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Second Order Rotatable Designs of Second Type using Symmetrical Unequal Block Arrangements with Two Unequal Block Sizes

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Abstract: In this paper, second order rotatable designs of second type using symmetrical unequal block arrangements with two unequal block sizes is suggested. This design is compared with second order rotatable designs of first type using symmetrical unequal block arrangements with two unequal block sizes (Raghavarao, 1963) on the basis of efficiency.
Keywords: Response surface methodology, Symmetrical unequal block arrangements with two unequal block sizes, Rotatability, Orthogonality, Efficiency.

I. INTRODUCTION

Response surface designs is a collection of mathematical and statistical techniques useful for analyzing problems where several independent variables influence a dependent variable or response. Box and Hunter (1957) introduced designs having spherical variance function are called rotatable designs. Das and Narasimham (1962) constructed rotatable designs using balanced incomplete block designs (BIBD). Raghavarao (1963) constructed SORD using symmetrical unequal block arrangements (SUBA) with two unequal block sizes. Draper and Guttman (1988) suggested an index of rotatability. Khuri (1988) introduced a measure of rotatability for response surface designs. Draper and Pukelshein (1990) developed another look at rotatability. Park et al. (1993) introduced new measure of rotatability for second order response surface designs. Das et al. (1999) developed modified response surface designs. Kim (2002) introduced extended central composite designs (CCD) with the axial points are indicated by two numbers. Kim and Ko (2004) developed slope rotatability of second type of CCD. Victorbabu and Vasundharadevi (2005) suggested modified second order response surface designs using BIBD. Victorbabu (2006) constructed modified SORD and second order slope rotatable designs using a pair of BIBD. Victorbabu et al. (2006) studied modified second order response surface designs using pairwise balanced designs (PBD). Victorbabu (2007) suggested a review on second order rotatable designs. Victorbabu et al. (2008) suggested modified second order response surface designs using CCD. Victorbabu and Vasundharadevi (2008) studied second order response surface designs using SUBA with two unequal block sizes. Victorbabu (2009) constructed modified SORD using a pair SUBA with two unequal block sizes. Park and Park (2010) suggested the extension of CCD for second order response surface models. Victorbabu and Surekha (2013) suggested measure of rotatability for second order response surface designs using incomplete block designs. Victorbabu and Surekha (2015) developed measure of rotatability for second order response surface designs using BIBD. Kim (2019) suggested modified slope rotatability using extended CCD. Chiranjeevi et al. (2021) extended the work of Kim (2002) and suggested second order rotatable designs of second type using CCD for $9 \leq v \leq 17$ (v: number of factors). Chiranjeevi and Victorbabu (2021) developed SORD second type using BIBD. Chiranjeevi and Victorbabu (2021) studied SORD of second type using PBD

In this paper, second order rotatable designs of second type using symmetrical unequal block arrangements with two unequal block sizes is suggested. This design is compared with second order rotatable designs of first type using symmetrical unequal block arrangements with two unequal block sizes (Raghavarao, 1963) on the basis of efficiency.

II. STIPULATIONS AND FORMULAS FOR SECOND ORDER ROTATABLE DESIGNS

Suppose we want use the second order polynomial response surface design $D = ((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i<j} \sum b_{ij} x_{iu} x_{ju} + \varphi_u \quad (2.1)$$

where x_{iu} represents the level of i^{th} factor (i=1,2,…,v) in the u^{th} run (u=1,2,…,N) of the experiment and φ_u are uncorrelated random error with mean zero and variance σ^2. Then 'D' is said to be second order rotatable designs (SORD) if the variance of $Y_u(x_1, x_2, \ldots, x_v)$ with respect to each of independent variable (x_i) is only a function of the distance $(d^2=\sum_{i=1}^{v} x_i^2)$ of the point $(x_1, x_2, \ldots, x_v)$ from the origin (center) of the design. Such a spherical variance function for estimation of responses in the second order polynomial model is achieved if the design points satisfy the following conditions (cf. Das and Giri 1999).

All odd order moments are must be zero. In their words when at least one odd power x's equal to zero.

$$1.\quad \sum x_{iu}=0, \sum x_{iu}x_{ju}=0, \sum x_{iu}x_{ju}^2=0, \sum x_{iu}x_{ju}x_{ku}=0,$$
$$\sum x_{iu}^3=0, \sum x_{iu}x_{ju}^3=0, \sum x_{iu}x_{ju}x_{ku}^2=0, \sum x_{iu}x_{ju}x_{ku}x_{lu}=0.$$
$$\text{for } i\neq j\neq k\neq l; \qquad (2.2)$$

$$2.\ \text{(i)} \sum x_{iu}^2=\text{constant}=N\mu_2 \qquad \text{(ii)} \sum x_{iu}^4=\text{constant}=cN\mu_4 \text{ for all } i \qquad (2.3)$$

$$3.\ \sum x_{iu}^2 x_{ju}^2=\text{constant}=N\mu_4; \text{for all } i\neq j \qquad (2.4)$$

$$4.\ \frac{\mu_4}{\mu_2^2}>\frac{v}{(c+v-1)} \qquad (2.5)$$

$$5.\ \sum x_{iu}^4=c\sum x_{iu}^2 x_{ju}^2 \qquad (2.6)$$

where c, μ_4 and μ_2 are constants.

