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Efficient Reliability Estimate for Weibull Class Models

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Abstract: Software quality is a major concern for all software developers. Quality of the software can be assessed using various measures such as - the failure intensity function, the mean time to failure, failure rate etc. Several methods of estimation, like - the maximum likelihood estimation, the least square estimation etc are used in estimating these quantities. However, the software reliability, which is the probability of failure free operation of the software over a specified period of time, is an important measure of the quality of the software that helps the developers of the software to ensure that the user requirements are met. Software reliability models having certain failure time distribution are used to find the estimators of reliability and software failure data obtained during testing are used to estimate these values. Since most of the software failures have Weibull failure time distribution, herein, it is intended to estimate the reliability for the Weibull class models. The technique of Blackwellization has been used to find the minimum variance unbiased estimator of the reliability function. Few case studies have been considered to explain the behavior of this estimator.
Keywords: Blackwellization, Minimum Variance Unbiased Estimator, Software reliability, Software reliability models, Weibull class models.

I. INTRODUCTION

Software plays a major role in every filed. Failing to have good quality software can result in economic damage to even loss of life. Thus, software quality is an essential concern of every software developer. Software failures occur randomly and the behavior of these failures with time are described by software reliability models. The software reliability models are often expressed in terms of failure times, which follow certain probability distribution. Depending on the distribution of the failure times, these software reliability models are categorized into several classes. Most of the models fall into Weibull class, where failure times are assumed to follow Weibull distribution. Several estimation procedures like - the method of maximum likelihood estimation, method of least squares, method of minimum variance unbiased estimation, etc are available. The method of minimum variance unbiased estimation is considered as the most efficient method of estimation, since it always provides an unbiased estimator and has the minimum variance among the class of all such unbiased estimators. However, because of the complexity involved in estimating it, the method is rarely used. Here, it is intended to obtain the minimum variance unbiased estimator of reliability using the technique of Blackwellization. The method is applied to Weibull class models, with known value of the parameter β.

A. Notations

1) $f(t)$: Probability density function (pdf) of T.
2) L : Likelihood function.
3) $f(x|y)$: Conditional pdf of X given Y.
4) $E(X)$: Expectation of X.
5) $E(U|S)$: Conditional expectation of U given S.
6) $W(\beta, \Phi)$: Weibull distribution with parameters β and Φ.
7) $\Gamma(n)$: Gamma function.

B. Terminologies

Software reliability [1]: It is the probability of failure free operation of a computer program in a specified environment for a specified period of time. Thus, if T denotes the failure time distribution of the given software, then, its reliability at time t, denoted by $R(t)$ is defined as $R(t)=P(T>t)$.

Software reliability models [2]: These describe the behavior of failure with time, by expressing failures as random processes in either times of failure or the number of failures, at fixed times.

Weibull class models: These are the software reliability models, where failure times are assumed to follow Weibull distribution denoted as $W(\beta,\Phi)$, with pdf

$$f(t) = \Phi\beta t^{\beta-1}e^{-\Phi t^{\beta}}, \quad t>0 \qquad (1)$$

where, β is the shape parameter, which is assumed to be known and Φ is the scale parameter, which also denotes the failure rate.
Method of MVUE [3]: If a statistic T based on a sample of size n is such that T is - (i) unbiased for the parameter θ and (ii) has the smallest variance among the class of all unbiased estimators of θ, then, T is called the Minimum Variance Unbiased Estimator (MVUE) of θ. It can be shown that such an MVUE is always unique. To find this MVUE, a procedure called Blackwellization is used.
Blackwellization [3]: The technique of obtaining MVUE from any unbiased estimator using the sufficient statistic is called Blackwellization. Here, an unbiased estimator say U is found for the parameter and then a complete sufficient estimator say S is found. The MVUE is then obtained as $E(U|S)$.
Lot of work has been done by many researchers in estimating various reliability measures of Weibull class models. Chris Bambey et al [4] proposed Baysian parameter and reliability estimate of two parameter Weibull failure time distribution. Nozer D. Singpurwalla, et al [5] described briefly, several well-known probability models for assessing the reliability of software. In the thesis entitled "Accurate Software Reliability Estimation", Jason Allen Denton [6] examined the impact of the parameter estimation technique on model accuracy and claimed that the maximum likelihood method provides estimators which are more reliable than the least squares method. Taehyoun et al [7] proposed an effective approach to estimate the parameters of software reliability growth model using a real valued genetic algorithm. Hiroyuki et al [8] proposed a new estimation method for the model parameters of a software reliability model based on the EM (Expectation-Maximization) principle. Suri et al [9] used the method of maximum likelihood estimation to estimate the parameters of Weibull distribution. Herein, it is intended to apply an efficient method of estimation, viz, the method of minimum variance unbiased estimation in estimating the reliability of the software. A procedure called Blackwellization, is used for this purpose. Even though the method exists in literature, it is hardly used because of the complexity involved in its computation. Herein, an effort is made to apply this technique to get the most efficient estimator of reliability.

