iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 9 **Issue:** I **Month of publication:** January 2021

DOI: https://doi.org/10.22214/ijraset.2021.32951

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International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue I Jan 2021- Available at www.ijraset.com

Lower Bounds of Functional Mean Code Word Length in Fuzzy Set

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Abstract: In the present paper, we find lower bound of functional mean code word length in fuzzy set with existing knowledge of different code word length. We know that lower bound of functional mean code word length is measure of fuzzy entropy which satisfy all different properties of itself. By finding lower bounds of different mean code word length, different measure of entropies in fuzzy set can be found which are very important in present .

Keywords: Mean code word length, Kraft's inequality, measure of entropy and fuzzy entropy, measure of directed divergence and fuzzy directed divergence,

I. INTRODUCTION

Let ($x_1, x_2, x_3, \ldots x_n$) be n inputs which have to be encoded in terms of an alphabet of size D. Let $l_1, l_2, l_3, \ldots, l_n$ be the n codeword lengths and let $p_1, p_2, \ldots, p_n$ be the probabilities ,the arithmetic mean L of codeword length is

$$L = \sum_{i=1}^{n} l_i p_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Shannon showed that the minimum value of L subject to Kraft's inequality $\sum_{i=1}^{n} D^{-l_i} \le 1$.....(2)

lies between S(P) and S(P)+1,where S(P) is given by $S(P) = -\sum_{i=1}^{n} p_i \log_D p_i$

After this Campbell considered more general exponential mean codeword length

$$L_\alpha = \frac{\alpha}{1-\alpha} \log\left[\sum p_i D^{l_i \frac{(1-\alpha)}{\alpha}}\right], \alpha > 0, \alpha \neq 1$$

and showed that subject to (2) the minimum value of L_α lies between $R_\alpha(P)$ and

$$R_\alpha(P)+1 \text{ where } R_\alpha(P) = \frac{1}{1-\alpha} \log \sum_{i=1}^{n} p_i^\alpha, \alpha > 0, \alpha \neq 1$$

Extending the scope of study, Kapur has several mean codeword length .

One of Kapur's mean codeword length is

$$L_\alpha^f = \frac{1}{\alpha-1} \log\left[\frac{\sum_{i=1}^{n} f(p_i) D^{(\alpha-1)l_i}}{\sum_{i=1}^{n} f(p_i)}\right]$$

Here we introduced lower bound of Kapur's mean codeword length with the help of Kraft's inequality in terms of fuzzy set.

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II. MAIN RESULT

LOWER BOUND OF KAPUR 'S MEAN CODEWORD LENGTH L_α^f SUBJECT TO KRAFT'S INEQUALITY

We know that according to Kapur's mean codeword length given in equation (1) ,the Fuzzy Mean Codeword length is

$$L_\alpha^f = \frac{1}{\alpha-1}\log\left[\frac{\sum_{i=1}^{n}\left\{ f(\mu_A(x_i)) + f(1-\mu_A(x_i))\right\} D^{(\alpha-1)l_i}}{\sum_{i=1}^{n}\left\{ f(\mu_A(x_i)) + f(1-\mu_A(x_i))\right\}}\right] \text{.............. (3)}$$

Now, we find the lower bound of this mean codeword length with respect to Kraft's inequality $\sum_{i=1}^{n} D^{-l_i} = k \leq 1$

.... (4)

Suppose $\left\{ f(\mu_A(x_i)) + f(1-\mu_A(x_i))\right\} = A_i$ & $D^{-l_i} = y_i$

