iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 8 **Issue:** X **Month of publication:** October 2020

DOI: https://doi.org/10.22214/ijraset.2020.31743

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 8 Issue X Oct 2020- Available at www.ijraset.com

Lindley Exponential Power Distribution with Properties and Applications

Ramesh Kumar Joshi[1], Vijay Kumar[2]
[1]*Department of Statistics, Trichandra Multiple Campus, Kathmandu, Nepal,*
[2]*Department of Mathematics and Statistics, DDU Gorakhpur University, Gorakhpur, India*

***Abstract:** In this article, we have purposed a new probability distribution created by using the Lindley-G family with baseline distribution as exponential power distribution called Lindley exponential power distribution. The mathematical and statistical properties of the proposed model are discussed. These include the shapes of the probability density, cumulative density and hazard rate functions, the quantile function, the skewness, and kurtosis. We discuss maximum likelihood estimation of the distribution parameters and asymptotic confidence interval based on maximum likelihood. All the computations are performed in R software. An application of the model is explored using a real data set and also AIC, BIC and AICC are calculated to assess the validity of the observed model. Finally the potentiality of LEP is compared with the fit attained by some other distributions.*

***Keywords:** Lindley distribution, Exponential power, Estimation, Maximum likelihood.*

I. INTRODUCTION

Lifetime distributions are generally used to study the length of the life of components of a system, a device, and in general, reliability and survival analysis. Lifetime distributions are frequently used in fields like life science, medicine, biology, engineering, insurance, etc. Many continuous probability distributions such as Cauchy, exponential, gamma, Weibull have been frequently used in statistical literature to analyze lifetime data. For a few years, most of the researchers are attracted towards one parameter Lindley distribution for its potential in modeling lifetime data, and it has been observed that this distribution has performed excellently in many applications. Researchers in the last few years has developed various extensions and modified form of the Lindley distribution which was developed by [15] in the context of Bayesian statistics as a counterexample to fiducial statistics. A detailed study on the Lindley distribution was done by [7].

A random variable T follows Lindley distribution with parameter θ and its probability density function (pdf) is given by

$$f(t)=\frac{\theta}{\theta+1}(1+t)e^{-\theta t};\ t>0,\theta>0 \qquad (1.1)$$

And its cumulative density function (CDF) is

$$F(t)=1-\frac{1+\theta+\theta t}{1+\theta}e^{-\theta t};\ t>0,\theta>0 \qquad (1.2)$$

Some of the modifications in the literature of Lindley distribution are given by [6] showed that the Lindley distribution is quite similar to the exponential distribution. Reference [9] has investigated the estimation of the parameters using hybrid censored data. The estimation of the model parameters for censored samples by [13], and this distribution was applied by [17] to calculate competing risks in lifetime data.

In the context of distribution theory, [8] introduced weighted Lindley distribution having two parameters and has shown that it is appropriate in modeling biological data for a mortality study. Reference [19] has introduced generalized Lindley, extended Lindley by [3], Reference [2] for exponentiated power Lindley, and The Lindley–Exponential distribution by [4]. Reference [12] has introduced a new extension of Lindley distribution. Further, we observed some continuous-discrete mixed approaches; Reference [22] have defined the discrete Poisson-Lindley. The Pareto Poisson Lindley distribution has introduced by [1]. Reference [26] introduced negative binomial Lindley distribution. Reference [5] have presented a new class of distributions to generate new distribution based on Lindley generator (Li-G) having additional shape parameter θ. The CDF and pdf of Li-G are respectively,

$$F(t;\theta,\lambda)=1-\left[1-G(t;\lambda)\right]^{\theta}\left[1-\frac{\theta}{\theta+1}\ln\overline{G}(t;\lambda)\right];\ t>0,\theta>0 \qquad (1.3)$$

and $$f(t;\theta,\lambda)=\frac{\theta^2}{\theta+1}g(t;\lambda)\left[1-G(t;\lambda)\right]^{\theta-1}\left[1-\ln\overline{G}(t;\lambda)\right];\ t>0,\theta>0 \qquad (1.4)$$

where $$g(t;\lambda)=\frac{dG(t;\lambda)}{dt},\overline{G}(t;\lambda)=1-G(t;\lambda)$$

The main aim of this study is to obtain a more flexible model by adding just one extra parameter to the exponential power distributio n using (1.3) and (1.4) to achieve a better fit to real data. We study properties of the LEP distribution and explore its applicability. T he contents of the proposed study are organized as follows. The new Lindley exponential power distribution is introduced and vario us distributional properties are discussed in Section 2. The maximum likelihood estimation procedure to estimate the model paramet ers and associated confidence intervals using the observed information matrix is discussed in Section 3. In Section 4, a real data set has been analyzed to explore the applications and suitability of the proposed distribution. In this section, we present the ML estimat ors of the parameters and approximate confidence intervals also AIC, BIC, CAIC and HQC are calculated to assess the validity of th e LEP model. Further we have performed goodness-of-fit tests using the Kolmogorov-Simnorov (K-S), the Anderson-Darling (AD) and the Cramer-Von Mises (CVM) statistics and found that the proposed model is better as compared to six other models. We expec t that this model may contribute in the field of survival analysis and theory of statistics. Finally, Section 5 ends up with some general concluding remarks.