II. RELIABILITY ESTIMATION

Consider the Weibull class models as given in (1). To find the minimum variance unbiased estimator of reliability, it is intended to obtain the unbiased estimator and the complete sufficient estimator of $R(t)$.

1) *Unbiased estimator of R(t):* The function defined by

$$U(t_1) = \begin{cases} 1 & \text{if } t_1>t \\ 0 & \text{otherwise} \end{cases}$$

is unbiased for $R(t)$, since $E(U(t_1))=1\cdot P(T_1>t)=R(t)$.

2) *Complete Sufficient Estimator*

The likelihood function of the sample $T_1, T_2,\ldots\ldots T_n$, of size n, taken from the distribution as given in (1) is

$$L = \Phi^n\beta^n\prod_{i=1}^{n}t_i^{\beta-1}e^{-\Phi\sum_{i=1}^{n}t_i^{\beta}} \qquad (2)$$

By using Factorization theorem, $\sum_{i=1}^{n}t_i^{\beta}$ is the complete sufficient estimator. Hence the minimum variance unbiased estimator of $R(t)$, denoted by $\tilde{R}(t)$ is obtained as

$$\tilde{R}(t) = E(U(t_1)\Big|\sum_{i=1}^{n}t_i^{\beta}) = \int_t^{\infty} f(t_1\Big|\sum_{i=1}^{n}t_i^{\beta})dt_1$$

i.e., $$\tilde{R}(t) = \int_t^{\infty}\frac{f(t_1,\sum_{i=1}^{n}t_i^{\beta})}{f(\sum_{i=1}^{n}t_i^{\beta})}dt_1 \qquad (3)$$

Since each T_i has $W(\beta, \Phi)$, each T_i^{β} has exponential distribution with parameter Φ and hence its pdf is given by

$$f(t_i^{\beta}) = \Phi e^{-\Phi t_i^{\beta}}, \; t_i^{\beta} > 0 \qquad (4)$$

Thus, $\sum_{i=1}^{n} t_i^{\beta}$ has gamma distribution with parameters n and Φ. Hence, the pdf of $\sum_{i=1}^{n} t_i^{\beta}$ is given by

$$f(\sum_{i=1}^{n} t_i^{\beta}) = \frac{1}{\Gamma(n)} \Phi^n e^{-\Phi \sum_{i=1}^{n} t_i^{\beta}} \left(\sum_{i=1}^{n} t_i^{\beta} \right)^{n-1} \qquad (5)$$

Using this, the pdf of $\sum_{i=2}^{n} t_i^{\beta}$ is obtained as $f(\sum_{i=2}^{n} t_i^{\beta}) = \frac{1}{\Gamma(n-1)} \Phi^{n-1} e^{-\Phi \sum_{i=2}^{n} t_i^{\beta}} \left(\sum_{i=2}^{n} t_i^{\beta} \right)^{n-2}$.

Also, using (4), the pdf of t_1 is obtained as $f(t_1^{\beta}) = \Phi e^{-\Phi t_1^{\beta}}, \; t_1^{\beta} > 0$.