Then eq^n(5.2) & eq^n(5.3) become $L_\alpha^f = \frac{1}{\alpha-1}\log\left[\frac{\sum_{i=1}^{n} A_i y_i^{(1-\alpha)}}{\sum_{i=1}^{n} A_i}\right]$..........(5)

$$\& \sum_{i=1}^{n} y_i = k \leq 1$$

Applying Lagrange's method, we get,

$$\frac{\partial}{\partial y_i}\left(\frac{1}{\alpha-1}\log\left[\frac{\sum_{i=1}^{n} A_i y_i^{(1-\alpha)}}{\sum_{i=1}^{n} A_i}\right] + \lambda\left(\sum_{i=1}^{n} y_i - k\right)\right) = 0$$

$$\Rightarrow \frac{-\sum_{i=1}^{n} A_i y_i^{-\alpha}}{\sum_{i=1}^{n} A_i y_i^{1-\alpha}} + \lambda = 0$$

$$\Rightarrow A_i y_i^{-\alpha} = \lambda \sum_{i=1}^{n} A_i y_i^{1-\alpha}$$

$$\Rightarrow A_i D^{\alpha l_i} = \lambda \sum_{i=1}^{n} A_i y_i^{1-\alpha}$$

$$\Rightarrow \frac{A_i^{1/\alpha}}{D^{-l_i}} = \left(\lambda \sum_{i=1}^{n} A_i y_i^{1-\alpha}\right)^{1/\alpha}$$

$$\Rightarrow \frac{A_1^{1/\alpha}}{D^{-l_1}} = \frac{A_2^{1/\alpha}}{D^{-l_2}} = ... = \frac{A_i^{1/\alpha}}{D^{-l_i}} = \left(\lambda \sum_{i=1}^{n} A_i y_i^{1-\alpha}\right)^{1/\alpha}$$

$$\Rightarrow \frac{A_1^{1/\alpha}}{D^{-l_1}} = \frac{A_2^{1/\alpha}}{D^{-l_2}} = ... = \frac{A_i^{1/\alpha}}{D^{-l_i}} = \frac{\sum_{i=1}^{n} A_i^{1/\alpha}}{\sum_{i=1}^{n} D^{-l_i}}$$

$$\Rightarrow \frac{A_1^{1/\alpha}}{D^{-l_1}} = \frac{A_2^{1/\alpha}}{D^{-l_2}} = ... = \frac{A_i^{1/\alpha}}{D^{-l_i}} = \frac{\sum_{i=1}^{n} A_i^{1/\alpha}}{k}$$

Minimum value of $D^{-l_i} = y_i = \dfrac{kA_i^{1/\alpha}}{\sum_{i=1}^{n} A_i^{1/\alpha}}$

Substituting this value in equation(5.4), we get minimum value of L_α^f

$$\text{Min}\, L_\alpha^f = \frac{1}{\alpha - 1}\log\left(\frac{\sum_{i=1}^{n} A_i \left\{\frac{kA_i^{1/\alpha}}{\sum_{i=1}^{n} A_i^{1/\alpha}}\right\}^{1-\alpha}}{\sum_{i=1}^{n} A_i}\right)$$

$$\text{Min}\ L_\alpha^f = \frac{1}{\alpha - 1}\log\left(\frac{\left(\sum_{i=1}^{n} A_i k^{1-\alpha} A_i^{\frac{1-\alpha}{\alpha}}\right)}{\left(\sum_{i=1}^{n} A_i^{1/\alpha}\right)^{1-\alpha}}\right)$$

$$\text{Min}\ L_\alpha^f = \frac{1}{\alpha - 1}\log\left(\frac{\sum_{i=1}^{n} A_i^{1/\alpha} k^{1-\alpha}}{\left(\sum_{i=1}^{n} A_i^{1/\alpha}\right)^{1-\alpha}}\right)$$

$$\text{Min}\ L_\alpha^f = \frac{1}{\alpha - 1}\log\left(\left(\sum_{i=1}^{n} A_i^{1/\alpha}\right)^{\alpha}\right) - \log k$$

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ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
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Min $L_{\alpha}^{f} = \frac{\alpha}{\alpha-1}\log\left(\sum_{i=1}^{n} A_i^{1/\alpha}\right) - \log k$