II. THE LINDLEY EXPONENTIAL POWER (LEP) DISTRIBUTION

Using the Li-G family (1.3) and (1.4) we are introduced the new distribution where the baseline distribution is the exponential power distribution. The exponential power distribution has introduced by [23] to analyze the software reliability data. The CDF and PDF of the exponential power distribution are respectively as

$$G(x)=1-\exp\left[1-e^{(\lambda x)^{\alpha}}\right];\ \alpha,\lambda>0,\ x>0 \qquad (2.1)$$

and $$g(x)=\alpha\lambda^{\alpha}x^{\alpha-1}e^{(\lambda x)^{\alpha}}\exp\left[1-e^{(\lambda x)^{\alpha}}\right];\ \alpha,\lambda>0,\ x>0 \qquad (2.2)$$

We obtained a new distribution with three parameters (α, λ, θ) for the random variable $X \sim L-EP(\alpha,\lambda,\theta)$ whose CDF is defined as

$$F(x)=1-\left[1-\left(\frac{\theta}{1+\theta}\right)\left(1-e^{(\lambda x)^{\alpha}}\right)\right]\exp\left[\theta\left(1-e^{(\lambda x)^{\alpha}}\right)\right];\ \alpha,\lambda,\theta>0,\ x>0 \qquad (2.3)$$

The PDF is given by

$$f(x)=\alpha\lambda^{\alpha}\left(\frac{\theta^2}{1+\theta}\right)x^{\alpha-1}e^{(\lambda x)^{\alpha}}\left[1-\left(1-e^{(\lambda x)^{\alpha}}\right)\right]\exp\left[\theta\left(1-e^{(\lambda x)^{\alpha}}\right)\right];\ \alpha,\lambda,\theta>0,\ x>0 \qquad (2.4)$$

The reliability/survival function of LEP distribution is

$$R(x)=\left[1-\left(\frac{\theta}{1+\theta}\right)\left(1-e^{(\lambda x)^{\alpha}}\right)\right]\exp\left[\theta\left(1-e^{(\lambda x)^{\alpha}}\right)\right];\ \alpha,\lambda,\theta>0,\ x>0 \qquad (2.5)$$

Suppose that an item has survived for a time t and we desire the probability that it will not survive for an additional time *dt* then, hazard rate function is,

$$h(t)=\lim_{dt\to 0}\frac{pr(t\le T<t+dt)}{dt.S(t)}=\frac{f(t)}{S(t)}=\frac{f(t)}{1-F(t)};\ 0<t<\infty \qquad (2.6)$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 8 Issue X Oct 2020- Available at www.ijraset.com

The hazard rate function (hrf) of L-EP distribution is

$$h(x)=\frac{f(x)}{R(x)}=\frac{\alpha\lambda^{\alpha}\left(\frac{\theta^2}{1+\theta}\right)x^{\alpha-1}e^{(\lambda x)^{\alpha}}\left[1-\left(1-e^{(\lambda x)^{\alpha}}\right)\right]}{\left[1-\left(\frac{\theta}{1+\theta}\right)\left(1-e^{(\lambda x)^{\alpha}}\right)\right]}\ ;\ \alpha,\lambda,\theta>0,\ x>0 \qquad (2.7)$$

Figure 1 (left panel) the PDF plot illustrate that f(x) can bear different shapes depending upon the values of shape parameters by keeping scale parameter fixed. The LEP distribution shows good statistical behavior based on probability density function. Fig. 1 (right panel) the HRF plot illustrate that h(x) can bear different shapes like increasing, decreasing and up-side bathtub for different values of shape parameters by keeping scale parameter fixed. The LEP distribution shows good statistical behavior based on hazard rate function.

