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An Inventory Model for Time-Varying Deteriorating Items and Weibull Distributed Ameliorating Items with Cubic Demand under Salvage Value and Shortages

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Abstract: It is widely observed that the items like fruits, flowers, green vegetables, dairy products etc are either kept in farms, in flower shops, in supermarkets or in cold storages. The demand of such items is very high and at the same time it is also decreased owing to spoilage or decay. So we cannot ignore the effect of amelioration and deterioration in the inventory management system. Here we developed an inventory model for both ameliorating and deteriorating items. The deterioration rate is time-varying and amelioration rate is two-parameter Weibull distributed. The assumption of constant demand rate may not be always appropriate for many inventory goods like milk, vegetables etc., the age of inventory has negative impact on demand due to loss of consumer confidence on quality of such products. Here demand rate is considered as a cubic function of time and shortages are allowed which are fully backlogged. The model is solved with salvages value associated to the units deteriorating during the cycle. Finally the model is illustrated with the help of a numerical example, some particular cases are derived and a comparative study of the optimal solutions is furnished graphically to analyze the optimal solutions towards different nature of demands.
Keywords: Inventory, deteriorating items, ameliorating items, Weibull distributed, time-varying, cubic demand, salvages value and shortages.
Subject classification: AMS Classification No. 90B05

I. INTRODUCTION

It is natural that goods like fruits, flowers, green vegetables, dairy products, radioactive substances etc deteriorate over time. Normally goods deteriorate during storage period. Several researchers have addressed the importance of the deterioration phenomenon in their field of applications; as a result, many inventory models with deteriorating items have been developed. But due to lack of considering the influence of demand, the ameliorating items for the amount of inventory is increasing gradually. Amelioration is a natural phenomenon observing in much life stock models. A few researchers have focused on ameliorating system. Professionals did not give much attention for fast growing animals like broiler, ducks, pigs etc. in the poultry farm, highbred fishes in berry (pond) which are known as ameliorating items. When these items are in storage, the stock increases (in weight) due to growth of the items and also decrease due to death, various diseases or some other factors. At the point when these things are away, the stock increases (in weight) because of development of the things. Furthermore the stock diminishes because of death, different illnesses or due to some different components. Hwang [1997] developed an inventory model for ameliorating items only. Again Hwang [2004] added to a stock model for both ameliorating and deteriorating things independently. Mallick et al. [2018] has considered a creation inventory model for both ameliorating and deteriorating items. Many researchers like Moon et al[2005], Law et al [2006], L-Q ji[2008], Valliathal et al[2010], Chen [2011], Nodoust [2017] are few noteworthy. In this paper, effort is given to discuss on an economic order quantity (EOQ) inventory model for both ameliorating and deteriorating items where the environment of Amelioration followed by Weibull Distribution to describe the different life spans effectively by utilizing the changes of parameters.

Biswaranjan-Mandal [2010] analyzed the EOQ inventory model for Weibull distributed deteriorating items under ramp type demand and shortages. Sahoo et al. [2010] formulated an EOQ model for price dependent demand rate and time-varying holding cost. Hung [2011] have used generalized type demand, deterioration and backorder rates. Mishra and Singh [2011] find an inventory model for ramp type demand, and time dependent deteriorating items with salvage value and shortages.

According to Mishra and Singh [2011] an inventory model for deteriorating items with uniform replenishment rate with power form demand, the rate of deterioration is cubic polynomial. Anil Kumar Sharma et.al. [2012] are considered an inventory model with time dependent holding cost. Babu Krishnaraj and Ramasamy [2012] find an inventory model with power demand pattern for Weibull deterioration rate without shortages. Mukesh kumar et al. [2012] are considered a deterministic inventory model for deteriorating items with price dependent used demand rate and time-varying holding cost under trade credit.

Tripathy and Pradhan [2012] examined is used salvage value and developed an inventory model for three parameter Weibull distribution deterioration rate under permissible delay in payments. Amutha and Chandrasekaran [2013] studied on deteriorating items with price dependent demand, three parameter Weibull distribution deterioration rate.

Pratibha Yadav [2013] have used cubic demand rate and production rate is variable with Weibull distribution. Sharma et al [2015], Biswaranjan Mandal [2020] and many others developed inventory models assuming demand rate as cubic function of time.