Min $L_{\alpha}^{f} = \frac{\alpha}{\alpha-1}\log\sum_{i=1}^{n}\left(\frac{\{f(\mu_A(x_i)) + f(1-\mu_A(x_i))\}^{1/\alpha}}{n}\right) + \frac{1}{\alpha-1}\log\frac{n^{\alpha}}{k^{\alpha-1}}$

Thus Min L_{α}^{f} lies between $M_{\alpha}^{f}(\mu_A(x_i))$ and $M_{\alpha}^{f}(\mu_A(x_i))$ +1

Where $M_{\alpha}^{f}(\mu_A(x_i)) = \frac{\alpha}{\alpha-1}\log\left(\frac{\sum_{i=1}^{n}\{f(\mu_A(x_i)) + f(1-\mu_A(x_i))\}^{1/\alpha}}{n}\right)$ ....(6)

which is generalized measure of entropy of order α for any function in terms of fuzzy set.

Generalized measure of entropy $M_{\alpha}^{f}(\mu_A(x_i))$ has different value for different function of

$\mu_A(x_i)$.

Suppose $f(\mu_A(x_i)) = \mu_A^{\beta}(x_i)$,then by eq^n(6)

$$M_{\alpha}^{f}(\mu_A(x_i)) = \frac{\alpha}{\alpha-1}\log\left(\frac{\sum_{i=1}^{n}\left\{\mu_A^{\beta}(x_i) + (1-\mu_A(x_i))^{\beta}\right\}^{1/\alpha}}{n}\right) \text{........ (7)}$$

$\alpha < 1, \beta > 1$

III. PROPERTIES OF $M_{\alpha}^{f}(\mu_A(x_i))$

1) $M_{\alpha}^{f}(\mu_A(x_i))$ is the minimum value of an exponentiated mean so it will always be non – negative.
2) $M_{\alpha}^{f}(\mu_A(x_i))$ is minimum iff A is a non fuzzy set.

   For $\mu_A(x_i) = 0$, $M_{\alpha}^{f}(\mu_A(x_i)) = 0$

   and when $\mu_A(x_i) = 1$ we get $M_{\alpha}^{f}(\mu_A(x_i)) = 0$
3) When A is most fuzzy set, and then $M_{\alpha}^{f}(\mu_A(x_i))$ is maximum.

   For maximum, $\frac{\partial M_{\alpha}^{f}(\mu_A(x_i))}{\partial \mu_A(x_i)} = 0$

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We can write eq^n (5.6) as

$$M_\alpha^f(\mu_A(x_i)) = \frac{\alpha}{\alpha-1}\log\left(\sum_{i=1}^{n}\left\{\mu_A^\beta(x_i)) + (1-\mu_A(x_i))^\beta\right\}^{1/\alpha}\right) - \frac{\alpha}{\alpha-1}\log n$$

$$\frac{\partial M_\alpha^f(\mu_A(x_i))}{\partial \mu_A(x_i)} = \frac{\beta}{\alpha-1}\left[\frac{(\mu_A^{\beta-1}(x_i) - (1-\mu_A(x_i))^{\beta-1})\left\{\mu_A^\beta(x_i)) + (1-\mu_A(x_i))^\beta\right\}^{\frac{1-\alpha}{\alpha}}}{\sum_{i=1}^{n}\left\{\mu_A^\beta(x_i)) + (1-\mu_A(x_i))^\beta\right\}^{1/\alpha}}\right]$$

.... (5.7)

$\dfrac{\partial M_\alpha^f(\mu_A(x_i))}{\partial \mu_A(x_i)} = 0$ gives $(\mu_A^{\beta-1}(x_i) - (1-\mu_A(x_i))^{\beta-1} = 0$

$$\Rightarrow \mu_A^{\beta-1}(x_i) = (1-\mu_A(x_i))^{\beta-1}$$

$$\Rightarrow \mu_A(x_i) = 1-\mu_A(x_i)$$

$$\Rightarrow \mu_A(x_i) = \frac{1}{2}$$

So at $\mu_A(x_i) = \dfrac{1}{2}$, $M_\alpha^f(\mu_A(x_i))$ has maximum or minimum value.