Considering the transformation $\sum_{i=1}^{n} t_i^{\beta} = t_1^{\beta} + \sum_{i=2}^{n} t_i^{\beta}$ and noting that the Jacobian of the inverse transformation is one, the joint pdf of t_1^{β} and $\sum_{i=1}^{n} t_i^{\beta}$ is obtained as

$$f(t_1^{\beta}, \sum_{i=1}^{n} t_i^{\beta}) = f(t_1^{\beta}, \sum_{i=2}^{n} t_i^{\beta}) |J| = f(t_1^{\beta}, \sum_{i=2}^{n} t_i^{\beta}).1$$

$$\therefore \; f(t_1^{\beta}, \sum_{i=1}^{n} t_i^{\beta}) = \frac{1}{\Gamma(n-1)} e^{-\Phi \sum_{i=1}^{n} t_i^{\beta}} (\sum_{i=1}^{n} t_i^{\beta} - t_1^{\beta})^{n-2}$$

To get $f(t_1, \sum_{i=1}^{n} t_i^{\beta})$ from $f(t_i^{\beta}, \sum_{i=1}^{n} t_i^{\beta})$, consider the transformation $x = t^{\frac{1}{\beta}}$, $y = u$. The Jacobian of the inverse transformation is $\frac{t_1}{\beta t_1^{\beta}}$. Using this, the joint pdf of t_1 and $\sum_{i=1}^{n} t_i^{\beta}$ is obtained as $f(t_1, \sum_{i=1}^{n} t_i^{\beta}) = \beta(n-1) \left(1 - \frac{t_1^{\beta}}{\sum_{i=1}^{n} t_i^{\beta}} \right)^{n-2} \frac{t_1^{\beta-1}}{\sum_{i=1}^{n} t_i^{\beta}}$

(6)

Using (5) and (6) in (3), the MVUE of reliability is obtained as

$$\tilde{R}(t) = \int_{t}^{\infty} \beta(n-1) \left(1 - \frac{t_1^{\beta}}{\sum_{i=1}^{n} t_i^{\beta}} \right)^{n-2} \frac{t_1^{\beta-1}}{\sum_{i=1}^{n} t_i^{\beta}} dt_1$$

This integral converges if $t_1 < \sqrt[\beta]{\sum_{i=1}^{n} t_i^{\beta}}$.

$$\text{Hence,} \quad \tilde{R}(t) = \int_{t}^{\sqrt[\beta]{\sum_{i=1}^{n} t_i^{\beta}}} \frac{\beta(n-1)}{\sum_{i=1}^{n} t_i^{\beta}} \left(1 - \frac{t_1^{\beta}}{\sum_{i=1}^{n} t_1^{\beta}} \right)^{n-2} t_1^{\beta-1} dt_1$$

With suitable substitution and simplification, the MVUE of reliability is finally obtained as

$$\tilde{R}(t) = \left(1 - \frac{t^{\beta}}{\sum_{i=1}^{n} t_i^{\beta}} \right)^{n-1}, \text{ where } t < \sqrt[\beta]{\sum_{i=1}^{n} t_i^{\beta}} \qquad (7)$$

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III. ANALYSIS

The behavior of reliability estimate can be observed through reliability curves by considering sample failure data. 3 case studies of Musa [1] and Lyu [2] have been considered. The reliability estimates are found for $\beta<1$, $\beta=1$ and $\beta>1$. With $\beta=1$, the model reduces to exponential.

1) *Case Study 1:* Consider the failure data of 15 failures given in Table 1, as given by Musa [1].

Table 1 : Failure data of 15 failures

Failure No	Failure time
1	10
2	19
3	32
4	43
5	58
6	70
7	88
8	103
9	125
10	150
11	169
12	199
13	231
14	256
15	296

Table 2 denotes the MVUE of reliability for $\beta=0.5$, $\beta=1$ and $\beta=2$.