Again Poonam Mishra and Shah [2008] studied an EOQ model for inventory management of time dependent deteriorating items with salvage value. Jaggi et al [1996] studied an EOQ model for deteriorating items with salvage value assuming deterioration and demand rate in constant behaviour. Karthikeyan et al [2015] developed a model to determine the optimum order quantity for constant deteriorating items with cubic demand and salvage value. Their model does not allow for time-varying deterioration and shortages, which would not make applicable in real word.

For these sort of situations, efforts have been made to develop a realistic inventory model with time-varying deterioration rate and two-parameter Weibull distributed ameliorating rate.

The demand rate is considered as a cubic function of time. The model is solved with salvages value associated to the units deteriorating during the cycle. Shortages are allowed and fully backlogged. Finally the model is illustrated with the help of a numerical example, some particular cases are derived and a comparative study of the optimal solutions is furnished graphically to analyze the optimal solutions towards different nature of demands.

II. NOTATIONS AND ASSUMPTIONS

The mathematical models are developed under the following notations and assumptions:

A. Notations

1) I(t) : On hand inventory level at time t.
2) R(t) : Demand rate.
3) Q : The maximum inventory level during the cycle.
4) $\theta(t)$: Time-varying deterioration rate.
5) A(t) : The ameliorating rate at time t.
6) T : The fixed length of each production cycle.
7) A_0 : The ordering cost per order during the cycle period.
8) p_c : The purchasing cost per unit item.
9) h_c : The holding cost per unit item.
10) d_c : The deterioration cost per unit item.
11) a_c : The cost of amelioration per unit item.
12) c_s : The shortage cost per unit item.
13) OC : Ordering cost per order.
14) PC : Purchasing cost over the cycle period.
15) HC : Holding cost over the cycle period.
16) CD: Cost due to deterioration over the cycle period.
17) AMC : Amelioration cost over the cycle period.
18) SV : Salvage value over the cycle period
19) CS : Cost due to shortage over the cycle period.
20) TC : Average total cost per unit time.

B. *Assumptions*

1) The inventory system included only one item.
2) The demand rate is time dependent cubic function

R(t) = a + bt + ct^2 + dt^3, a, b, c, d $\geq$ 0 where a is the initial demand rate, b is the initial rate of change of demand, c is the rate at which the demand rate increases and d is the rate at which the change in the demand rate itself increases.

3) The time-varying deterioration rate is given by

$\theta(t) = \theta_o t, 0 \leq \theta_o << 1$.

4) A(t) is the amelioration rate following Weibull distributed

$A(t) = \alpha\beta t^{\beta-1}, 0 \leq \alpha << 1, \beta \geq 1$, where α is the shape parameter and β is the scale parameter.

5) Lead time is zero.
6) Shortages are allowed and fully backlogged.
7) Replenishment rate is infinite.
8) The time horizon is infinite.
9) The salvage value kd_c , $0 \leq k < 1$ is associated with deteriorated units during a cycle time.

III. FORMULATION AND SOLUTION OF THE MODEL

In this model, we consider an inventory model starting with no shortage. Replenishment occurs at time t=0 and the inventory level attains its maximum. From t = 0 to t = t_1 the stock will be diminished due to the effect of deterioration, amelioration and demand, and ultimately falls to zero at t =t_1. The shortages occur during time period [t_1, T] which are fully backlogged. The behaviour of the model at any time t can be described by the following differential equations:

$$\frac{dI(t)}{dt} + (\theta(t) - A(t))I(t) = -R(t), 0 \leq t \leq t_1 \quad (3.1)$$

And $$\frac{dI(t)}{dt} = -R(t), t_1 \leq t \leq T \quad (3.2)$$

The initial condition is $I(0) = Q$ and $I(t_1) = 0$ (3.3)

Putting the values of $\theta(t) = \theta_o t, 0 \leq \theta_o << 1$, $A(t) = \alpha\beta t^{\beta-1}, 0 \leq \alpha << 1, \beta \geq 1$ and

R(t) = a + bt + ct^2 + dt^3, a, b, c, d $\geq$ 0, we get

$$\frac{dI(t)}{dt} + (\theta_o t - \alpha\beta t^{\beta-1})I(t) = -(a + bt + ct^2 + dt^3), 0 \leq t \leq t_1 \quad (3.4)$$

And $$\frac{dI(t)}{dt} = -(a + bt + ct^2 + dt^3), t_1 \leq t \leq T \quad (3.5)$$