For this, we differentiate eq^n(5.7) again

$$\frac{\partial^2 M_\alpha^f(\mu_A(x_i))}{\partial(\mu_A(x_i))^2} = \frac{\beta(\beta-1)}{\alpha-1}\left(\mu_A^{\beta-2}(x_i) + (1-\mu_A(x_i))^{\beta-2}\right)\left[\frac{\left\{\mu_A^\beta(x_i)) + (1-\mu_A(x_i))^\beta\right\}^{\frac{1-\alpha}{\alpha}}}{\sum_{i=1}^{n}\left\{\mu_A^\beta(x_i)) + (1-\mu_A(x_i))^\beta\right\}^{1/\alpha}}\right]$$

$$+\frac{\beta}{\alpha-1}\left(\mu_A^{\beta-1}(x_i) - (1-\mu_A(x_i))^{\beta-1}\right)\frac{d}{dx}\left[\frac{\left\{\mu_A^\beta(x_i)) + (1-\mu_A(x_i))^\beta\right\}^{\frac{1-\alpha}{\alpha}}}{\sum_{i=1}^{n}\left\{\mu_A^\beta(x_i)) + (1-\mu_A(x_i))^\beta\right\}^{1/\alpha}}\right]$$

Which is less than zero for $\alpha < 1, \beta > 1$

So ,at $\mu_A(x_i) = \dfrac{1}{2}, M_\alpha^f(\mu_A(x_i))$ has maximum value.

$$\left[M_\alpha^f(\mu_A(x_i))\right]_{Max} = \frac{\beta-1}{1-\alpha}\log 2$$

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4) Since $M_{\alpha}^{f}(\mu_A(x_i))$ is an increasing function of $\mu_A(x_i)$ for $0 \le \mu_A(x_i) \le \frac{1}{2}$ .i.e

$$\left\{M_{\alpha}^{f}(\mu_A(x_i))/\mu_A(x_i)=0\right\}=0$$

$$\text{and}\left\{M_{\alpha}^{f}(\mu_A(x_i))\Big/\mu_A(x_i)=\frac{1}{2}\right\}=\frac{\beta-1}{1-\alpha}\log 2>0\ for\,\alpha<1,\beta>1$$

5) Since $M_{\alpha}^{f}(\mu_A(x_i))$ is decreasing function of $\mu_A(x_i)$ for $\frac{1}{2} \le \mu_A(x_i) \le 1$

$$\text{i.e.}\left\{M_{\alpha}^{f}(\mu_A(x_i))\Big/\mu_A(x_i)=\frac{1}{2}\right\}=\frac{\beta-1}{1-\alpha}\log 2 \text{ and}$$

$$\left\{M_{\alpha}^{f}(\mu_A(x_i))/\mu_A(x_i)=1\right\}=0$$

So, $M_{\alpha}^{f}(\mu_A(x_i))$ is a concave function.

6) $M_{\alpha}^{f}(\mu_A(x_i))$ does not change when $\mu_A(x_i)$ is replaced by $1-\mu_A(x_i)$

$$M_{\alpha}^{f}(\mu_A(x_i)) = M_{\alpha}^{f}(1-\mu_A(x_i))$$

Now we study the monotonic behavior of $M_{\alpha}^{f}(\mu_A(x_i))$. For this, different values of $M_{\alpha}^{f}(\mu_A(x_i))$ by parameters has been calculated and further the generalized measure has been presented graphically.