Table 2 : MVUE of reliability for case study 1

Failure number	Failure time (t)	$\tilde{R}(t)$ ($\beta=0.5$)	$\tilde{R}(t)$ ($\beta=1$)	$\tilde{R}(t)$ ($\beta=2$)
1	10	0.7481	0.9268	0.9959
2	19	0.6693	0.8653	0.9853
3	32	0.5925	0.7831	0.9589
4	43	0.5441	0.7193	0.9270
5	58	0.4920	0.6400	0.8710
6	70	0.4579	0.5825	0.81743
7	88	0.4152	0.5052	0.7261
8	103	0.3854	0.4482	0.6439
9	125	0.3485	0.3753	0.5203
10	150	0.3138	0.3059	0.3865
11	169	0.2913	0.2613	0.2957
12	199	0.2609	0.2030	0.1792
13	231	0.2338	0.1543	0.0935
14	256	0.2157	0.1241	0.0512
15	296	0.1908	0.0869	0.0160

Fig. 1denotes the 3 reliability curves for $\beta=0.5$, $\beta=1$ and $\beta=2$.

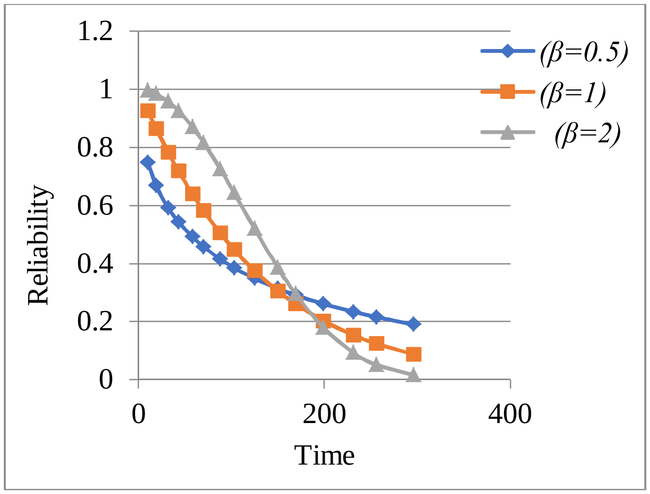

Fig. 1. Reliability curves for case study 1.

2) Case study 2: Consider the failure data of 10 failures given in Table 3, as given by Lyu [2].

Table 3: Failure data of 10 failures

Failure No	Failure time
1	7
2	18
3	26
4	36
5	51
6	73
7	93
8	118
9	146
10	181

Table 4 denotes the MVUE of reliability for $\beta=0.5$, $\beta=1$ and $\beta=2$.

Table 4: MVUE of reliability for case study 2

Failure number	Failure time (t)	$\tilde{R}(t)$ ($\beta=0.5$)	$\tilde{R}(t)$ ($\beta=1$)	$\tilde{R}(t)$ ($\beta=2$)
1	7	0.7380	0.9189	0.9949
2	18	0.6112	0.8033	0.9669
3	26	0.5516	0.7276	0.9321
4	36	0.4944	0.6419	0.8735
5	51	0.4296	0.5301	0.7607
6	73	0.3604	0.3973	0.5658
7	93	0.3133	0.3032	0.3893
8	118	0.2675	0.2137	0.2078
9	146	0.2277	0.1420	0.0795
10	181	0.1894	0.0829	0.0141

Fig. 2 denotes the 3 reliability curves for $\beta=0.5$, $\beta=1$ and $\beta=2$.

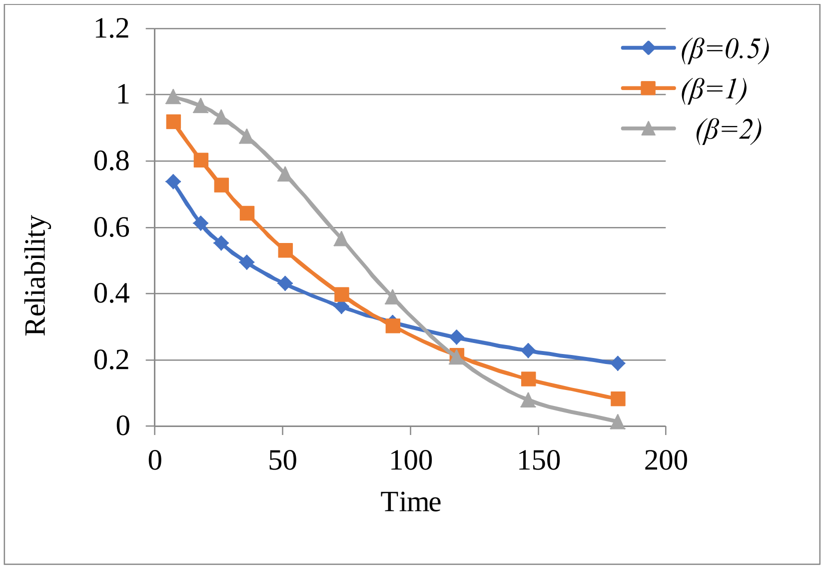

Fig. 2. Reliability curves for case study 2.