Now solving the equations (3.4) and (3.5) using the initial condition (3.3) and neglecting the second and higher powers of θ_o and α [since $O(\theta_o{}^2)$ and $O(\alpha^2)$ are very small as $0 \leq \theta_o, \alpha << 1$], we get

$$\begin{aligned} I(t) = {} & a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{d}{4}(t_1^4 - t^4) + \frac{a\theta_o}{6}(t_1^3 - 3t_1 t^2 + 2t^3) \\ & + \frac{b\theta_o}{8}(t_1^4 - 2t^2{}_1 t^2 + t^4) + \frac{c\theta_o}{30}(3t_1^5 - 5t^3{}_1 t^2 + 2t^5) + \frac{d\theta_o}{24}(2t_1^6 - 3t^4{}_1 t^2 + t^6), \\ & - \frac{a\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}) - \frac{b\alpha}{\beta+2}(t_1^{\beta+2} - t^{\beta+2}) - \frac{c\alpha}{\beta+3}(t_1^{\beta+3} - t^{\beta+3}) - \frac{d\alpha}{\beta+4}(t_1^{\beta+4} - t^{\beta+4}) \\ & + a\alpha t^{\beta}(t_1 - t) + \frac{b\alpha}{2}t^{\beta}(t_1^2 - t^2) + \frac{c\alpha}{3}t^{\beta}(t_1^3 - t^3) + \frac{d\alpha}{4}t^{\beta}(t_1^4 - t^4), \; 0 \leq t \leq t_1 \end{aligned} \quad (3.6)$$

And I(t) = $a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{d}{4}(t_1^4 - t^4)$, $t_1 \le t \le T$ (3.7)

Since I(0) = Q, we get from equation (3.6) the following expression

$$Q = at_1 + \frac{b}{2}t_1^2 + (\frac{c}{3} + \frac{a\theta_o}{6})t_1^3 + (\frac{d}{4} + \frac{b\theta_o}{8})t_1^4 + \frac{c\theta_o}{10}t_1^5 + \frac{d\theta_o}{12}t_1^6 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{b\alpha}{\beta+2}t_1^{\beta+2} - \frac{c\alpha}{\beta+3}t_1^{\beta+3} - \frac{d\alpha}{\beta+4}t_1^{\beta+4} \quad (3.8)$$

A. *Cost Components*

The total cost over the period [0,T] consists of the following cost components :

1) Ordering cost **(OC)** over the period [0,T] = A_0 (fixed)
2) Purchasing cost (**PC**) over the period [0,T] = p_c I(0) = p_c Q

$$= p_c\{at_1 + \frac{b}{2}t_1^2 + (\frac{c}{3} + \frac{a\theta_o}{6})t_1^3 + (\frac{d}{4} + \frac{b\theta_o}{8})t_1^4 + \frac{c\theta_o}{10}t_1^5 + \frac{d\theta_o}{12}t_1^6 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{b\alpha}{\beta+2}t_1^{\beta+2} - \frac{c\alpha}{\beta+3}t_1^{\beta+3} - \frac{d\alpha}{\beta+4}t_1^{\beta+4}\}$$

3) Holding cost **(HC)** for carrying inventory over the period [0,T] = $h_c\int_0^{t_1} I(t)dt$

$$= h_c\{\frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + (\frac{c}{4} + \frac{a\theta_o}{12})t_1^4 + (\frac{d}{5} + \frac{b\theta_o}{15})t_1^5 + \frac{c\theta_o}{18}t_1^6 + \frac{d\theta_o}{21}t_1^7 - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3} - \frac{c\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\beta+4} - \frac{d\alpha\beta}{(\beta+1)(\beta+5)}t_1^{\beta+5}\}$$

4) Cost due to deterioration **(CD)** over the period [0,T] = $d_c\int_0^{t_1}\theta_o tI(t)dt$

$$= d_c\theta_o\{\frac{a}{6}t_1^3 + \frac{b}{8}t_1^4 + \frac{c}{10}t_1^5 + \frac{d}{12}t_1^6\}$$

5) Salvage cost **(SV)** over the period [0,T] = k $d_c\theta_o\{\frac{a}{6}t_1^3 + \frac{b}{8}t_1^4 + \frac{c}{10}t_1^5 + \frac{d}{12}t_1^6\}$
6) The amelioration cost **(AMC)** over the period [0,T] = $a_c\int_0^{t_1}\alpha\beta t^{\beta-1}I(t)dt$

$$= a_c\alpha\{\frac{a}{\beta+1}t_1^{\beta+1} + \frac{b}{\beta+2}t_1^{\beta+2} + \frac{c}{\beta+3}t_1^{\beta+3} + \frac{d}{\beta+4}t_1^{\beta+4}\}$$