We have from eq^n (7)

$$M_{\alpha}^{f}(\mu_A(x_i)) = \frac{\alpha}{\alpha-1}\log\left(\frac{\sum_{i=1}^{n}\left\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right\}^{1/\alpha}}{n}\right) \text{ where } \alpha<1,\beta>1\ \&\ \alpha\neq 0$$

a) Suppose $\alpha=\frac{1}{2},\beta=2$

$\mu_A(x_i)$	$M_{\alpha}^{f}(\mu_A(x_i))$
0	0
0.1	0.172
0.2	0.334
0.3	0.473
0.4	0.568
0.5	0.602
0.6	0.568
0.7	0.473
0.8	0.334
0.9	0.172
1	0

Table 1.1

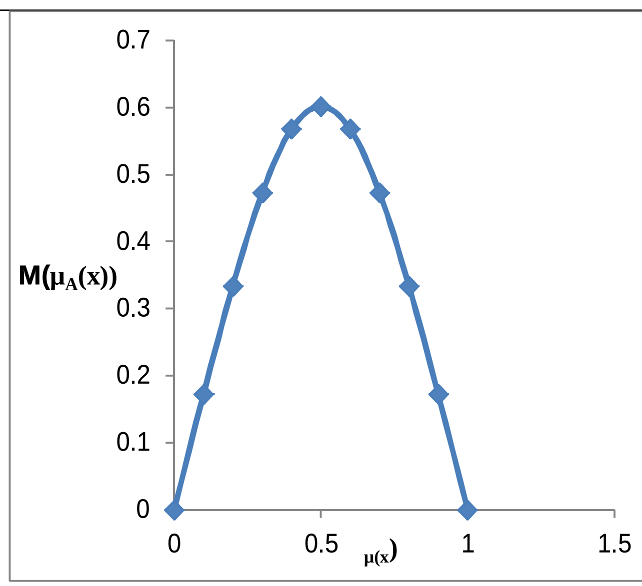

Figure 1.1

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b) $\alpha = -\frac{1}{2}, \beta = 2$

$\mu_A(x_i)$	$M_\alpha^f(\mu_A(x_i))$
0	0
0.1	0.057
0.2	0.111
0.3	0.158
0.4	0.189
0.5	0.201
0.6	0.189
0.7	0.158
0.8	0.111
0.9	0.057
1	0

Table 1.2

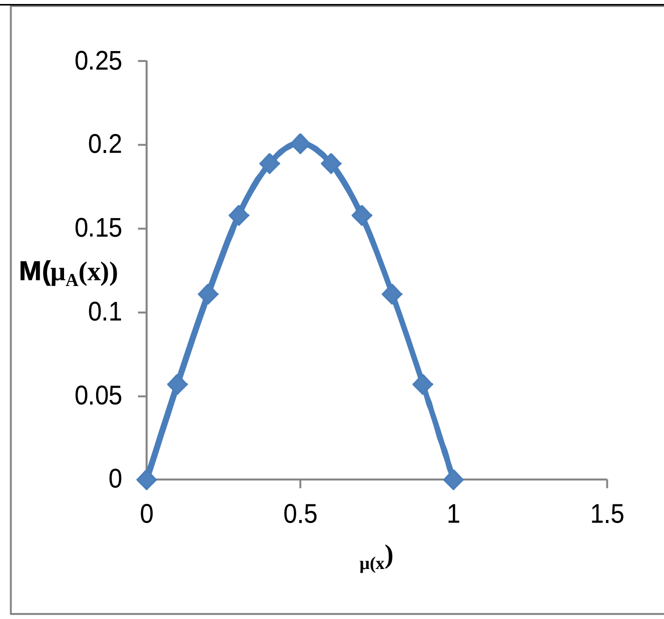

Figure 1.2

c) $\alpha = -\frac{1}{2}, \beta = 3$

$\mu_A(x_i)$	$M_\alpha^f(\mu_A(x_i))$
0	0
0.1	0.091
0.2	0.189
0.3	0.288
0.4	0.369
0.5	0.401
0.6	0.369
0.7	0.288
0.8	0.189
0.9	0.091
1	0

