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# Five Dimensional Plane Symmetric Cosmological Model in Perfect Fluid with Cosmological Constant Λ.

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***Abstract:*** ***In this paper, we have investigated five dimensional plane symmetric cosmological model in the frame of scalar field in presence of binary mixture of perfect fluid and dark energy given by cosmological constant. The perfect fluid is chosen to be one obeying equation of state*** $P = \gamma\rho$ ***with*** $\gamma \in [0,1]$***. The exact solution of Einstein's field equation are obtained corresponding to higher dimensional plane symmetric cosmological model. Models with power-law and exponential expansion have discussed in detail.***

***Keywords: Plane Symmetric cosmological model, Perfect fluid, Cosmological constant, Dark energy.***

## I. INTRODUCTION

The astronomical observation including red shift Supernovae of the type $I_a$ [3,14], Wilkinson Microwave Anisotropy probe (WMAP) [5,17], Cosmic Microwave Background (CMB) radiation [15] and large-scale structure [1,2] have indicated that our universe is undergoing an accelerating expansion at present. All such consequence strongly implies existence of an additional component to the matter distribution of the universe with negative pressure the so-called dark energy. The observational evidence indicate that our universe contains 4.9 % ordinary baryonic matter, 26.8% dark matter and 68.3% dark energy.

At the present state of evolution of the universe, on whole, is spherically symmetric and isotropic. But in its early stages of evolution, it could not have had a such smoothed out picture. So we consider plane symmetric, inflationary model which is less restrictive than symmetry and isotropy. It provides avenue to study inhomogeneities. Inhomogeneous cosmological model plays an important role in understanding some essential features of the universe such as formation of galaxies during early stages of evolution and process of homogenization.

Saha[6],Saha and Boyadjiev [9] have studied the role of a term Λ in the evolution of Binachi type-I space time in presence of spinor and/or scalar field with a perfect fluid. Khalatnikov and Kamensnchik [11] studied the Einstein's equations for Bianchi type-I universe in the presence of dust,stiff matter and cosmological constant. Saha[7] studied a self - consistent Bianchi type-I cosmological model in presence of binary mixture of perfect fluid and dark energy given by cosmological constant. Singh and Chaubey [18], K S Adhav et al.[12] studied Bianchi type-V model of the universe with a binary mixture of perfect fluid and dark energy. K S Adhav et al.[13] investigated Bianchi type-I universe with binary mixture of perfect fluid and dark energy. Katore,et al.[16] investigated planesymmetric cosmological model with perfect fluid and dark energy. Recently Gupta Apurav and Sancheti M [10] investigated inhomogeneous plane symmetric cosmological model with cosmological constant. The study of higher dimensional cosmology is important because it reveal the nature of the universe at early stages of the evolution.

In present paper, author focused to investigate the role of the term Λ in evolution of higher dimensional plane symmetric cosmological model in presence of scalar field with a perfect fluid obeys the equation of state $P = \gamma\rho$ with $\gamma \in [0,1]$. The Exact Solution has been obtained in quadrature form. The case of power - law and Exponential expansion have been discussed in detail. The outline of the paper is as follows. The metric and field equation with solution are describe in section 2, section 3 Universe as Binary Mixture of Perfect fluid and dark energy, Models with power-law and exponential expansion discussed in section 4 and Finally conclusion are summarized in section 5.

## II. BASIC EQUATIONS

We consider plane symmetric higher dimensional Bianchi type-I space time describe by the metric

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2dz^2 - C^2dw^2 \quad (1)$$

As the matrix potential A,B,C are function of cosmic function t only. Here extra coordinate can be taken as space-like.

The energy momentum tensor of the source is given by

$$T_i^j = (\rho + P)u^iu_j - P\delta_j^i \quad (2)$$

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Where, $u^j$ the flow vector satisfying the equation $g_{ij}u^i u^j = 1$

Here $\rho$ is the energy density of a perfect fluid and / or dark density , while $P$ is the corresponding pressure. P and $\rho$ are related by equation of state .