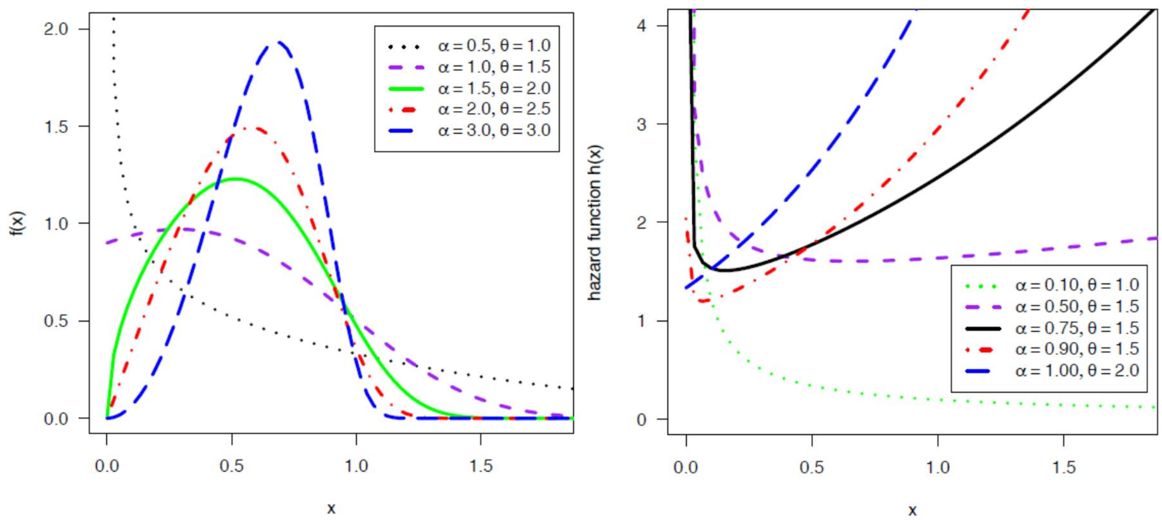

Fig 1. Graph of PDF (left panel) and hazard function (right panel) for λ = 1 and different values of α and θ.

The Quantile function of LEP is given by

$$p-1+\left[1-\left(\frac{\theta}{1+\theta}\right)\left(1-e^{(\lambda x)^{\alpha}}\right)\right]\exp\left[\theta\left(1-e^{(\lambda x)^{\alpha}}\right)\right]=0,\ 0<p<1 \qquad (2.8)$$

In descriptive statistics, the measures of skewness and kurtosis play an important role in data analysis. The coefficient of Bowley's skewness measure based on quartiles is given by

$$S_k(B)=\frac{Q(3/4)+Q(1/4)-2Q(1/2)}{Q(3/4)-Q(1/4)}, \qquad (2.9)$$

and the coefficient of Moor's kurtosis measures based on octiles [18] is given by

$$K_u(M)=\frac{Q(0.875)-Q(0.625)+Q(0.375)-Q(0.125)}{Q(3/4)-Q(1/4)}, \qquad (2.10)$$

III. MAXIMUM LIKELIHOOD ESTIMATION (MLE)

In this section, we discuss the maximum likelihood estimators (MLE's) of the LEP distribution.

Let $\underline{x}=(x_1,\ldots,x_n)$ be a random sample of size 'n' from L-EP(α, λ, θ), then the likelihood function L(α, λ, θ/ $\underline{x}$) can be written as,

$$L(\alpha,\lambda,\theta\,|\,\underline{x})=\prod_{i=1}^{n}\alpha\lambda^{\alpha}\left(\frac{\theta^2}{1+\theta}\right)x_i^{\alpha-1}e^{(\lambda x_i)^{\alpha}}\left[1-\left(1-e^{(\lambda x_i)^{\alpha}}\right)\right]\exp\left[\theta\left(1-e^{(\lambda x_i)^{\alpha}}\right)\right]$$

It is easy to deals with log-likelihood estimator of the parameters l(α,λ,θ) is

$$l = n\ln\alpha + \alpha n\ln\alpha + 2n\ln\theta - n\ln(1+\theta) + (\alpha-1)\sum_{i=1}^{n}\ln x_i + (\lambda x_i)^{\alpha} + \theta\sum_{i=1}^{n}\left(1-e^{(\lambda x_i)^{\alpha}}\right) + \sum_{i=1}^{n}\ln\left\{1-\left(1-e^{(\lambda x_i)^{\alpha}}\right)\right\} \quad (3.1)$$

To estimate the unknown parameters of the LEP (α,λ,θ), we have to solve the following nonlinear equations equating to zero.

$$\frac{\partial l}{\partial\alpha} = \frac{n}{\alpha} + n\ln\lambda + \sum_{i=1}^{n}\ln x_i + 2\sum_{i=1}^{n}(\lambda x_i)^{\alpha}\ln(\lambda x_i) - \theta\sum_{i=1}^{n}(\lambda x_i)^{\alpha} e^{(\lambda x_i)^{\alpha}}\ln(\lambda x_i)$$

$$\frac{\partial l}{\partial\lambda} = \frac{n\alpha}{\lambda} + \frac{2\alpha}{\lambda}\sum_{i=1}^{n}(\lambda x_i)^{\alpha} - \frac{\alpha\theta}{\lambda}\sum_{i=1}^{n}(\lambda x_i)^{\alpha} e^{(\lambda x_i)^{\alpha}} \quad (3.2)$$

$$\frac{\partial l}{\partial\theta} = n + \frac{n(2+\theta)}{\theta(\theta+1)} - \sum_{i=1}^{n} e^{(\lambda x_i)^{\alpha}}$$