The variances and covariances of the estimated parameters are

$$V(\hat{b}_0)=\frac{\mu_4(c+v-1)\sigma^2}{N[\mu_4(c+v-1)-v\mu_2^2]},$$

$$V(\hat{b}_i)=\frac{\sigma^2}{N\mu_2},$$

$$V(\hat{b}_{ij})=\frac{\sigma^2}{N\mu_4},$$

$$V(\hat{b}_{ii})=\frac{\sigma^2}{(c-1)N\mu_4}\left[\frac{\mu_4(c+v-2)-(v-1)\mu_2^2}{\mu_4(c+v-1)-v\mu_2^2}\right],$$

$$Cov(\hat{b}_0,\hat{b}_{ii})=\frac{-\mu_2\sigma^2}{N[\mu_4(c+v-1)-v\mu_2^2]},$$

$$Cov(\hat{b}_{ii},\hat{b}_{jj})=\frac{(\mu_2^2-\mu_4)\sigma^2}{(c-1)N\mu_4[\mu_4(c+v-1)-v\mu_2^2]} \quad \text{and other covariances vanish.} \qquad (2.7)$$

The variance of the estimated response at the point $(x_{10}, x_{20}, \ldots, x_{v0})$ is

$$V(\hat{y}_0)=V(\hat{b}_0)+\left[V(\hat{b}_i)+2Cov(\hat{b}_0,\hat{b}_{ii})\right]d^2+V(\hat{b}_{ii})d^4+$$
$$\sum x_{i0}^2 x_{j0}^2\left[V(\hat{b}_{ij})+2Cov(\hat{b}_{ii},\hat{b}_{jj})-2V(\hat{b}_{ii})\right] \qquad (2.8)$$

The coefficient of $\sum x_{i0}^2 x_{j0}^2$ in the above equation (2.8) is simplified to $(c-3)\sigma^2/(c-1)N\lambda_4$.

A second order response surface design D is said to be SORD, if in this design c=3 and all the other conditions (2.2) to (2.7) hold.

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III. SORD OF FIRST TYPE USING SYMMETRICAL UNEQUAL BLOCK ARRANGEMENTS WITH TWO UNEQUAL BLOCK SIZES (CF. RAGHAVARAO (1963)).

Let ($v, b, r, k_1, k_2, b_1, b_2, \lambda$) denote parameters of SUBA with two unequal block sizes, $b_1+b_2=b$, $k=\sup(k_1, k_2)$ and $2^{t(k)}$ denote a fractional replication of 2^k in +1 or -1 levels in which no interaction with less than five factors is confounded. $[1-(v, b, r, k_1, k_2, b_1, b_2, \lambda)]$ denote the design points generated from transpose of the incidence matrix of SUBA with two unequal block sizes. Let $[1-(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from SUBA with two unequal block sizes by "multiplication" (cf. Raghavarao (1971), pp 298-300), $(a, 0, 0, \ldots, 0)2^1$ denote the design points generated from (a, 0, 0, …, 0) point set. Let U denote the union of the design points generated from different sets of points, n_0 denote the number of central points. The method of construction of SORD of first type using SUBA with two unequal block sizes is given in the following result

1) *Result:* The design points, $[1-(v,b,r,k_1,k_2,b_1,b_2,\lambda)]2^{t(k)} U (a,0,\ldots,0)2^1 U (n_0)$ will give a v-dimensional SORD of first type using SUBA with two unequal block sizes in $N=b2^{t(k)}+2v+n_0$ design points, with

$$a^4 = \frac{2^{t(k)}(3\lambda - r)}{2}.$$

The condition for the design becomes an orthogonal design.
From equation 2 (i) of (2.3) and (3) of (2.4), we have

$$\sum x_{iu}^2 = r2^{t(k)} + 2a^2 = N\mu_2$$

$$\sum x_{iu}^2 x_{ju}^2 = \lambda 2^{t(k)} = N\mu_4$$

For the convenience N is replaced by M
By using the orthogonality condition we have

$$\mu_2^2 = \mu_4$$

$$\left(\frac{r2^{t(k)} + 2a^2}{M}\right)^2 = \frac{\lambda 2^{t(k)}}{M}$$

then we can obtain

$$a^2 = \left(\frac{\sqrt{\lambda 2^{t(k)} M} - r2^{t(k)}}{2}\right) (\text{for orthogonality}) \tag{3.1}$$