The Einstein's field equations for the metric (1) with the help of Eq. (2) are given by using equation (2) we find,

$$T_1^1 = T_2^2 = T_3^3 = T_4^4 = -P \, , T_5^5 = \rho$$

in a co-moving system of co-ordinates.

The Einstein's field equation with Λ can be written in the form

$$R_j^i - \frac{1}{2} R g_j^i + \Lambda = -8\Pi P + \Lambda \qquad (3)$$

From the equation (1) - (3) line element leads to the given system of equations

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = -8\pi P + \Lambda \qquad (4)$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + 2\frac{\dot{A}\dot{C}}{AC} + \left(\frac{\dot{A}}{A}\right)^2 = -8\pi P + \Lambda \qquad (5)$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \left(\frac{\dot{A}}{A}\right)^2 = -8\pi P + \Lambda \qquad (6)$$

$$2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} + \left(\frac{\dot{A}}{A}\right)^2 = -8\pi\rho + \Lambda \qquad (7)$$

Here, K is the gravitational constant and overhead dot (.) denotes differentiation with respect to t. Let V be the function of time 't' denoted by

$$V = A^2BC \qquad (8)$$

From equation (4) and (5) ,we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \qquad (9)$$

Then from equation (8) and (9) , we have

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = 0$$

Which on integration gives

$$\frac{A}{B} = l_1 \exp\left(m_1 \int \frac{dt}{V}\right)$$

From equation (5) and (6), we get

$$\frac{d}{dt}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\frac{\dot{V}}{V} = 0$$

Which on integration gives

$$\frac{B}{C} = l_2 \exp\left(m_2 \int \frac{dt}{V}\right) \qquad (12)$$

From eqn. (4) and (6)

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right)\frac{\dot{V}}{V} = 0$$

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Which on integration gives

$$\frac{A}{C} = l_3 \exp\left(m_3 \int \frac{dt}{V}\right) \quad (13)$$

Where, $m_3 = m_1 + m_2$, $l_3 = l_1 l_2$

Using eqn. (11), (12) and (13), the value of A, B, C can be written explicitly as

$$A = D_1 V^{\frac{1}{4}} exp\left(X_1 \int \frac{dt}{V}\right) \quad (14)$$

$$B = D_2 V^{\frac{1}{4}} exp\left(X_2 \int \frac{dt}{V}\right) \quad (15)$$

$$C = D_3 V^{\frac{1}{4}} exp\left(X_3 \int \frac{dt}{V}\right) \quad (16)$$

Where $D_i$ (i = 1, 2,3) satisfies the relation $D_1^2 D_2 D_3 = 1$ and $2X_1 + X_2 + X_3 = 0$

On adding equation (5), (6), 2 times eqn. (4) and 4 times eqn. (6), we get

$$6\left(\frac{\ddot{A}}{A}\right) + 3\left(\frac{\ddot{B}}{B}\right) + 3\left(\frac{\ddot{C}}{C}\right) + 12\left(\frac{\dot{A}\dot{B}}{AB}\right) + 12\left(\frac{\dot{A}\dot{C}}{AC}\right) + 6\left(\frac{\dot{B}\dot{C}}{BC}\right) + 6\left(\frac{\dot{A}}{A}\right)^2 = 32\pi(\rho - P) + 8\Lambda \quad (17)$$

By using the eqn.

$$\frac{\ddot{V}}{V} = \frac{32}{3}\pi(\rho - P) + \frac{8}{3}\Lambda \quad (18)$$

Energy conservation eqn. is given by

$$\dot{\rho} = -\frac{\dot{V}}{V}(\rho + P) \quad (19)$$

Using Eqn. (18),(19)

$$\dot{V} = \sqrt{\left(\frac{32}{3}\pi\rho V^2 + C_1 + \frac{4}{3}\Lambda V\right)} \quad (20)$$

Where $C_1$ is the constant of integration.