Table1.3

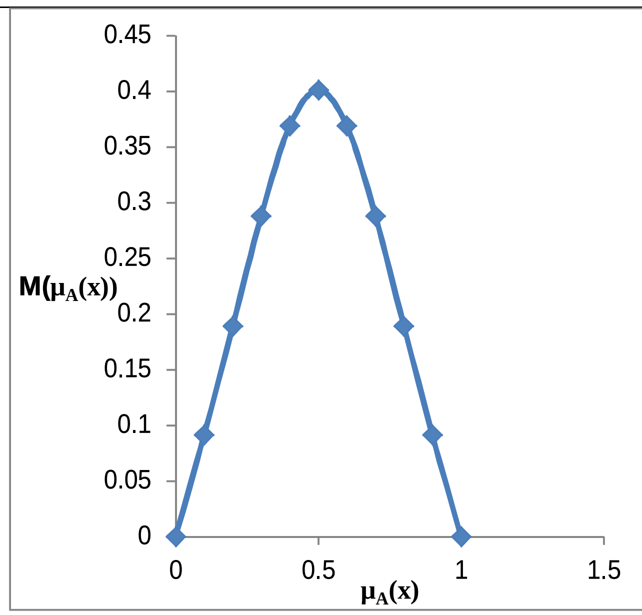

Figure1.3

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d) $\alpha = -1, \beta = 3$

$\mu_A(x_i)$	$M_\alpha^f(\mu_A(x_i))$
0	0
0.1	0.069
0.2	0.142
0.3	0.216
0.4	0.276
0.5	0.301
0.6	0.276
0.7	0.216
0.8	0.142
0.9	0.069
1	0

Table 1.4

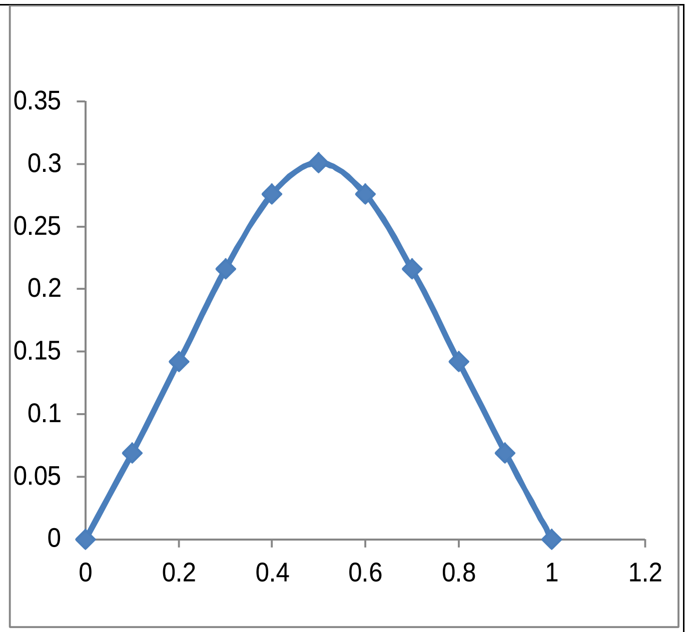

Figure 1.4

Thus, we found that value of $M_\alpha^f(\mu_A(x_i))$ given by eq^n(7) satisfy all properties of fuzzy entropy. Also by assuming different values of $f(\mu_A(x_i))$, we get different fuzzy mean codeword length whose lower bounds are expressed as fuzzy entropies.

IV. CONCLUSION

In this paper , by developing new fuzzy code word length ,we found lower bound of this fuzzy code word length which are expressed as measure of fuzzy entropies . We get this by existing knowledge. This generalisation are very important in present .Also we have proved properties of above fuzzy entropy and with the help of data, we have represented it graphically.