7) Cost due to shortage **(CS)** over the period [0,T] = $c_s\int_{t_1}^{T}(T-t)R(t)dt$

$$= \int_{t_1}^{T}(T-t)R(t)dt = \int_{t_1}^{T}(T-t)(a+bt+ct^2+dt^3)dt$$

$$= c_s\{\frac{a}{2}(T^2 - 2Tt_1 + t_1^2) + \frac{b}{6}(T^3 - 3Tt_1^2 + 2t_1^3) + \frac{c}{12}(T^4 - 4Tt_1^3 + 3t_1^4) + \frac{d}{20}(T^5 - 5Tt_1^4 + 4t_1^5)\}$$

The average total cost per unit time of the system during the cycle [0,T] will be

$$TC(t_1) = \frac{1}{T}\,[\,OC + PC + HC + CD - SV + AMC + CS\,]$$

$$= \frac{1}{T}[\, A_0 + p_c\{ at_1 + \frac{b}{2}t_1^2 + (\frac{c}{3}+\frac{a\theta_o}{6})t_1^3 + (\frac{d}{4}+\frac{b\theta_o}{8})t_1^4 + \frac{c\theta_o}{10}t_1^5 + \frac{d\theta_o}{12}t_1^6 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{b\alpha}{\beta+2}t_1^{\beta+2}$$
$$- \frac{c\alpha}{\beta+3}t_1^{\beta+3} - \frac{d\alpha}{\beta+4}t_1^{\beta+4}\} + h_c\{\frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + (\frac{c}{4}+\frac{a\theta_o}{12})t_1^4 + (\frac{d}{5}+\frac{b\theta_o}{15})t_1^5 + \frac{c\theta_o}{18}t_1^6 + \frac{d\theta_o}{21}t_1^7$$
$$- \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3} - \frac{c\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\beta+4} - \frac{d\alpha\beta}{(\beta+1)(\beta+5)}t_1^{\beta+5}\}$$
$$+ d_c\theta_o(1-k)\{\frac{a}{6}t_1^3 + \frac{b}{8}t_1^4 + \frac{c}{10}t_1^5 + \frac{d}{12}t_1^6\} + a_c\alpha\{\frac{a}{\beta+1}t_1^{\beta+1} + \frac{b}{\beta+2}t_1^{\beta+2} + \frac{c}{\beta+3}t_1^{\beta+3} + \frac{d}{\beta+4}t_1^{\beta+4}\} + c_s$$
$$\{\frac{a}{2}(T^2 - 2Tt_1 + t_1^2) + \frac{b}{6}(T^3 - 3Tt_1^2 + 2t_1^3) + \frac{c}{12}(T^4 - 4Tt_1^3 + 3t_1^4) + \frac{d}{20}(T^5 - 5Tt_1^4 + 4t_1^5)\}\,]$$

(3.9)

For minimum, the necessary condition is $\frac{dTC(t_1)}{dt_1} = 0$

This gives $p_c\{ a + bt_1 + (c+\frac{a\theta_o}{2})t_1^2 + (d+\frac{b\theta_o}{2})t_1^3 + \frac{c\theta_o}{2}t_1^4 + \frac{d\theta_o}{2}t_1^5 - a\alpha t_1^{\beta} - b\alpha t_1^{\beta+1}$

$$- c\alpha t_1^{\beta+2} - d\alpha t_1^{\beta+3}\} + h_c\{at_1 + bt_1^2 + (c+\frac{a\theta_o}{3})t_1^3 + (d+\frac{b\theta_o}{3})t_1^4 + \frac{c\theta_o}{3}t_1^5 + \frac{d\theta_o}{3}t_1^6$$
$$- \frac{a\alpha\beta}{(\beta+1)}t_1^{\beta+1} - \frac{b\alpha\beta}{(\beta+1)}t_1^{\beta+2} - \frac{c\alpha\beta}{(\beta+1)}t_1^{\beta+3} - \frac{d\alpha\beta}{(\beta+1)}t_1^{\beta+4}\} + \frac{d_c\theta_o(1-k)}{2}\{at_1^2 + bt_1^3 + ct_1^4 + dt_1^5\}$$
$$+ a_c\alpha\{at_1^{\beta} + bt_1^{\beta+1} + ct_1^{\beta+2} + dt_1^{\beta+3}\} + c_s\{a(t_1 - T) + bt_1(t_1 - T) + ct_1^2(t_1 - T) + dt_1^3(t_1 - T)\} = 0$$

(3.10)

For minimum the sufficient condition $\frac{d^2TC(t_1)}{dt_1^2} > 0$ would be satisfied.

