215.
[6] Timothy B. Hallett, Simon Gregson, Owen Mugurungi, Elizabeth Gonese, Geoff P. Garnett [2009], 'Assessing evidence for behaviour change affecting the course of HIV epidemics: A new mathematical modelling approach and application to data from Zimbabwe', Epidemics Vol.1, PP 108–117.
[7] Huiyan Kang, Xinchu Fu [2015], 'Epidemic spreading and global stability of an SIS model with an infective vector on complex networks', Communications in Nonlinear Science and Numerical Simulation, Vol 27, PP 30-39.
[8] Thomas Wetere Tulu, BopingTian, Zunyou Wu [2017], 'Mathematical Modeling, Analysis and Markov Chain Monte Carlo simulation of Ebola Epidemics' Results in Physics Vol 7, PP 962–968.
[9] Meghendra Singh, AchlaMarathe, Madhav V. Marathe, Samarth Swarup [2018]. "Behavior Model Calibration for Epidemic Simulations". AAMAS 2018, July 10-15, 2018, Stockholm, Sweden, PP. 1640-1648.
[10] Jürgen Hackl and ThibautDubernet [2019]. "Epidemic Spreading in Urban Areas Using Agent-Based Transportation Models". MDPI, Future internet, PP. 1-14.

©IJRASET: All Rights are Reserved



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089 (24*7 Support on Whatsapp)