Furthermore, considering that the pressure (P) and density ($\rho$) obeying an equation of state of type $P = f(P)$, we conclude that the $\rho$ and P are functions of V. Hence the right-hand side of eqn. (20) is the function of V.

$$\ddot{V} = \frac{32}{3}\pi(\rho - P)V + \frac{8}{3}\Lambda V = F(V)$$

From mechanical point of view Eq. (20) can be interpreted as an equation of motion of single particle with unit mass under the force F(V). Then

$$\dot{V} = \pm\sqrt{2(\epsilon - U(V))} \quad (22)$$

Here $\epsilon$ can be viewed as the energy and U(V) as the potential of force F. Comparing equation (19) and (21), we $\epsilon = C_1$ and

$$U(V) = -\left(\frac{32}{3}\pi\rho V^2 + \frac{4}{3}\Lambda\right) \quad (23)$$

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Finally, we write the solution of the equation (20) in quadrature form,

$$\int \frac{dV}{\sqrt{2\left(\frac{32}{3}\pi\rho V^2 + \frac{4}{3}\Lambda V + C_1\right)}} = t + t_0 \quad (24)$$

Where, integration constant $t_0$ can be taken to be zero, since it only gives a shift in time. Hence, let us take $t_0 = 0$.

$$\int \frac{dV}{\sqrt{2\left(\frac{32}{3}\pi\rho V^2 + \frac{4}{3}\Lambda V + C_1\right)}} = t \quad (25)$$

## III. UNIVERSE AS BINARY MIXTURE OF PERFECT FLUID AND DARK ENERGY

We consider five-dimensional plane symmetric universe filled with perfect fluid which obey the equation of state.

$$P = \gamma\rho \quad (26)$$

Where $\gamma$ is the constant and lies in the interval $\gamma \in [0, 1]$ Depending on its numerical value $\gamma$ de- scribe following type of universe

$$\gamma = 0 \text{ (Dust Universe)} \quad (27)$$

$$\gamma = \frac{1}{3} \text{ (radiation Universe)} \quad (28)$$

$$\gamma \in \left(\frac{1}{3}, 1\right) \text{(Hard Universe)} \quad (29)$$

$$\gamma = 1 \text{(Zelvoich Universe or stiff Universe)} \quad (30)$$

In a comoving frame the conservation law of energy momentum tensor lead to the balance equation for the energy density

$$\dot{\rho} = -\frac{\dot{V}}{V}(\rho + P) \quad (31)$$

From the equation (19) we get,

$$P = \frac{\gamma\rho_0}{V^{1+\gamma}} \quad (32)$$

$$\rho = \frac{\rho_0}{V^{1+\gamma}} \quad (33)$$

here $\rho_0$ is the integration constant. Therefore, equation (25) gives

$$\int \frac{dV}{\sqrt{2C_1 + \frac{64}{3}\pi\rho_0 V^{1-\gamma} + \frac{8}{3}\Lambda V^2}} \quad (34)$$

### A. *Case I*

Let us consider the $\gamma = 1$ for (Zeldovich universe) for $C_1 = 0$, equation (34) reduces to

$$\int \frac{dV}{\sqrt{\frac{64}{3}\pi\rho_0 + \frac{8}{3}\Lambda V^2}} \quad (35)$$

This gives the

$$V = \frac{\sqrt{8\pi\rho_0}}{\sqrt{\Lambda}} \sinh\sqrt{\frac{8}{3}\Lambda}\, t \quad (36)$$

Put the values of V in eqn. (14), (15) & (16)

$$A = D_1 (K_1)^{\frac{1}{4}} sinh^{\frac{1}{4}} \sqrt{\frac{8}{3}\Lambda}\, t \left[\operatorname{csch}\sqrt{\frac{8}{3}\Lambda}\, t + \coth\sqrt{\frac{8}{3}\Lambda}\, t\right]$$

$$B = D_2 (K_1)^{\frac{1}{4}} sinh^{\frac{1}{4}} \sqrt{\frac{8}{3}\Lambda}\, t \left[\operatorname{csch}\sqrt{\frac{8}{3}\Lambda}\, t + \coth\sqrt{\frac{8}{3}\Lambda}\, t\right]$$