Clearly, it is difficult to solve (3.2), so by using the computer software R, Mathematica, Matlab or any other program and Newton-Raphson's iteration method one can solve these equations. Let us denote the parameter vector by $\underline{\delta} = (\alpha, \lambda, \theta)$ and the corresponding MLE of $\underline{\delta}$ as $\hat{\underline{\delta}} = (\hat{\alpha}, \hat{\lambda}, \hat{\theta})$, then the asymptotic normality results in, $\left(\hat{\underline{\delta}} - \underline{\delta}\right) \to N_3\left[0, \left(I(\underline{\delta})\right)^{-1}\right]$ where $I(\underline{\delta})$ is the Fisher's information matrix given by,

$$I(\underline{\delta}) = -\begin{pmatrix} E\left(\frac{\partial^2 l}{\partial\alpha^2}\right) & E\left(\frac{\partial^2 l}{\partial\alpha\partial\lambda}\right) & E\left(\frac{\partial^2 l}{\partial\alpha\partial\theta}\right) \\ E\left(\frac{\partial^2 l}{\partial\lambda\partial\alpha}\right) & E\left(\frac{\partial^2 l}{\partial\lambda^2}\right) & E\left(\frac{\partial^2 l}{\partial\lambda\partial\theta}\right) \\ E\left(\frac{\partial^2 l}{\partial\alpha\partial\theta}\right) & E\left(\frac{\partial^2 l}{\partial\lambda\partial\theta}\right) & E\left(\frac{\partial^2 l}{\partial\theta^2}\right) \end{pmatrix}$$

Further differentiating (3.2) we get,

$$\frac{\partial^2 l}{\partial\alpha^2} = -\frac{n}{\alpha^2} - \theta\sum_{i=1}^{n}(\lambda x_i)^{2\alpha} e^{(\lambda x_i)^{\alpha}}\left[\ln(\lambda x_i)\right]^2 - \theta\sum_{i=1}^{n}(\lambda x_i)^{\alpha} e^{(\lambda x_i)^{\alpha}}\left[\ln(\lambda x_i)\right]^2$$

$$\frac{\partial^2 l}{\partial\lambda^2} = \frac{\alpha}{\lambda^2}\sum_{i=1}^{n}\left[\theta(\lambda x_i)^{\alpha} e^{(\lambda x_i)^{\alpha}} - 2(\lambda x_i)^{\alpha} - 1\right] - \frac{\alpha^2}{\lambda^2}\sum_{i=1}^{n}\left[\theta(\lambda x_i)^{2\alpha} e^{(\lambda x_i)^{\alpha}} + \theta(\lambda x_i)^{\alpha} e^{(\lambda x_i)^{\alpha}} - 2(\lambda x_i)^{\alpha}\right]$$

$$\frac{\partial^2 l}{\partial\theta^2} = \frac{n}{(\theta+1)^2} - \frac{2n}{\theta^2}$$

$$\frac{\partial^2 l}{\partial\alpha\partial\lambda} = \frac{n}{\lambda} + \frac{2}{\lambda}\sum_{i=1}^{n}(\lambda x_i)^{\alpha} - \frac{\theta}{\lambda}\sum_{i=1}^{n}(\lambda x_i)^{\alpha} e^{(\lambda x_i)^{\alpha}} - \frac{\alpha\theta}{\lambda}\sum_{i=1}^{n}(\lambda x_i)^{2\alpha} e^{(\lambda x_i)^{\alpha}}\ln(\lambda x_i)$$
$$-\frac{\theta}{\lambda}\sum_{i=1}^{n}(\lambda x_i)^{\alpha} e^{(\lambda x_i)^{\alpha}}\ln(\lambda x_i) + \frac{2}{\lambda}\sum_{i=1}^{n}(\lambda x_i)^{\alpha}\ln(\lambda x_i)$$

$$\frac{\partial^2 l}{\partial\alpha\partial\theta} = -\sum_{i=1}^{n}(\lambda x_i)^{\alpha} e^{(\lambda x_i)^{\alpha}} \ln(\lambda x_i)$$

$$\frac{\partial^2 l}{\partial\lambda\partial\theta} = -\frac{\alpha}{\lambda}\sum_{i=1}^{n}(\lambda x_i)^{\alpha} e^{(\lambda x_i)^{\alpha}}$$