3) *Case study 3:* Another failure data of 17 failures as given by Musa [1] and the corresponding reliability estimates for $\beta=0.5$, $\beta=1$ and $\beta=2$ are given in Table 5.

Table 5 : MVUE of reliability for case study 3

Failure number	Failure time (t)	$\tilde{R}(t)$ ($\beta=0.5$)	$\tilde{R}(t)$ ($\beta=1$)	$\tilde{R}(t)$ ($\beta=2$)
1	932	0.7256	0.9036	0.9907
2	4035	0.5094	0.6418	0.8394
3	4696	0.4824	0.5961	0.7885
4	4893	0.4750	0.5831	0.7725
5	6369	0.4265	0.4938	0.6441
6	6524	0.4220	0.4852	0.6301
7	7882	0.3864	0.4156	0.5073
8	8170	0.3796	0.4021	0.4818
9	9339	0.3542	0.3515	0.3825
10	10400	0.3338	0.3107	0.3010
11	10542	0.3312	0.3056	0.2908
12	11036	0.3225	0.2885	0.2570
13	11696	0.3116	0.2669	0.2156
14	11905	0.3082	0.2604	0.2034
15	12266	0.3026	0.2496	0.1834
16	12954	0.2924	0.2300	0.1490
17	14000	0.2779	0.2031	0.1057

Fig. 3 denotes the 3 reliability curves for $\beta=0.5$, $\beta=1$ and $\beta=2$.

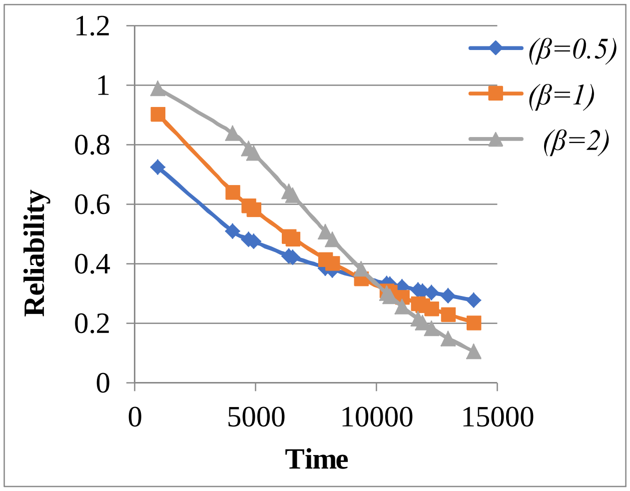

Fig. 3. Reliability curves for case study 3.

The three reliability curves exhibit the same behavior in all the three cases. As β tends to approach zero, the curve tends to incline horizontally, while as β tends to grow, the curve tends to incline vertically. The actual reliability in the beginning is one. It can be seen from all the three case studies that, the higher values of β are closer to one in the beginning than the lower values of β. As time increases, higher values of β tend to approach faster to 0 than lower values of β . These reliability estimates help the software developers to decide the stopping schedule of testing and also to decide whether the user requirements are met. These estimates also help the users of the software to decide whether or not to accept the software. Also, based on the reliability estimates, the software developers can have an idea of the durability of the software in the long run, prior to its release.

IV. CONCLUSION

The reliability of the software is one of the most measurable qualities of the software. Reliability decreases with increasing time and hence its estimation is a useful tool for the developers to decide about its quality and thereby its release. Software failure data obtained during testing can be used to estimate this reliability. Even though method of maximum likelihood estimation is the commonly used method because of its simplicity, it is not as efficient as the minimum variance unbiased estimators, as maximum likelihood estimators need not be unbiased and efficient. MVUE are always unbiased and efficient, since they have minimum variance among the class of all unbiased estimators. Application of Blackwellization to find this minimum variance unbiased estimator always provides unique estimator and the most efficient estimator. This technique has been applied here to obtain the most efficient estimate of reliability. This efficient estimator helps the developers and users of the software in their decision making problems.