and the condition for the design become rotatability .
From equation 2 (ii) of (2.3) and 3 of (2.4), we have

$$\sum x_{iu}^4 = r2^{t(k)} + 2a^4 = 3N\mu_4$$

$$\sum x_{iu}^2 x_{ju}^2 = \lambda 2^{t(k)} = N\mu_4$$

For the convenience N is replace by M
Then the rotatability condition equation (2.6), we have

$$\sum x_{iu}^4 = c \sum x_{iu}^2 x_{ju}^2$$

$$\Rightarrow r2^{t(k)} + 2a^4 = 3(\lambda 2^{t(k)})$$

$$\Rightarrow a^4 = \left(\frac{(3\lambda - r)2^{t(k)}}{2}\right) (\text{for rotatability}) \tag{3.2}$$

A. Proposed Method Of SORD Of Second Type Using Symmetrical Unequal Block Arrangements With Two Unequal Block Sizes

Let $(v, b, r, k_1, k_2, b_1, b_2, \lambda)$ denote a parameters of SUBA with two unequal block sizes, $b_1+b_2=b$, $k=\sup(k_1, k_2)$ and $2^{t(k)}$ denote a fractional replication of 2^k in +1 or -1 levels in which no interaction with less than five factors are confounded. $[1-(v, b, r, k_1, k_2, b_1, b_2, \lambda)]$ denote the design points generated from transpose of the incidence matrix of SUBA with two unequal block sizes. Let $[1-(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from SUBA with two unequal block sizes by "multiplication" (cf. Raghavarao (1971), pp 298-300). We use the additional set of points like $(\pm a_1,0,...,0),(0,\pm a_1,0,...,0),...,(0,0,...,\pm a_1)$ and $(\pm a_2,0,...,0),(0,\pm a_2,0,...,0),...,(0,0,...,\pm a_2)$ are two sets of axial points. Here $(a_1,0,0,...,0)2^1U(a_2,0,0,...,0)2^1$ denote the 4v design points generated from $(a_1,0,0,...,0)U(a_2,0,0,...,0)$ point set. Let U denote the union of the design points generated from different sets of points, and (n_0) denote the number of central points. The method of construction of SORD of second type using SUBA with two unequal block sizes is given in the following theorem.

1) *Theorem (3.1):* The design points $[1-(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}U(a_1,0,...,0)2^1U(a_2,0,...,0)2^1U(n_0)$ will give a v-dimensional SORD of second type using SUBA with two unequal block sizes in $N=b2^{t(k)}+4v+n_0$ design points, with

$$a_1^4+a_2^4=\frac{2^{t(k)}(3\lambda-r)}{2}\ \text{(for rotatability)}.$$

2) *Proof:* For the design points generated from second order rotatable designs of second type using SUBA with two unequal block sizes, simple symmetry conditions (2.2) are true. Further, conditions (2.3) and (2.4) are true as follows:

$$\sum x_{iu}^2=r2^{t(k)}+2(a_1^2+a_2^2)=N\mu_2 \quad (3.3)$$

$$\sum x_{iu}^4=r2^{t(k)}+2(a_1^4+a_2^4)=cN\mu_4 \quad (3.4)$$

$$\sum x_{iu}^2x_{ju}^2=\lambda 2^{t(k)}=N\mu_4 \quad (3.5)$$

Solving equations (3.4) and (3.5), we get $a_1^4+a_2^4=\frac{2^{t(k)}(3\lambda-r)}{2}$ (for rotatability).

Example 3.1. We illustrate the theorem (3.1) to obtain SORD of second type using SUBA with two unequal block sizes with parameters $(v=6,b=11,r=7,k_1=3,k_2=4,b_1=2,b_2=9,\lambda=4)$. The design points $[1-(6,11,7,3,4,2,9,4)]2^{t(4)}U(a_1,0,0,...,0)2^1U(a_2,0,0,...,0)2^1U(n_0=1)$ will give a v-dimensional SORD of second type using SUBA with two unequal block sizes in N=201 design points with one central point. From (3.3), (3.4) and (3.5) we have

$$\sum x_{iu}^2=112+2(a_1^2+a_2^2)=N\mu_2 \quad (3.6)$$

$$\sum x_{iu}^4=112+2(a_1^4+a_2^4)=cN\mu_4 \quad (3.7)$$

$$\sum x_{iu}^2x_{ju}^2=64=N\mu_4 \quad (3.8)$$

From (3.7) and (3.8), we can obtain the rotatability value $a_1^4+a_2^4=40$, here we assume for an arbitrary value $a_1=1$, then we get $a_2=2.4989$ and c=3 (for rotatability).