Let $t_1 = t_1^*$ be the optimum value of t_1.

The optimal values Q^* of Q and TC^* of TC are obtained by putting the value $t_1 = t_1^*$ from the expressions (3.8) and (3.9).

IV. PARTICULAR CASES

1) If the demand rate is quadratic function of time then $d = 0$

From (3.8), the total amount of inventory Q becomes

$$Q = at_1 + \frac{b}{2}t_1^2 + (\frac{c}{3}+\frac{a\theta_o}{6})t_1^3 + \frac{b\theta_o}{8}t_1^4 + \frac{c\theta_o}{10}t_1^5 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{b\alpha}{\beta+2}t_1^{\beta+2} - \frac{c\alpha}{\beta+3}t_1^{\beta+3} \quad (4.1)$$

From (3.9), the average total cost per unit time of the system during the cycle [0,T] becomes

$$TC(t_1) = \frac{1}{T}[\, A_0 + p_c\{ at_1 + \frac{b}{2}t_1^2 + (\frac{c}{3}+\frac{a\theta_o}{6})t_1^3 + \frac{b\theta_o}{8}t_1^4 + \frac{c\theta_o}{10}t_1^5 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{b\alpha}{\beta+2}t_1^{\beta+2} - \frac{c\alpha}{\beta+3}t_1^{\beta+3}\} +$$
$$h_c\{\frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + (\frac{c}{4}+\frac{a\theta_o}{12})t_1^4 + \frac{b\theta_o}{15}t_1^5 + \frac{c\theta_o}{18}t_1^6 - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3} - \frac{c\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\beta+4}\}$$

$$+d_c\theta_o(1-k)\{\frac{a}{6}t_1^3+\frac{b}{8}t_1^4+\frac{c}{10}t_1^5\}+\ a_c\alpha\{\frac{a}{\beta+1}t_1^{\beta+1}+\frac{b}{\beta+2}t_1^{\beta+2}+\frac{c}{\beta+3}t_1^{\beta+3}\}+$$

$$c_s\ \{\frac{a}{2}(T^2-2Tt_1+t_1^{\ 2})+\frac{b}{6}(T^3-3Tt_1^{\ 2}+2t_1^{\ 3})+\ \frac{c}{12}(T^4-4Tt_1^{\ 3}+3t_1^{\ 4})\}\ ] \qquad (4.2)$$

The equation (3.10) becomes

$$p_c\ \{\ a+bt_1+(c+\frac{a\theta_o}{2})t_1^2+\frac{b\theta_o}{2}t_1^3+\frac{c\theta_o}{2}t_1^4-a\alpha t_1^{\ \beta}-b\alpha t_1^{\ \beta+1}-c\alpha\, t_1^{\ \beta+2}\ \}$$

$$+\ h_c\{at_1+bt_1^2+(c+\frac{a\theta_o}{3})t_1^3+\frac{b\theta_o}{3}t_1^4+\frac{c\theta_o}{3}t_1^5-\frac{a\alpha\beta}{(\beta+1)}t_1^{\ \beta+1}-\frac{b\alpha\beta}{(\beta+1)}t_1^{\ \beta+2}-\frac{c\alpha\beta}{(\beta+1)}t_1^{\ \beta+3}\}$$

$$+\frac{d_c\theta_o(1-k)}{2}\{at_1^2+bt_1^3+ct_1^4\}+\ a_c\alpha\{at_1^{\ \beta}+bt_1^{\ \beta+1}+ct_1^{\ \beta+2}\}$$

$$+\ c_s\{a(t_1-T)+bt_1(t_1-T)+ct_1^{\ 2}(t_1-T)\}=0 \qquad (4.3)$$

This gives the optimum value of t_1.

2) If the demand rate is linear trended function of time then c = 0 and $d=0$

From (3.8), the total amount of inventory Q becomes

$$Q=\ at_1+\frac{b}{2}t_1^2+\frac{a\theta_o}{6}t_1^3+\frac{b\theta_o}{8}t_1^4\ -\frac{a\alpha}{\beta+1}t_1^{\ \beta+1}-\frac{b\alpha}{\beta+2}t_1^{\ \beta+2} \qquad (4.4)$$

