

# Seismic Surface Wave Analysis for Layering Information of the Crust Using Sikim Earthquake

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**Abstract**— Seismic surface wave analysis has performed for Sikim earthquake of magnitude 6.9 occurred on 18 September 2011 of 12:40:48 UTC. Group velocity dispersion has determined by graphical method. A model taking subsurface layer parameters is also constructed to compute the group velocity dispersion by modified Haskell matrix method. Group velocity dispersion by graphical method is then interpreted from model parameters. Some statistical analysis of the model are studied and presented in this research. Interpreted information of the crust for the Sikim region shows that there are four major subsurface layers of thickness 1.0 km, 2.7 km, 3.5 km and 5.5 km.

**Keywords**— Seismic wave, earthquake data, dispersion, period, Crustal layers.

## I. INTRODUCTION

Earth's crust is formed by different layers. When earthquake occurred, seismic wave is propagated into these layers and it can be collected various information about earth's crust by seismic surface wave dispersion analysis. This analysis of the Sikim earthquakes can be used for getting information of the earth's crust using simple models of continental or oceanic crust. Ewing and Press (1952) first introduced such model for the oceanic crust using rayleigh wave dispersion and later a theoretical dispersion relation has been presented for a two-layer model of continental crust ([7]).

There are two types of modeling technique that are commonly being used in determination of the crustal layers from seismic surface wave dispersion. Direct modeling technique determines the layering information of the crust from observed surface wave dispersion ([2], [3], [5], [11]). On other hand, the most widely used indirect modeling technique deals with trial-and-error procedures. Dispersion is computed for different model parameters to see how the computed dispersion matches with observed dispersion ([4]-[5]).

In this research, seismic surface wave analysis have been performed to compute group velocity dispersion for layering information of the crust using graphical method ([8]) for the up-down component of the Sikim earthquake recorded at Dhaka Meteorological Department on 18 September 2011 at 12:40:48 UTC.

## II. MATERIAL AND METHODS

### A. Earthquake data

Earthquake of magnitude 6.9 occurred on 18 September 2011 at Sikim, India-Nepal border region is 643.3 km far from the capital city Dhaka, Bangladesh and also not so far from the Himalayan frontal arch. Table 1 lists the source parameters of the selected event which is geographically located at India-Nepal border region. The event was recorded at Dhaka Meteorological Department seismic station located at 23.78° N and 90.38° E and the station is equipped with a three component digital broad-band sensor which can record up-down, north-south and east-west components. The recorded earthquake seismic wave is shown in Fig. 1.

TABLE 1: Earthquake Source parameters

Date	Origin Time	Location	Depth (Km)	Distance of epicenter (Km)	Mw
18 <sup>th</sup> September 2011	12:40:48(UTC) 18:40:48(BST)	27.723°N, 88.06°E	19.7	496.39	6.9

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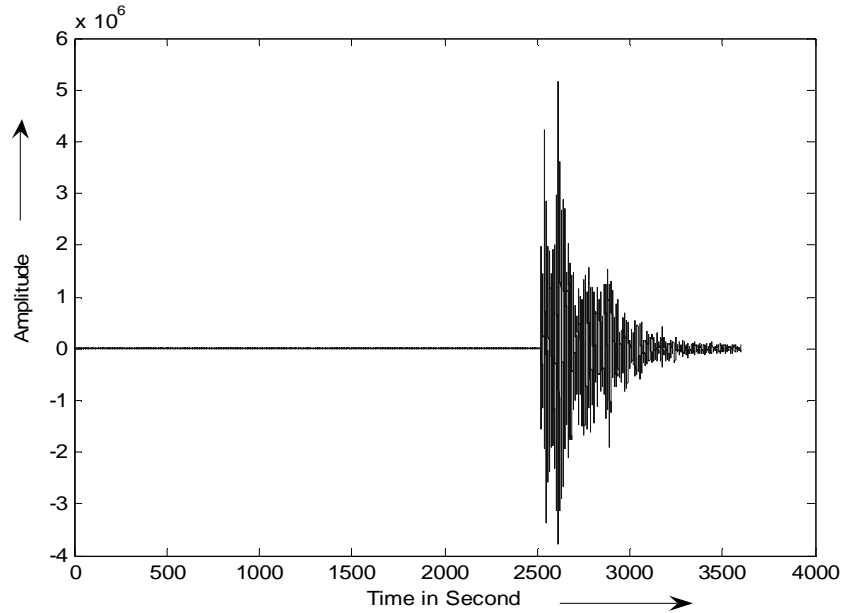


Fig. 1 Up-down ground accelerated earthquake seismic wave for Sikkim earthquake

### III.METHOD

Group velocity dispersion is considered as the factor, which has relationship with crustal structure. Group velocity from recorded earthquake wave and multilayered crustal model can be obtained respectively by graphical method and modified Haskell matrix method as explained below.