The non-singularity condition (2.5), we have

$$\frac{0.3184}{0.3184} > \frac{6}{3+6-1} \Rightarrow 1.0000 > 0.75$$

Hence the non singularity condition is also satisfied.
The variances and covariances of the estimated parameters are

$$V(\hat{b}_0)=0.0199\sigma^2$$
$$V(\hat{b}_i)=0.0088\sigma^2$$
$$V(\hat{b}_{ij})=0.0156\sigma^2$$
$$V(\hat{b}_{ii})=0.0078\sigma^2$$
$$Cov(\hat{b}_0,\hat{b}_{ii})= -0.0044\sigma^2$$
$$Cov(\hat{b}_{ii},\hat{b}_{jj})=0 \quad (3.9)$$

The variance of the estimated response at the point $(x_{10}, x_{20},\ldots,x_{v0})$ is

$$V(\hat{Y})=0.0199\sigma^2+d^4(0.0078\sigma^2) \quad (3.10)$$

Table 3.1 gives the values of the variance of the estimated responses for different factors by using PBD

Table 3.1 The variance of estimated response for different factors

$(v, b, r, k_1,k_2,b_1,b_2, \lambda)$	n_0	N	$a_1^4+a_2^4$	a_1	a_2	$V(\hat{Y})$
(6,11,7,3,4,2,9,4)	1	201	40	1	2.4989	$(0.0199+0.0078d^4)\sigma^2$
(9,12,7,3,6,3,9,4)	1	421	80	1	2.9813	$(0.0131+0.0039d^4)\sigma^2$
(10,11,5,4,5,5,6,2)	1	217	8	1	1.6266	$(0.0276+0.0002d^2+ 0.0156\ d^4)\sigma^2$
(12,15,7,4,6,3,12,3)	1	529	32	1	2.3596	$(0.0132+0.0052\ d^4)\sigma^2$

IV. STUDY OF ORTHOGONALITY IN SORD OF SECOND TYPE USING SUBA WITH TWO UNEQUAL BLOCK SIZES (CF. KIM (2002).

An orthogonal design is one in which the terms in the fitted model are uncorrelated with one another and thus the parameter estimates are uncorrelated. In this case, the variance of the predicated response at any point x in the experimental region, is expressible as a weighted sum of the variance of the parameter estimates in the model. For second order moments $\sum x_{iu}^2$ and $\sum x_{iu}^2 x_{ju}^2$ is impossible to obtain. This is because the moments $\sum x_{iu}^2$ and $\sum x_{iu}^2 x_{ju}^2$ are necessarily positive. Hence, we consider the model with the pure quadratic terms correlated for their means. In regard to orthogonality, this model is often used for the sake of simplicity in the calculation. A design is said to be orthogonal we shall investigate the restriction $\left(\sum x_{iu}^2\right)^2 = N\sum x_{iu}^2 x_{ju}^2$ i.e $\left(N\mu_2^2\right)^2 = N\left(N\mu_4\right)$ i.e $\mu_2^2=\mu_4$ to get SORD of second type using SUBA with two unequal block sizes

$$a_1^2+a_2^2=\frac{\sqrt{N(\lambda 2^{t(k)})}-r2^{t(k)}}{2} \quad (4.1)$$

It must be established the equation (4.1) makes SORD of second type using SUBA with two unequal block sizes an orthogonal system. However $N=2^{t(v)}+4v+n_0$, the value of (4.1) depends on v, n_0 and the design points of SORD of second type. The following table 1 gives the values of orthogonality of second order response surface methodology using various parameters of SORD of second type and n_0, the value of '$a_1^2+a_2^2$' makes orthogonal second order response surface designs by using SORD of second type using SUBA with two unequal block sizes.

Let $(v, b, r, k_1, k_2, b_1, b_2, \lambda)$ denote a parameters of SUBA with two unequal block sizes, $b_1+b_2=b$, $k=\sup(k_1, k_2,)$ and $2^{t(k)}$ denote a fractional replication of 2^k in +1 or -1 levels in which no interaction with less than five factors are confounded. $[1-(v, b, r, k_1, k_2, b_1, b_{2,}\lambda)]$ denote the design points generated from transpose of the incidence matrix of SUBA with two unequal block sizes. Let $[1-(v, b, r, k_1, k_2, b_1, b_{2,}\lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from SUBA with two unequal block sizes by "multiplication" (cf. Raghavarao (1971), pp 298-300). We use the additional set of points like $(\pm a_1,0,\ldots,0),(0,\pm a_1,0,\ldots,0),\ldots,(0,0,\ldots,\pm a_1)$ and $(\pm a_2,0,\ldots,0),(0,\pm a_2,0,\ldots,0),\ldots,(0,0,\ldots,\pm a_2)$ are two sets of axial points. Here $(a_1,0,0,\ldots,0)2^1U(a_2,0,0,\ldots,0)2^1$ denote the 4v design points generated from $(a_1,0,0,\ldots,0)U(a_2,0,0,\ldots,0)$ point set. Let U denote the union of the design points generated from different sets of points, and (n_0) denote the number of central points. The method of study of Orthogonality of SORD of second type using SUBA with two unequal block sizes is given in the following theorem.