In practice, it is useless that the MLE has asymptotic variance $\left(I\left(\underline{\delta}\right)\right)^{-1}$ because we don't know $\underline{\delta}$. Hence we approximate the asymptotic variance by plugging in the estimated value of the parameters. The common procedure is to use observed fisher information matrix O($\hat{\underline{\delta}}$) as an estimate of the information matrix $I\left(\underline{\delta}\right)$ given by

$$O\left(\hat{\underline{\delta}}\right) = -\begin{pmatrix} \frac{\partial^2 l}{\partial\alpha^2} & \frac{\partial^2 l}{\partial\alpha\partial\lambda} & \frac{\partial^2 l}{\partial\alpha\partial\theta} \\ \frac{\partial^2 l}{\partial\alpha\partial\lambda} & \frac{\partial^2 l}{\partial\lambda^2} & \frac{\partial^2 l}{\partial\lambda\partial\theta} \\ \frac{\partial^2 l}{\partial\alpha\partial\theta} & \frac{\partial^2 l}{\partial\lambda\partial\theta} & \frac{\partial^2 l}{\partial\theta^2} \end{pmatrix}_{|(\hat{\alpha},\hat{\lambda},\hat{\theta})} = -H\left(\underline{\delta}\right)_{|(\underline{\delta}=\hat{\underline{\delta}})}$$

where H is the Hessian matrix.

The Newton-Raphson algorithm to maximize the likelihood produces the observed information matrix. Therefore, the variance-covariance matrix is given by,

$$\left[-H\left(\underline{\delta}\right)_{|(\underline{\delta}=\hat{\underline{\delta}})}\right]^{-1} = \begin{pmatrix} \mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha},\hat{\lambda}) & \mathrm{cov}(\hat{\alpha},\hat{\theta}) \\ \mathrm{cov}(\hat{\alpha},\hat{\lambda}) & \mathrm{var}(\hat{\lambda}) & \mathrm{cov}(\hat{\theta},\hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha},\hat{\theta}) & \mathrm{cov}(\hat{\theta},\hat{\lambda}) & \mathrm{var}(\hat{\theta}) \end{pmatrix}$$

Hence from the asymptotic normality of MLEs, approximate 100(1-α) % confidence intervals for α, λ, θ can be constructed as,

$\hat{\alpha} \pm Z_{\alpha/2}\sqrt{\mathrm{var}(\hat{\alpha})}$, $\hat{\lambda} \pm Z_{\alpha/2}\sqrt{\mathrm{var}(\hat{\lambda})}$ and $\hat{\theta} \pm Z_{\alpha/2}\sqrt{\mathrm{var}(\hat{\theta})}$, where $Z_{\alpha/2}$ is the upper percentile of standard normal variate.

IV. ILLUSTRATION WITH A REAL DATA SET

In this section, we illustrate the applicability of Lindley exponential power distribution using a real data set. The data set we have taken is the Aircraft Windshield data sets used by [24]. The data represents the service times for a particular model windshield that had not failed at the time of observation. The unit for measurement is 1000 hours.

0.046, 1.436, 2.592, 0.140, 1.492, 2.600, 0.150, 1.580, 2.670, 0.248, 1.719, 2.717, 0.280, 1.794, 2.819, 0.313, 1.915, 2.820, 0.389, 1.920, 2.878, 0.487, 1.963, 2.950, 0.622, 1.978, 3.003, 0.900, 2.053, 3.102, 0.952, 2.065, 3.304, 0.996, 2.117, 3.483, 1.003, 2.137, 3.500, 1.010, 2.141, 3.622, 1.085, 2.163, 3.665, 1.092, 2.183, 3.695, 1.152, 2.240, 4.015, 1.183, 2.341, 4.628, 1.244, 2.435, 4.806, 1.249, 2.464, 4.881, 1.262, 2.543, 5.140

By maximizing the likelihood function in (3.1), we have computed the maximum likelihood estimates directly by using optim() function in R software [20] and [21]. We have obtained the estimated value of α, λ and θ as $\hat{\alpha}$ = 0.9772214, $\hat{\lambda}$ =0.3946060 and $\hat{\theta}$ = 0.1938969 corresponding Log-Likelihood value is -98.11712. In Table 1 we have demonstrated the MLE's with their standard errors (SE) for α, λ and θ.

Table 1: MLE and SE

Parameter	MLE	SE
alpha	0.9772	0.5295
lambda	0.3946	0.4169
theta	0.9612	1.3452

In Fig. 2 have displayed the profile log-likelihood functions of α, λ and θ and it is proven that MLEs are unique.

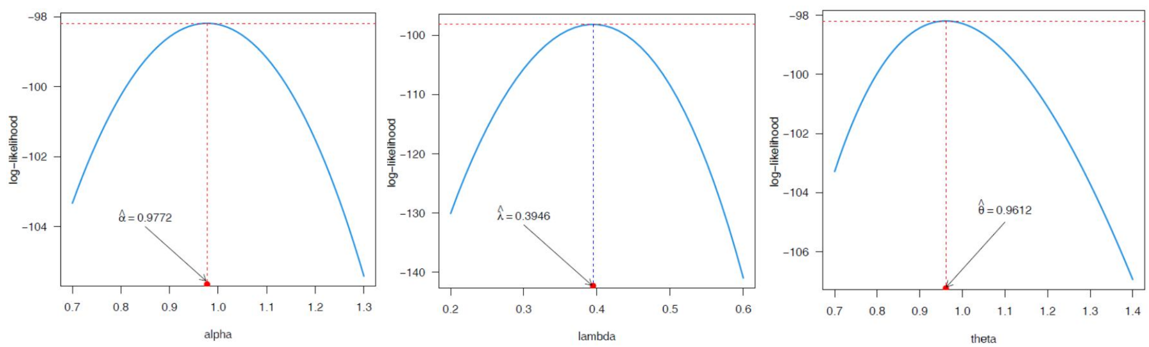

Fig 2. Profile log-likelihood functions of α, λ and θ.