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$$C = D_3 (K_1)^{\frac{1}{4}} sinh^{\frac{1}{4}} \sqrt{\frac{8}{3}\Lambda}\, t \left[ \text{csch} \sqrt{\frac{8}{3}\Lambda}\, t + \coth \sqrt{\frac{8}{3}\Lambda}\, t \right]$$

Where, $K_1 = \frac{\sqrt{8\pi\rho_0}}{\Lambda}$ and Where $D_i$ (i = 1, 2,3 ) satisfies the relation $D_1^2 D_2 D_3 = 1$ and $2X_1 + X_2 + X_3 = 0$

$$\rho = \frac{\Lambda}{8\pi sinh^2 \sqrt{\frac{8}{3}\Lambda}\, t} \tag{37}$$

$$P = \frac{\Lambda}{8\pi sinh^2 \sqrt{\frac{8}{3}\Lambda}\, t} \tag{38}$$

*B. Case II*

Let us Consider $\gamma = 0$ (Dust Universe) for $C_1 = 0$ eqn. (34) reduces to

$$\frac{1}{\sqrt{\frac{8}{3}\Lambda}} \int \frac{dV}{\sqrt{\left(V - \frac{K}{2}\right)^2 + \left(\frac{K}{2}\right)^2}} = t \tag{39}$$

This gives the

$$V = \frac{K}{2} \sinh \sqrt{\frac{8}{3}\Lambda}\, t + \frac{K}{2} \tag{40}$$

Put the values of V in eqn. (14) (15) & (16)

$$A = D_1 \left( \frac{K}{2} \sinh \sqrt{\frac{8}{3}\Lambda}\, t + \frac{K}{2} \right)^{\frac{1}{4}} e^{\left( X_1 \int \frac{dt}{\frac{K}{2} \sinh \sqrt{\frac{8}{3}\Lambda}\, t + \frac{K}{2}} \right)}$$

$$B = D_2 \left( \frac{K}{2} \sinh \sqrt{\frac{8}{3}\Lambda}\, t + \frac{K}{2} \right)^{\frac{1}{4}} e^{\left( X_2 \int \frac{dt}{\frac{K}{2} \sinh \sqrt{\frac{8}{3}\Lambda}\, t + \frac{K}{2}} \right)}$$

$$C = D_3 \left( \frac{K}{2} \sinh \sqrt{\frac{8}{3}\Lambda}\, t + \frac{K}{2} \right)^{\frac{1}{4}} e^{\left( X_3 \int \frac{dt}{\frac{K}{2} \sinh \sqrt{\frac{8}{3}\Lambda}\, t + \frac{K}{2}} \right)}$$

From the eqn. (32) &(40) we have

$$\rho = \frac{\rho_0}{\frac{K}{2} \sinh \sqrt{\frac{8}{3}\Lambda}\, t + \frac{K}{2}} \tag{41}$$

$$P = 0 \tag{42}$$

## IV. MODELS WITH CONSTANT DECLARATION PARAMETER

*A. Model for Exponential Expansion*

Thus, extra conditions are needed to solve the system completely. For that we have used two different volumetric expansion law $V = \alpha e^{4\beta t}$ Where $\alpha, \beta$ are positive constant. Then

$$A = D_1 \alpha^{\frac{1}{4}} e^{\beta t} \exp\left( \frac{-X_1}{4\alpha\beta}\, e^{-4\beta t} \right) \tag{43}$$

$$B = D_2 \alpha^{\frac{1}{4}} e^{\beta t} \exp\left( \frac{-X_2}{4\alpha\beta}\, e^{-4\beta t} \right) \tag{44}$$

$$C = D_3 \alpha^{\frac{1}{4}} e^{\beta t} \exp\left( \frac{-X_3}{4\alpha\beta}\, e^{-4\beta t} \right) \tag{45}$$