1) *Theorem (4.1):* The design points, $[1-(v, b, r, k_1, k_2, b_1, b_{2,}\lambda)]2^{t(k)}U(a_1,0,\ldots,0)2^1U(a_2,0,\ldots,0)2^1U(n_0)$ will give a v-dimensional SORD of second type using SUBA with two unequal block sizes in $N=b2^{t(k)}+4v+n_0$ design points, with

$$a_1^2+a_2^2=\frac{\sqrt{N(\lambda 2^{t(k)})}-r2^{t(k)}}{2}(\text{for orthogonality}).$$

2) *Proof:* For the design points generated from second order rotatable designs of second type using SUBA with two unequal block sizes, simple symmetry conditions (2.2) are true. Further, conditions (2.3) and (2.4) are true as follows

$$\sum x_{iu}^2=r2^{t(k)}+2(a_1^2+a_2^2)=N\mu_2 \quad (4.2)$$

$$\sum x_{iu}^4=r2^{t(k)}+2(a_1^4+a_2^4)=cN\mu_4 \quad (4.3)$$

$$\sum x_{iu}^2x_{ju}^2=\lambda 2^{t(k)}=N\mu_4 \quad (4.4)$$

Solving equations (4.2) and (4.4) using $\mu_2^2=\mu_4$

$$\left(\frac{r2^{t(k)}+2(a_1^2+a_2^2)}{N}\right)^2=\frac{\lambda 2^{t(k)}}{N}$$

then we can obtain

$$a_1^2+a_2^2=\frac{\sqrt{N(\lambda 2^{t(k)})}-r2^{t(k)}}{2}(\text{for orthogonality}).$$

Example 4.1. We illustrate the theorem (4.1) second order rotatable designs of second type using SUBA with two unequal block sizes with parameters $(v=6,b=11,r=7,k_1=3,k_2=4,b_1=2,b_2=9,\lambda=4)$. The design points $[1\text{-}(6,11,7,3,4,2,9,4)]2^{t(4)}U(a_1,0,0,...,0)2^1U(a_2,0,0,...,0)2^1U(n_0=1)$ will give a v-dimensional SORD of second type using SUBA with two unequal block sizes in N=201 design points with one central point. From (3.3), (3.4) and (3.5) we have

$$\sum x_{iu}^2=112+2(a_1^2+a_2^2)=N\mu_2 \qquad (4.5)$$

$$\sum x_{iu}^4=112+2(a_1^4+a_2^4)=cN\mu_4 \qquad (4.6)$$

$$\sum x_{iu}^2x_{ju}^2=64=N\mu_4 \qquad (4.7)$$

From (4.5) and (4.7), using $\mu_2^2=\mu_4$, we can obtain the orthogonality value

$$\Rightarrow a_1^2+a_2^2=\frac{\sqrt{(201)\times(64)}-112}{2}$$

$$=0.7098.$$

The values of orthogonality of SORD of second type using SUBA with two unequal block sizes for $6\leq v\leq 12$ with central points are given in the following table 4.1

Table 4.1: values of orthogonality of SORD of second type using SUBA with two unequal block sizes

(6,11,7,3,4,2,9,4)			(9,12,7,3,6,3,9,4)			(10,11,5,4,5,5,6,2)			(12,15,7,4,6,3,12,3)		
n_0	N	$a_1^2+a_2^2$	n_0	N	$a_1^2+a_2^2$	n_0	N	$a_1^2+a_2^2$	n_0	N	$a_1^2+a_2^2$
1	201	0.7098	1	421	4.0689	1	217	1.6653	1	529	0.6765
2	202	0.8506	2	422	4.2067	2	218	1.7612	2	530	0.7829
3	203	0.9912	3	423	4.3443	3	219	1.8569	3	531	0.8893
4	204	1.1314	4	424	4.4818	4	220	1.9524	4	532	0.9956
5	205	1.2713	5	425	4.6190	5	221	2.0476	5	533	1.1017

V. EFFICIENCY COMPARISON FOR SORD OF SECOND TYPE USING PBD WITH SORD OF FIRST TYPE USING SUBA WITH TWO UNEQUAL BLOCK SIZES

In this section, SORD of second type using SUBA with two unequal block sizes is used as the basis for estimating specific coefficient in the response surface model, SORD of second type using SUBA with two unequal block sizes is compared with SORD of first type using SUBA with two unequal block sizes. This comparison criterion is based on the precision at which the coefficient is estimated. It is consider that the numbers of experimental plots are required at same way.