From (3.9), the average total cost per unit time of the system during the cycle [0,T] becomes

$$TC(t_1)=\frac{1}{T}[\ A_0\ +\ p_c\ \{\ at_1+\frac{b}{2}t_1^2+\frac{a\theta_o}{6}t_1^3+\frac{b\theta_o}{8}t_1^4\ -\frac{a\alpha}{\beta+1}t_1^{\ \beta+1}-\frac{b\alpha}{\beta+2}t_1^{\ \beta+2}\ \}$$

$$+\ h_c\{\frac{a}{2}t_1^2+\frac{b}{3}t_1^3+\frac{a\theta_o}{12}t_1^4+\frac{b\theta_o}{15}t_1^5-\frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}-\frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}\}$$

$$+d_c\theta_o(1-k)\{\frac{a}{6}t_1^3+\frac{b}{8}t_1^4\}+\ a_c\alpha\{\frac{a}{\beta+1}t_1^{\ \beta+1}+\frac{b}{\beta+2}t_1^{\ \beta+2}\}$$

$$+\ c_s\ \{\frac{a}{2}(T^2-2Tt_1+t_1^{\ 2})+\frac{b}{6}(T^3-3Tt_1^{\ 2}+2t_1^{\ 3})\ \}\ ] \qquad (4.5)$$

The equation (3.10) becomes

$$p_c\ \{\ a+bt_1+\frac{a\theta_o}{2}t_1^2+\frac{b\theta_o}{2}t_1^3-a\alpha t_1^{\ \beta}-b\alpha t_1^{\ \beta+1}\ \}+\ h_c\{at_1+bt_1^2+\frac{a\theta_o}{3}t_1^3+\frac{b\theta_o}{3}t_1^4-\frac{a\alpha\beta}{(\beta+1)}t_1^{\ \beta+1}\ -\frac{b\alpha\beta}{(\beta+1)}t_1^{\ \beta+2}\}+$$

$$\frac{d_c\theta_o(1-k)}{2}\{at_1^2+bt_1^3\}+\ a_c\alpha\{at_1^{\ \beta}+bt_1^{\ \beta+1}\}+\ c_s\{a(t_1-T)+bt_1(t_1-T)\}=0$$

(4.6)

This gives the optimum value of t_1.

3) If the demand rate is constant then b = 0, c=0 and $d=0$

From (3.8), the total amount of inventory Q becomes

$$Q=\ at_1+\frac{a\theta_o}{6}t_1^3\ -\frac{a\alpha}{\beta+1}t_1^{\ \beta+1} \qquad (4.7)$$

From (3.9), the average total cost per unit time of the system during the cycle [0,T] becomes

$$TC(t_1) = \frac{1}{T}[\ A_0 + p_c\{at_1 + \frac{a\theta_o}{6}t_1^3 - \frac{a\alpha}{\beta+1}t_1^{\beta+1}\} + h_c\{\frac{a}{2}t_1^2 + \frac{a\theta_o}{12}t_1^4 - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}\} + \frac{d_c\theta_o(1-k)a}{6}t_1^3 + \frac{a_c\alpha a}{\beta+1}t_1^{\beta+1} + \frac{c_s a}{2}(T^2 - 2Tt_1 + t_1^2)\ ] \quad (4.8)$$

The equation (3.10) becomes

$$p_c\{1 + \frac{\theta_o}{2}t_1^2 - \alpha t_1^{\beta}\} + h_c\{t_1 + \frac{\theta_o}{3}t_1^3 - \frac{\alpha\beta}{(\beta+1)}t_1^{\beta+1}\} + \frac{d_c\theta_o(1-k)}{2}t_1^2 + a_c\alpha t_1^{\beta} + c_s(t_1 - T) = 0 \quad (4.9)$$

This gives the optimum value of t_1.

V. NUMERICAL EXAMPLE

To illustrate the developed inventory model, let the values of parameters be as follows:

A_0 = \$500 per order; a = 30; b = 20; c = 10; d = 3; θ_o = 0.01; α = 0.001; β = 2; k = 0.1; p_c = \$5 per unit; h_c = \$12 per unit; d_c = \$4 per unit; a_c = \$7 per unit; c_s = \$15 per unit; T = 1 year

Solving the equation (3.10) with the help of computer using the above parameter values, we find the following optimum outputs t_1^* = 0.37 year; Q^* = 12.66 units and TC^* = Rs 729.20

It is checked that this solution satisfies the sufficient condition for optimality.

A. *Comparison Of Inventory Models Between Varying Demand Rates*

The comparative study is also furnished to illustrate the special cases of the inventory model by varying demand rates.