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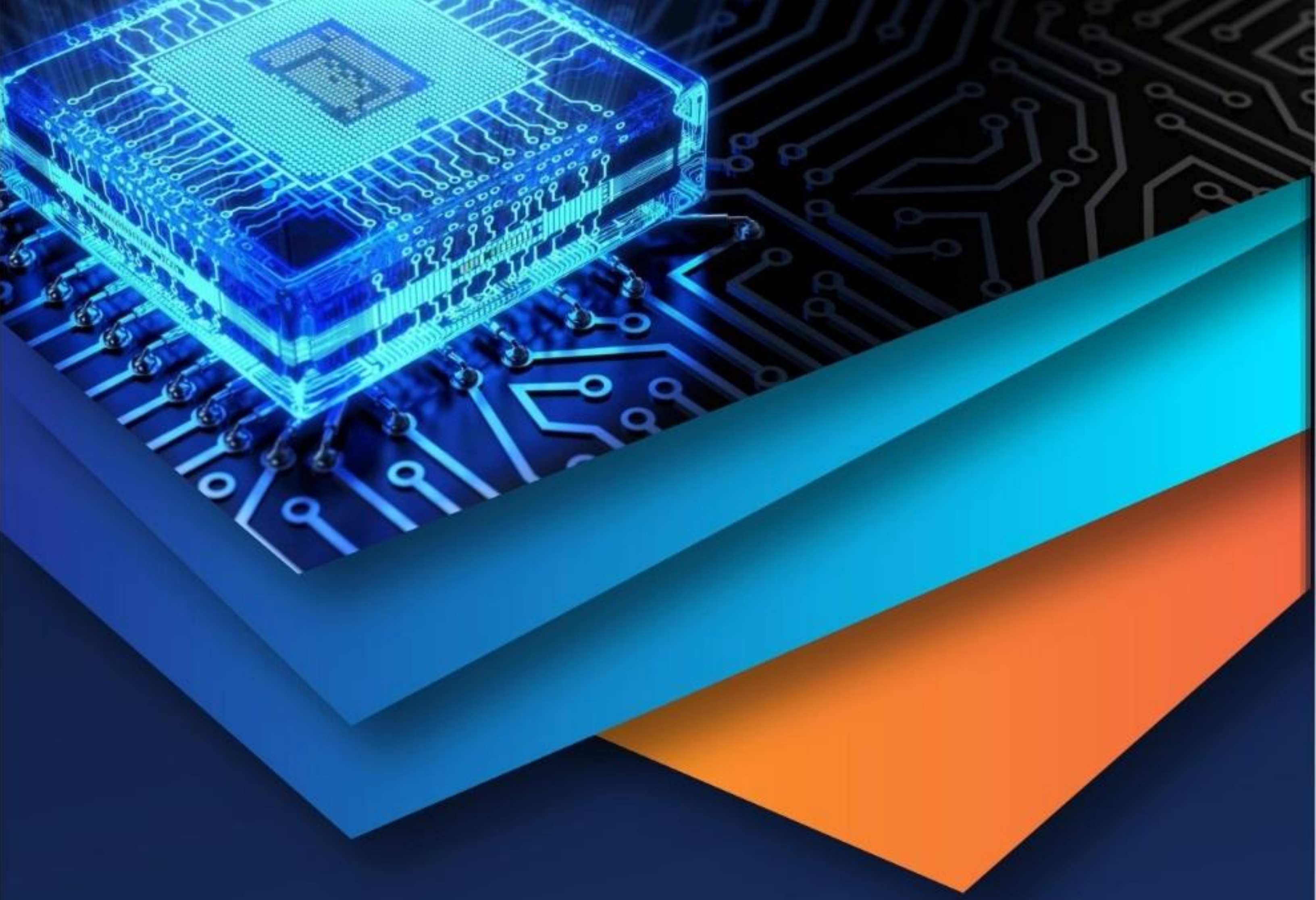

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