For example in terms of estimating mixed quadratic coefficient b_{ij} $(i\neq j)$, two experimental designs, lets try to compare D_1 and D_2. The number of experimental plots required in D_1 and D_2 are M and N respectively. The relative efficiency of D_1 and D_2 is given by the following equation (see Myers (1976), section 7.2).

$$E\left(D_1/D_2\right)=\frac{\{Var(b_{ij})\,\text{in}\,D_2\}N}{\{Var(b_{ij})\,\text{in}\,D_1\}M} \qquad (5.1)$$

Where $N=b2^{t(k)}+4v+n_0$ (Design points in SORD of second type using SUBA with two unequal block sizes)

$M=b2^{t(k)}+2v+m_0$ (Design points in SORD of first type using SUBA with two unequal block sizes)

In this case, in order to compare fairly, the experimental system should make the second product equal to value of $\frac{\sum x_{iu}^2}{N}$. It must be scaled and for this the following scaling criteria is used.

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$$(i)\ \frac{1}{N}\sum x_{iu}=0$$

$$(ii)\ \frac{1}{N}\sum x_{iu}^2=1,\ (i=1,2,...,v) \qquad (5.2)$$

A. Comparison in Mixed Quadratic Coefficient $b_{ij}\ (i \neq j)$

According to equation (2.7) $V(\hat{b}_{ij})=\frac{\sigma^2}{N\mu_4}$, but this before scaling equation (5.2) will be write in SORD of second type using SUBA with two unequal block sizes. From (ii) of (5.2) $\frac{(r2^{t(k)}+2a_1^2+2a_2^2)}{N}$ each time making equation (5.2) will be is equal to 1, i.e. (ii) =1, then the scaling factor g

$$g=\left\{\left(\frac{b2^{t(K)}+4v+n_0}{r2^{t(k)}+2a_1^2+2a_2^2}\right)\right\}^{1/2} \qquad (5.3)$$

However the $V(b_{ij})$ is multiplied by with the scaling factor 'g' than the $V(b_{ij})=\frac{\sigma^2}{N\lambda_4}.\frac{1}{g^4}$ that is

$$V(b_{ij})=\frac{\sigma^2}{N\lambda_4}\left\{\left[\frac{r2^{t(k)}+2a_1^2+2a_2^2}{b2^{t(k)}+4v+n_0}\right]\right\}^2$$

According to equation (5.1) the relative efficiency SORD of second type using SUBA with two unequal block sizes versus SORD of first type using SUBA with two unequal block sizes in the mixed quadratic coefficient b_{ij} is obtained as follows

$$E\left(\frac{\text{SORD of second type using SUBA with two unequal block sizes}}{\text{SORD of first type using SUBA with two unequal block sizes}}\right)=\frac{\frac{\sigma^2}{N\lambda_4}\left(\frac{r2^{t(k)}+2a^2}{b2^{t(k)}+2v+m_0}\right)^2(b2^{t(k)}+2v+m_0)}{\frac{\sigma^2}{N\lambda_4}\left(\frac{r2^{t(k)}+2a_1^2+2a_2^2}{b2^{t(k)}+4v+n_0}\right)^2(b2^{t(k)}+4v+n_0)}$$

$$=\frac{(r2^{t(k)}+2a^2)^2(b2^{t(k)}+4v+n_0)}{(r2^{t(k)}+2a_1^2+2a_2^2)^2(b2^{t(k)}+2v+m_0)} \qquad (5.4)$$

From equation (5.4) the condition that $E\left(\frac{\text{SORD of second type using SUBA with two unequal block sizes}}{\text{SORD of first type using SUBA with two unequal block sizes}}\right)>1$, than the SORD of second type using SUBA with two unequal block sizes is more efficient than SORD of first type using SUBA with two unequal block sizes.

$$a_1^2+a_2^2<\frac{1}{2}\left\{(r2^{t(k)}+2a^2)\sqrt{\frac{b2^{t(k)}+4v+n_0}{b2^{t(k)}+2v+m_0}}-r2^{t(k)}\right\} \qquad (5.5)$$

From the values of (3.1) and (4.1) substitute in (5.4) and then we get the value of greater than 1. From this orthogonal SORD of second type using SUBA with two unequal block sizes has the same degree of efficiency as orthogonal SORD of first type using SUBA with two unequal block sizes, and consider the efficiency of SORD of second type using SUBA with two unequal block sizes is giving the better efficiency than SORD of first type using SUBA with two unequal block sizes. Now, the efficiency comparison of SORD of second type using SUBA with two unequal block sizes versus SORD of first type using SUBA with two unequal block sizes with rotatability.