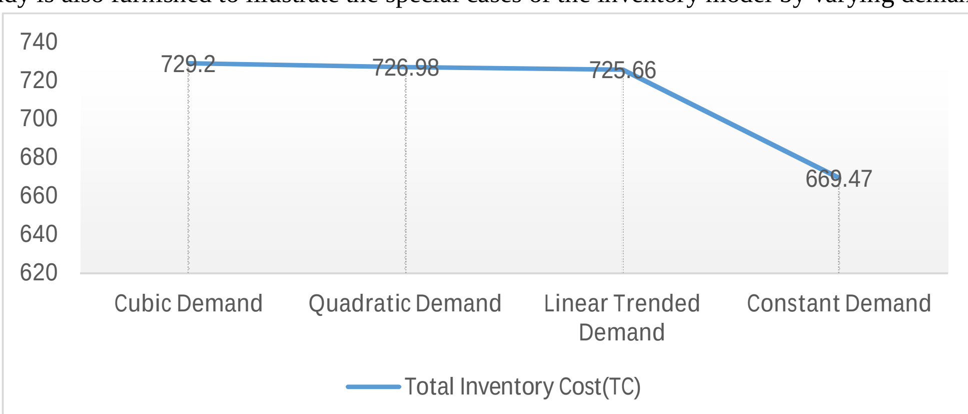

Fig. 1: Demand Rate vs Optimal Inventory Total Cost

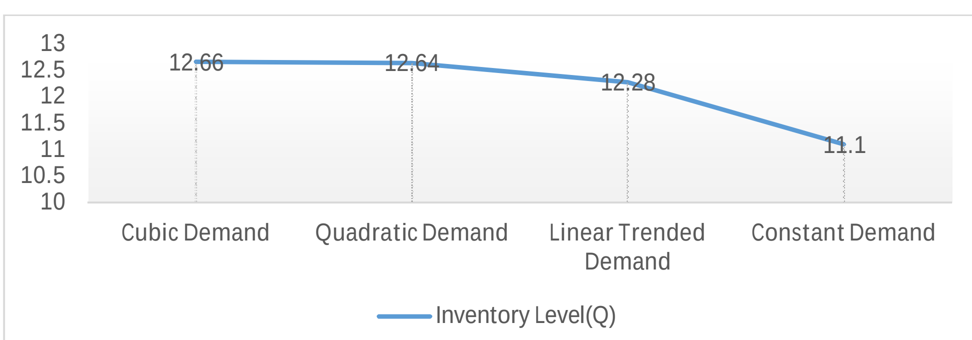

Fig. 2: Demand Rate vs Optimal Inventory Level

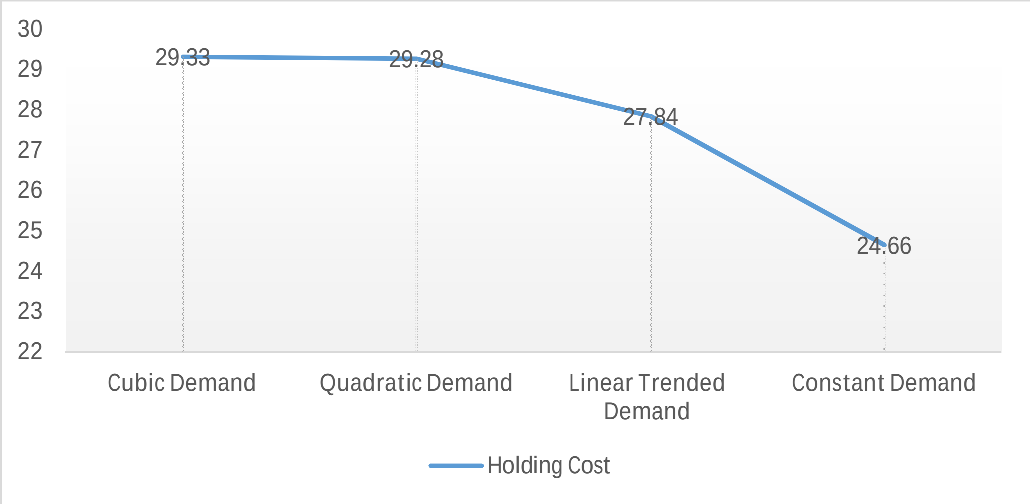

Fig 3: Demand Rate vs Holding Cost

B. Concluding Remarks

In this study, an inventory model has been framed for time-varying deteriorating items under Weibull distributed amelioration environment with cubic demand rate in nature. The salvage value has been incorporated for deteriorating items. Shortages are allowed which are fully backlogged. The models are developed analytically as well as computationally with graphical representation.