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The directional Hubble parameter's $H_x, H_y, H_z$ *and* $H_w$ *have given by*

$$H_x = H_y = \beta + \frac{X_1}{\alpha} e^{-4\beta t} \quad (46)$$

$$H_z = \beta + \frac{X_2}{\alpha} e^{-4\beta t} \quad (47)$$

$$H_w = \beta + \frac{X_3}{\alpha} e^{-4\beta t} \quad (48)$$

Therefore, Mean Hubble parameter H is given by,

$$\mathrm{H} = \frac{1}{4}\left[2\left(\frac{\dot{A}}{A}\right) + \left(\frac{\dot{B}}{B}\right) + \left(\frac{\dot{C}}{C}\right)\right]$$

$$H = \beta \quad (49)$$

Anisotropy parameter of the expansion is

$$\Delta = \frac{1}{4}\sum_{i=1}^{i=4}\left(\frac{H_i - H}{H}\right)^2 = 0 \quad (50)$$

The dynamical scalar is given by,

$$\theta = 4H = 4\beta \quad (51)$$

Deceleration parameter q is given by $q = \frac{d}{dt}\left(\frac{1}{H} - 1\right) = -1$

*B. Model for Power Law Expansion*

Here we take $V = \alpha t^{4n}$ where α and n are constant

$$A = D_1 \alpha^{\frac{1}{4}} t^n \exp\left(\frac{X_1}{\alpha}\frac{t^{1-4n}}{1-4n}\right) \quad (52)$$

$$B = D_2 \alpha^{\frac{1}{4}} t^n \exp\left(\frac{X_2}{\alpha}\frac{t^{1-4n}}{1-4n}\right) \quad (53)$$

$$C = D_3 \alpha^{\frac{1}{4}} t^n \exp\left(\frac{X_3}{\alpha}\frac{t^{1-4n}}{1-4n}\right) \quad (54)$$

The directional Hubble parameter's $H_x, H_y, H_z$ *and* $H_w$ *have given by*

$$H_x = H_y = \frac{n}{t} + \frac{X_1}{\alpha t^{4n}} \quad (55)$$

$$H_z = \frac{n}{t} + \frac{X_2}{\alpha t^{4n}} \quad (56)$$

$$H_w = \frac{n}{t} + \frac{X_3}{\alpha t^{4n}} \quad (57)$$

Therefore, Mean Hubble parameter H is given by

$$H = \frac{n}{t} \quad (58)$$

Anisotropy parameter of the expansion is

$$\Delta = \frac{1}{4}\sum_{i=1}^{i=4}\left(\frac{H_i - H}{H}\right)^2 = 0$$

The dynamical scalar is given by,

$$\theta = 4H = \frac{4n}{t} \quad (59)$$

Deceleration parameter q is given by $q = \frac{d}{dt}\left(\frac{1}{H} - 1\right) = -(n-1)$

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## V. CONCLUSIONS

The Higher dimensional plane symmetric cosmological model is studied in presence of perfect fluid and a term Λ in framework of scaler - tensor theory of gravitation. The exact solution for corresponding Einstein's field equations are obtained by considering a constant declaration parameter that lead two different aspect of the volumetric expansion namely a power law and an exponential volumetric expansion.