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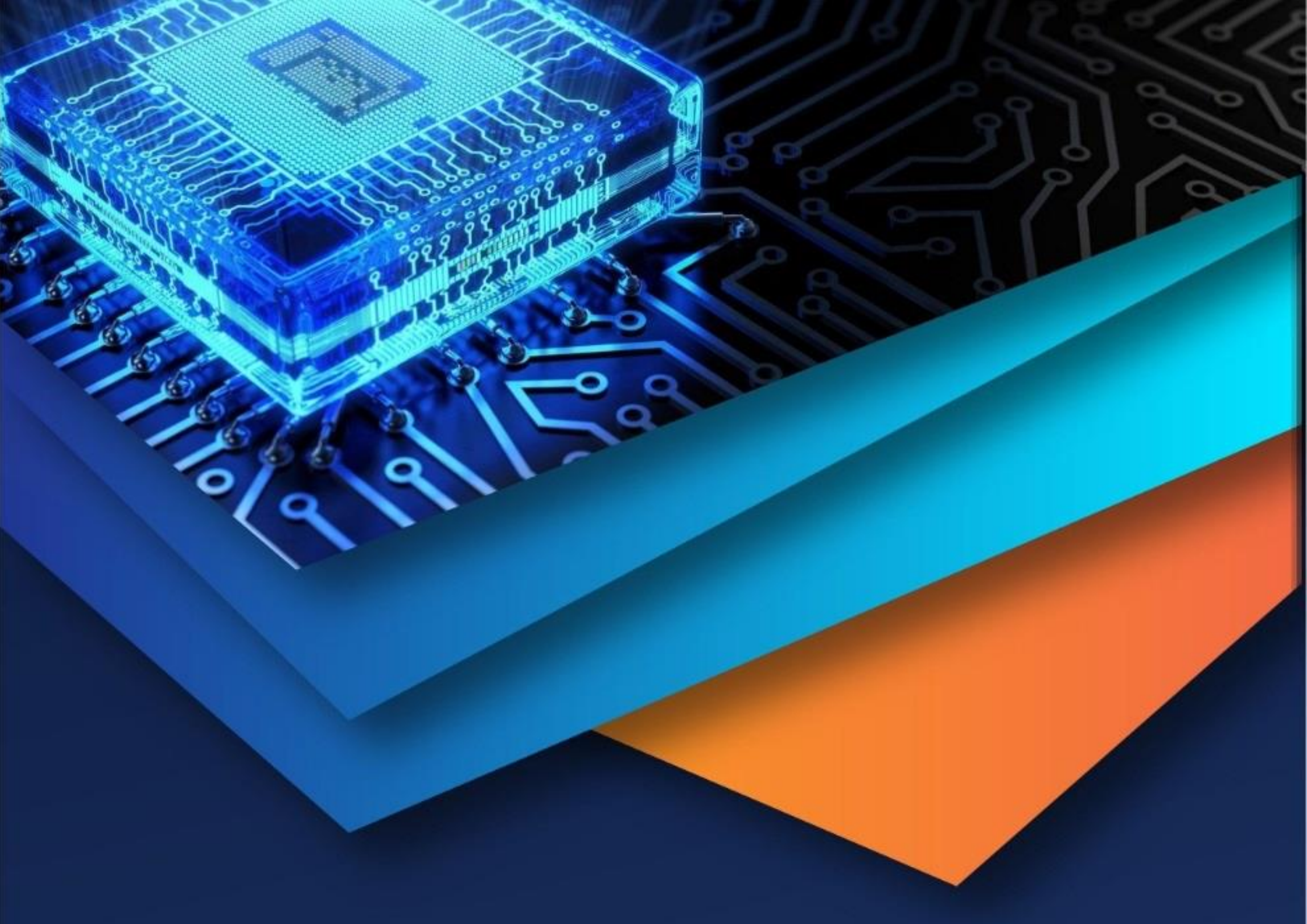

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