Efforts are given on comparative study graphically between optimal inventory total cost, optimal inventory level and holding cost considering cubic, quadratic, linear and constant nature of demand rates. Analyzing Fig. 1, Fig. 2 and Fig. 3, it is observed that optimality of inventory total cost, inventory level and holding cost are moderately changing for cubic, quadratic and linear trended demand rates, whereas these are changing significantly towards the model with constant demand in nature. The total cost is minimum for constant demand rate in compare to other natures of demand rates. So demand parameters are most important for estimation of optimal solution of the inventory model and we need adequate attention to estimate these parameters.

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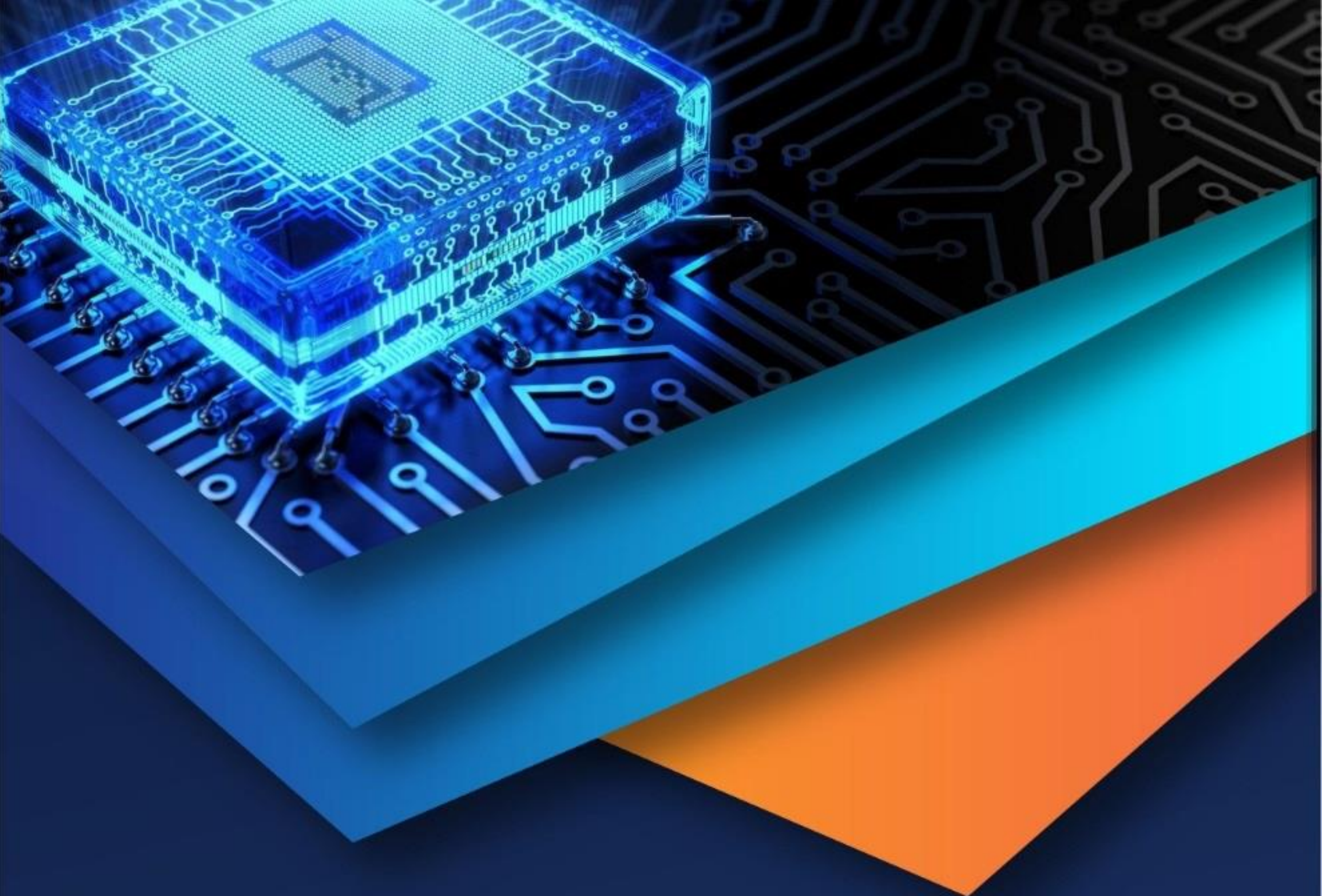

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