The Q-Q plots and P-P plot of LEP distribution are displayed in Fig. 3. It is observed that the distribution fits the data excellently.

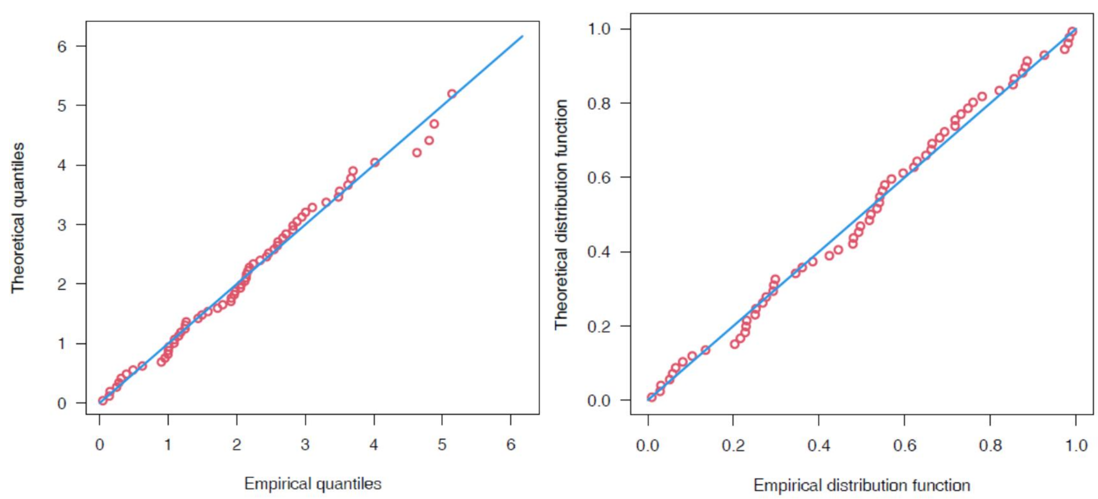

Fig 3. The Q-Q plot (left panel) and P-P plot (right panel) of LEP distribution

For the comparison propose we have fitted the LEP distribution and some selected distributions which are as follows,

A. Weibull Extension (WE) Distribution

The probability density function of Weibull extension (WE) distribution by [25] with three parameters (α, β, λ) is

$$f_{WE}(x;\alpha,\beta,\lambda) = \lambda\beta\left(\frac{x}{\alpha}\right)^{\beta-1}\exp\left(\frac{x}{\alpha}\right)^{\beta}\exp\left\{-\lambda\alpha\left(\exp\left(\frac{x}{\alpha}\right)^{\beta}-1\right)\right\} \quad ; \; x>0$$

$\alpha > 0, \beta > 0$ and $\lambda > 0$

B. Generalized Exponential Extension (GEE) Distribution

The probability density function of GEE distribution by [16] with the parameters α , β and λ is

$$f_{GEE}\left(x;\alpha,\beta,\lambda\right)=\alpha\beta\lambda\left(1+\lambda x\right)^{\alpha-1}exp\left\{1-\left(1+\lambda x\right)^{\alpha}\right\}\left[1-exp\left\{1-\left(1+\lambda x\right)^{\alpha}\right\}\right]^{\beta-1}\quad ;\ x\geq 0.$$

C. Weighted Lindley Distribution

Reference [8] has introduced a two-parameter weighted Lindley distribution and its PDF can be written as,

$$f_{WL}(x)=\frac{\theta^{\alpha+1}}{(\alpha+\theta)\Gamma(\alpha)}x^{\alpha-1}(1+x)e^{-\theta x}\quad ;x\geq 0,\alpha>0,\theta>0.$$

D. Generalized Exponential (GE) Distribution

The probability density function of generalized exponential distribution by [10] is.

$$f_{GE}\left(x;\alpha,\lambda\right)=\alpha\ \lambda\ e^{-\lambda\ x}\left\{1-e^{-\lambda\ x}\right\}^{\alpha-1};\left(\alpha,\lambda\right)>0,x>0$$

E. Lindley-Exponential Distribution (LE)

The probability density function of Lindley–Exponential distribution [4] is,

$$f_{LE}(x)=\lambda\left(\frac{\theta^2}{1+\theta}\right)e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\theta-1}\left\{1-\ln\left(1-e^{-\lambda x}\right)\right\};\ \lambda,\theta>0,\ x>0$$

The negative log-likelihood value and the value of AIC, BIC, CAIC and HQIC are presented in Table 2. We conclude that the propo sed model produces a better fit to the data taken than other models.