Substituting the values of (3.2) into (5.5) and we evaluated that the SORD of second type using SUBA with two unequal block sizes will be more efficient than the previous SORD of first type using SUBA with two unequal block sizes with rotatability.

$$a_1^2+a_2^2<\frac{1}{2}\left\{\left(r2^{t(k)}+2\sqrt{\frac{(3\lambda-r)2^{t(k)}}{2}}\right)\sqrt{\frac{b2^{t(k)}+4v+n_0}{b2^{t(k)}+2v+m_0}}-r2^{t(k)}\right\} \quad (5.6)$$

For example, in SORD of first type using SUBA with two unequal block sizes when $(v=6,b=11,r=7,k_1=3,k_2=4,b_1=2,b_2=9,\lambda=4)$ and $m_0=1$ then we get M =189 then a=2.5149 and $n_0=1$, then the equation (5.6)

$$a_1^2+a_2^2<8.2727 \quad (5.7)$$

Among the rotatability of second type of SUBA with two unequal block sizes, it is easy to find an experimental plan that satisfies the equation (5.7). For example in SORD of second type using SUBA with two unequal block sizes with $a_1=1$, $a_2=2.4989$ satisfy the rotatability property equation (3.7), and $a_1^2+a_2^2=7.2445$ as it satisfies the equation (5.7) as well, it is more efficient than the rotatability SORD of first type using SUBA with two unequal block sizes. Then the relative efficiency of SORD of second type using SUBA with two unequal block sizes versus SORD of first type using SUBA with two unequal block sizes equation (5.4) is as follows.

$$\frac{(112+2(6.3247))^2(176+24+1)}{(112+2(0.7098))^2(176+12+1)}=1.2812$$

B. Comparison in the Pure Quadratic Coefficient b_{ii}

Now this time in terms of estimating the pure quadratic coefficient b_{ii}, the efficiency of SORD of second type using SUBA with two unequal block sizes g is comparing with SORD of first type using SUBA with two unequal block sizes, here the scaling factor the equation (4.2) is applied. The relative efficiency SORD of second type using SUBA with two unequal block sizes versus SORD of first type using SUBA with two unequal block sizes is as follows based on the equation (5.1)

$$E\left(\frac{\text{SORD of second type using SUBA with two unequal block sizes}}{\text{SORD of first type using SUBA with two unequal block sizes}}\right)=\frac{\sigma^2 e_1\left(\frac{r2^{t(k)}+2a^2}{b2^{t(k)}+2v+m_0}\right)^2\left(b2^{t(k)}+2v+m_0\right)}{\sigma^2 e_2\left(\frac{r2^{t(k)}+2a_1^2+2a_2^2}{b2^{t(k)}+4v+n_0}\right)^2\left(b2^{t(k)}+4v+n_0\right)}$$

$$=\frac{e_1\left(r2^{t(k)}+2a^2\right)^2\left(b2^{t(k)}+4v+n_0\right)}{e_2\left(r2^{t(k)}+2a_1^2+2a_2^2\right)^2\left(b2^{t(k)}+2v+m_0\right)} \quad (5.8)$$

Where,

$e_1=v(b_{ii})$ in SORD of first type using SUBA with two unequal block sizes.

$e_2=v(b_{ii})$ in SORD of second type using SUBA with two unequal block sizes .

For example, in SORD of second type using SUBA with two unequal block sizes of design $(v=6,b=11,r=5,k_1=3,k_2=4,b_1=2,b_2=9,\lambda=4)$, $n_o=1$, $a_1=1$, $a_2=2.4989$, $e_2=0.0078$ and in SORD of first type using SUBA with two unequal block sizes m_0=1, a=2.5149, e_1= 0.0087, lets us compare the relative efficiency of SORD of second type using SUBA with two unequal block sizes versus SORD of first type using SUBA with two unequal block sizes, if you get the equation (5.8) as 1.4327, then we conclude that the SORD of second type using SUBA with two unequal block sizes is more efficient than SORD of first type using SUBA with two unequal block sizes.

C. Comparison in Terms Estimating the first Order Coefficient b_i

It can be discussed in the same process of the $V(b_i)$ by multiplying the scaling factor then $V(b_i)=\frac{\sigma^2}{(r2^{t(k)}+2a_1^2+2a_2^2)}$ is to be multiplied by $1/g^2$ then we get, $V(b_i)=\frac{\sigma^2}{(r2^{t(k)}4v+n_0)}$ similarly the $V(b_{i)}$ is multiplied by scaling factor in SORD of first type using SUBA with two unequal block sizes the we obtained $V(b_i)=\frac{\sigma^2}{(r2^{t(k)}+2v+n_0)}$ so finally we compare the relative efficiency of $E\left(\frac{\text{SORD of second type using SUBA with two unequal block sizes}}{\text{SORD of first type using SUBA with two unequal block sizes}}\right)$ and it obtained 1, then the efficiency of SORD of second type using SUBA with two unequal block sizes is more efficient than SORD of first type using SUBA with two unequal block sizes

VI. CONCLUSION

In this paper, second order rotatable designs of SORD of second type using SUBA with two unequal block sizes in which the axial points are indicated by two numbers a_1 and a_2 and it is called as SORD of second type using SUBA with two unequal block sizes. The variance covariance of the estimated parameters are studied and we evaluated for the SORD of second type using SUBA with two unequal block sizes is most orthogonal for second order response surface designs and the results of the orthogonality are given in the table 4.1.