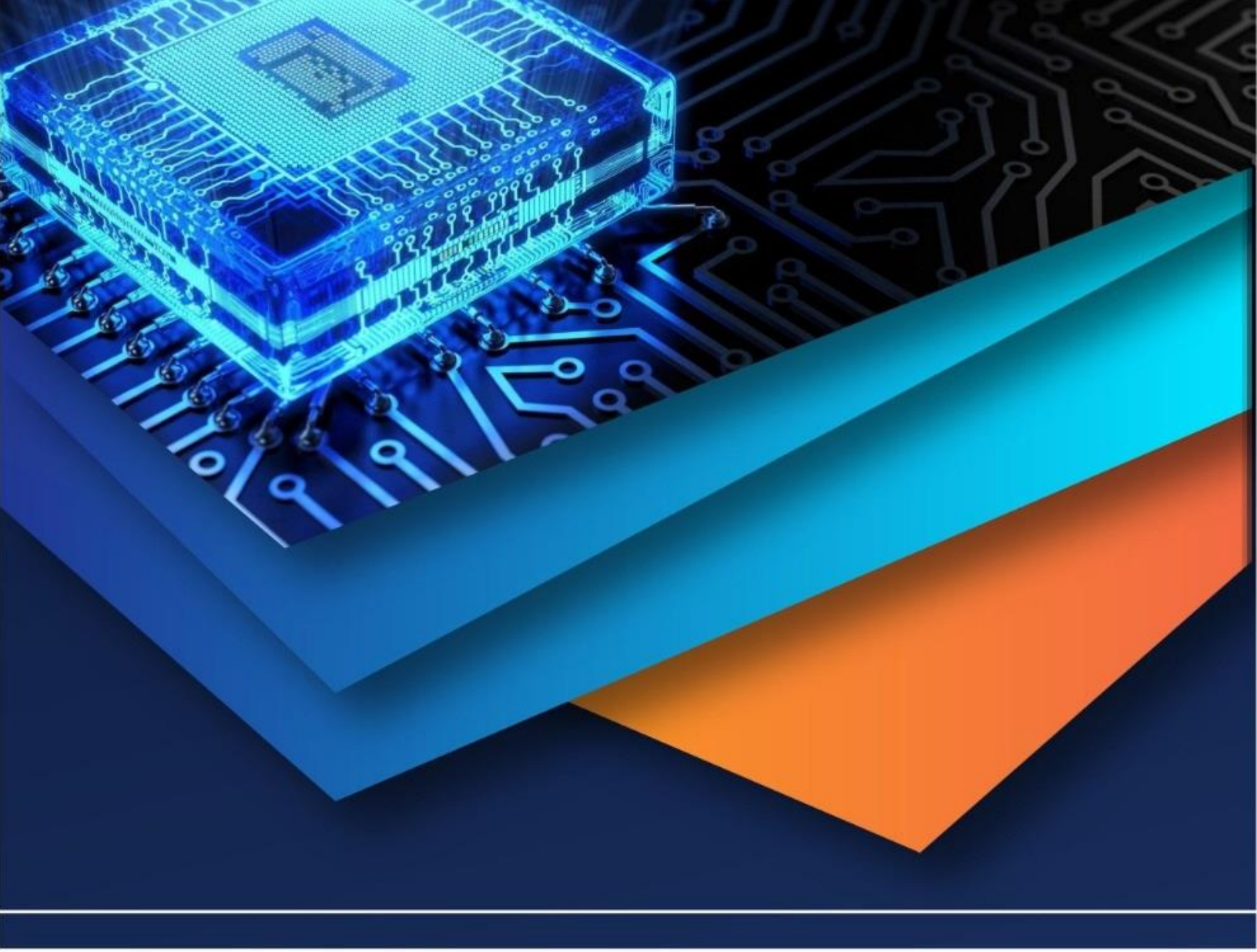
In the Exponential Volumetric expansion, the universe starts with zero volume, at the initial epoch and expands exponentially approaching to infinite volume. The universe expands homogeneously. The universe accelerates with highest rate q = -1. The shear scalar tends to zero for all t. The anisotropy parameter Δ = 0 shows that model is isotropic for all time. Hence the present model is isotropic and accelerated which is consistent to the current observation.

In the power law expansion, both scale factors vanish at t = 0, start evolving with time and finally as time t tends to infinity they diverge to infinity. This is consistent with big bang model. The value of anisotropy parameter shows that model is isotropic for all time. The deceleration parameter appears with negative sign which implies accelerating expansion of the universe.

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