Table 2
Log-likelihood, AIC, BIC, CAIC and HQIC

Model	-LL	AIC	BIC	CAIC	HQIC
LEP	98.1878	202.3756	208.8050	202.7823	204.9043
WE	98.2657	202.5313	208.9608	202.9381	205.0601
GEE	98.6627	203.3254	209.7548	203.7322	205.8541
WL	101.7626	207.5252	211.8114	207.7252	209.2110
GE	103.5466	211.0933	215.3796	211.2933	212.7791
LE	104.5060	213.0120	217.2983	213.2120	214.6978

The histogram and the fitted density functions are displayed in Fig. 4 which compares the distribution function for the different models with the empirical distribution function produces the same. Therefore, for the given data sets illustrates the proposed distribution gets better fit and more reliable results from other alternatives.

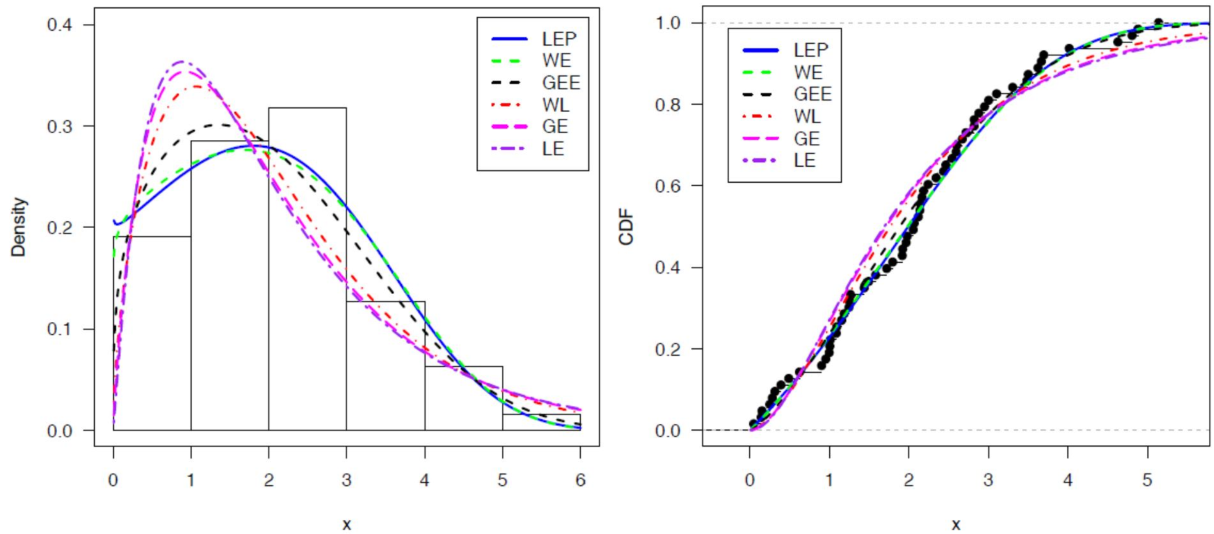

Fig 4. The Histogram and the PDF of fitted distributions (left panel) and Empirical CDF with estimated CDF (right panel).

In Table 3 we have displayed the value of the test statistics the Kolmogorov-Simnorov (KS), the Anderson-Darling (AD) and the Cramer-Von Mises (CVM) statistics and their corresponding *p*-value of different models. The result verifies that the proposed model has the minimum value of the test statistic and higher *p*-value thus we conclude that the LEP distribution is good in the view of goodness-of-fit.

Table 3
The Goodness-of-Fit Statistics and Their Corresponding p-Value

Model	*KS(p-value)*	*AD(p-value)*	*CVM(p-value)*
LEP	0.0661(0.9293)	0.0405(0.9319)	0.2724(0.9572)
WE	0.0694(0.9013)	0.0460(0.9018)	0.3020(0.9364)
GEE	0.0958(0.5771)	0.0684(0.7633)	0.4040(0.8442)
WL	0.1303(0.2154)	0.1665(0.3436)	0.9554(0.3812)
GE	0.1438(0.1335)	0.2333(0.2114)	1.3165(0.2270)
LE	0.1488(0.1108)	0.2596(0.1767)	1.4828(0.1807)

V. CONCLUSION

In this study, we have introduced a new extension of the Lindley model called Lindley exponential power (LEP) distribution. Some statistical properties of the LEP model have been discussed. From the graphical analysis of PDF and HRF, the proposed model is versatile and increasing, decreasing and up side bathtub hazard function. We have calculated the maximum likelihood estimation of the model parameters and the corresponding information matrix of the MLE's. We have also studied the application of LEP distribution using a real data set and found quite useful and behaves better in terms of fitting than some selected models. It may be an alternative model for the practitioners in the area of theory and applied statistics.