The comparison between the SORD of second type using SUBA with two unequal block sizes versus SORD of first type using SUBA with two unequal block sizes for different coefficients are studied then we conclude that the SORD of second type using SUBA with two unequal block sizes is more efficient than SORD of first type using SUBA with two unequal block sizes. It is convenient to use the practical situations and give the more efficiency when compared to SORD of first type using SUBA with two unequal block sizes.

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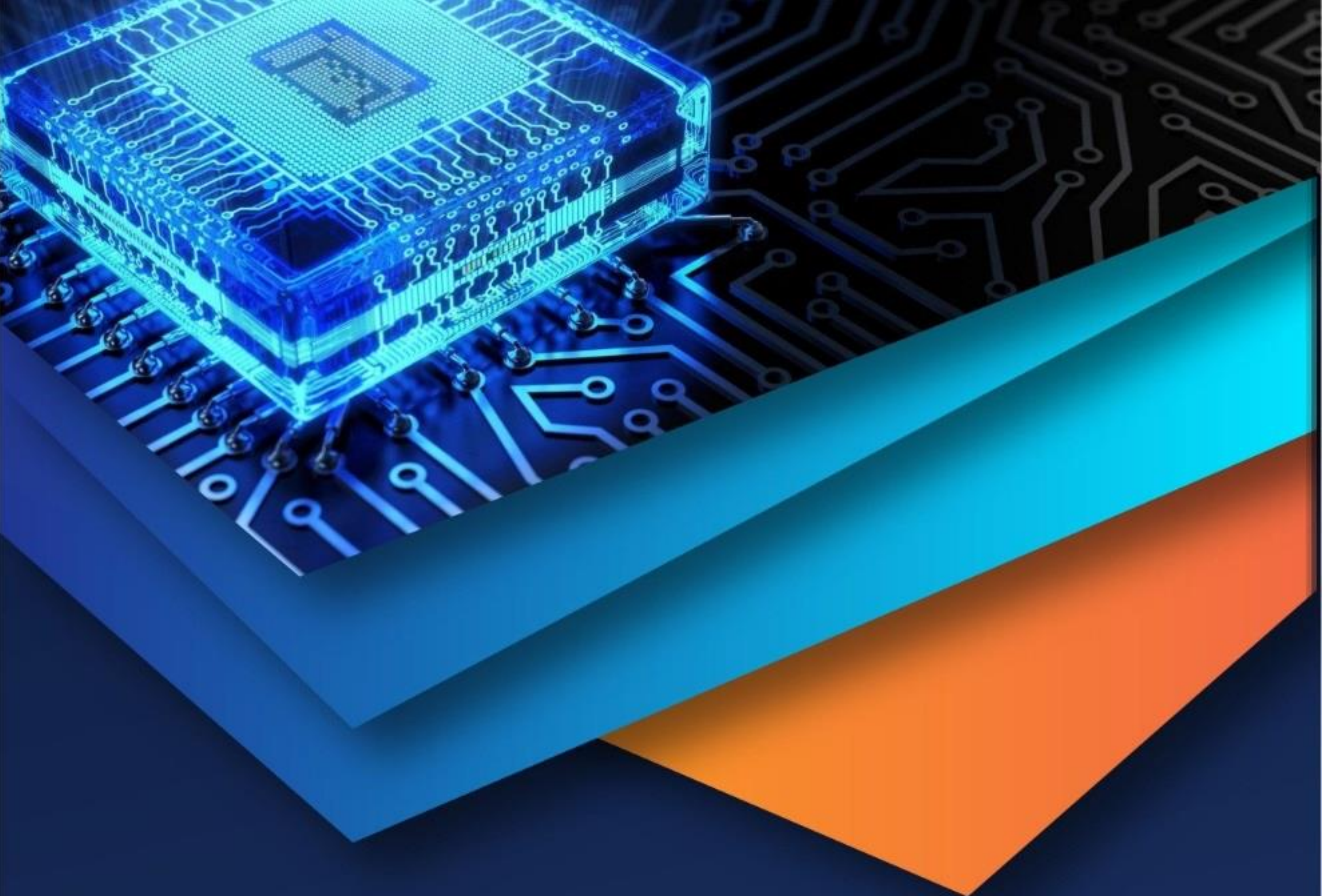

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