REFERENCES

[1] Asgharzadeh, A., Bakouch, H. S., & Esmaeili, L. (2013). Pareto Poisson–Lindley distribution with applications. Journal of Applied Statistics, 40(8), 1717-1734.
[2] Ashour, S. K., & Eltehiwy, M. A. (2015). Exponentiated power Lindley distribution. Journal of advanced research, 6(6), 895-905.
[3] Bakouch, H.S., Al-Zahrani, B.M., Al-Shomrani, A.A., Marchi, V.A., Louzada, F. (2012). An extended Lindley distribution. J. Korean Stat. Soc. 41, 75–85.
[4] Bhati, D., Malik, M. A., & Vaman, H. J. (2015). Lindley–Exponential distribution: properties and applications. Metron, 73(3), 335-357.
[5] Cakmakyapan, S., & Gamze, O. Z. E. L. (2016). The Lindley family of distributions: Properties and applications. Hacettepe Journal of Mathematics and Statistics, 46(6), 1113-1137.
[6] Ghitany, M., Al-Mutairi, D. And Nadarajah, S. (2008a). Zero-truncated Poisson–Lindley distribution and its application. Math Comput Simulat 79: 279-287.
[7] Ghitany, M.E., Atieh, B., Nadarajah, S. (2008). Lindley distribution and its application. Math. Comput. Simul. 78, 493–506.

[8] Ghitany, M. E., Alqallaf, F., Al-Mutairi, D. K., & Husain, H. A. (2011). A two-parameter weighted Lindley distribution and its applications to survival data. Mathematics and Computers in simulation, 81(6), 1190-1201.
[9] Gupta, P.K. And Singh, B. (2013). Parameter estimation of Lindley distribution with hybrid censored data. Int J Syst Assur Eng Manag 4: 378-385.
[10] Gupta, R. D. & Kundu, D. (1999). Generalized exponential distributions, Australian and New Zealand Journal of Statistics, 41(2), 173 - 188.
[11] Hinkley, D. (1977). On quick choice of power transformation. Journal of the Royal Statistical Society: Series C (Applied Statistics), 26(1), 67-69.
[12] Ibrahim, M., Yadav, A. S., Yousof, H. M., Goual, H., & Hamedani, G. G. (2019). A new extension of Lindley distribution: modified validation test, characterizations and different methods of estimation. Communications for Statistical Applications and Methods, 26(5), 473-495.
[13] Krishna, H. And Kumar, K. (2011). Reliability estimation in Lindley distribution with progressively type II right censored sample. Math Comput Simulat 82: 281-294.
[14] Kumar, V. and Ligges, U. (2011). reliaR : A package for some probability distributions, http://cran.r-project.org/web/packages/reliaR/index.html.
[15] Lindley, D.V. (1958). Fiducial distributions and Bayes' theorem, Journal of the Royal Statistical Society Series B, 20, p. 102-107.
[16] Lemonte, A. J. (2013). A new exponential-type distribution with constant, decreasing, increasing, upside-down bathtub and bathtub-shaped failure rate function. Computational Statistics & Data Analysis, 62, 149-170.
[17] Mazucheli, J. And Achcar, J. (2011). The Lindley distribution applied to competing risks lifetime data. Comput Meth Prog Bio 104: 188-192.
[18] Moors, J. J. A. (1988). A quantile alternative for kurtosis. Journal of the Royal Statistical Society: Series D (The Statistician), 37(1), 25-32.
[19] Nadarajah, S., Bakouch, H.S. And Tahmasbi, R. (2011). A generalized Lindley distribution. Sankhya B 73: 331-359.
[20] R Core Team (2020). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.
[21] Rizzo, M. L. (2008). Statistical computing with R. Chapman & Hall/CRC.
[22] Sankaran, M. (1970). The discrete Poisson–Lindley distribution. Biometrics 26: 145-149.
[23] Srivastava, A. K., & Kumar, V. (2011). Analysis of software reliability data using exponential power model. IJACSA Editorial.
[24] Tahir, M. H., Cordeiro, G. M., Mansoor, M., & Zubair, M. (2015). The Weibull-Lomax distribution: properties and applications. Hacettepe Journal of Mathematics and Statistics, 44(2), 461-480.
[25] Tang, Y., Xie, M., & Goh, T. N. (2003). Statistical analysis of a Weibull extension model. Communications in Statistics-Theory and Methods, 32(5), 913-928.
[26] Zamani, H. & Ismail, N. (2010). Negative binomial-Lindley distribution and its application. J Math Stat 6: 4-9.