#### A. Graphical Method

Graphical method is basically a technique of group velocity dispersion determination. In this method the travel times ( $t$ ) of some chosen phases along the surface wave train are measured and plotted on a graph versus the order number ( $n$ ) of the chosen phases. Usually the travel times of the wave crests and troughs are read. The  $(n, t)$  curve built by these points is then approximated with linear segments. The period is determined by the slope of these lines and the corresponding travel times are read from the midpoints of the segments ([8]). The group velocity,  $U_g$  of seismic surface wave can be obtained as:

$$U_g = \Delta/t \quad (1)$$

Where  $\Delta$  is epicentral distance and  $t$  is the travel time.

### IV.MODIFIED HASKELL MATRIX METHOD

Modified Haskell matrix method for the case of  $n-1$  homogeneous, isotropic elastic layers over a half-space matrix can be written as ([13]):

$$J = \widehat{E}^n A^{n-1} \dots A^m \dots A^2 \cdot A^1 \quad (2)$$

Where  $A^m$  is the 4X4 Haskell matrix for the m'th layer and  $\widehat{E}^n$  is the half-space inversion matrix. Then the secular function (dispersion relation) can be written as:

$$\Delta(T, C) = J \Big|_{12}^2 = J_{11} J_{22} - J_{12} J_{21} = 0 \quad (3)$$

Hence two columns or rows of  $J$  are necessary for this result and it requires a 2X4 matrix to store the product as each layer. It has been seen that Haskell matrix poses a loss of significant figures in the secular function ([6], [10], [12]). In order to minimize the losses a 6X6 matrix is employed where the elements are second order sub-determinant of the Haskell matrix. The matrix can be

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written as:

$$R_i^m = \sum_{j=1}^6 B_{ij}^m R_j^{m-1} \tag{4}$$

Where  $B_{ij}^m = A^m \Big|_{kl}^{ij}$  (which is a matrix conversion from 4X4 to 6X6 Hence the secular function,  $\Delta(T, c) = R^n B^n = 0$ ) (5)

$B^n$  is the matrix of sub-determinants of the half-space  $\widehat{E}^n$ .

However, B matrix shows that

$$R_i^1 = B_{i1}^1 \tag{6}$$

and  $R_3^m = R_4^m$  [Using Eqn. (4)]

This phenomenon leads to define a 5X5 matrix  $\widehat{B}$  rather than 6X6 matrix and the Modified  $\widehat{B}$  matrix can be expressed as:

$$\widehat{B} = \begin{bmatrix} B_{11} & B_{12} & 2B_{13} & B_{15} & B_{16} \\ B_{21} & B_{22} & 2B_{23} & B_{25} & B_{26} \\ B_{31} & B_{32} & (2B_{33}-1) & B_{35} & B_{36} \\ B_{51} & B_{52} & 2B_{53} & B_{55} & B_{56} \\ B_{61} & B_{62} & 2B_{63} & B_{65} & B_{66} \end{bmatrix} \tag{7}$$

and similarly  $R_i^m$  is thus reduced from a six dimensional vector to five.

Dispersion relation (Equation 5) can be solved numerically according to the model parameters ( $V_p$ ,  $V_s$ ,  $\rho$  and thickness) in the form of group velocity versus time period plot. On other hand same plot can also be obtained from the recorded earthquake data using graphical method. Hence crustal interpretations are now possible in an indirect way by matching both dispersion relations.

### V. MODEL ERROR ESTIMATION

The aim of the current research is to study the crustal structure / shallow depth hence the depth of 54.0 km is being considered here.

The S-wave velocities of the layers are free to change during the inversion. Consequently, the P-wave velocities are estimated using the  $V_p/V_s$  ratio 1.732. Poisson's ratio ( $\sigma$ ) in each layer was assumed to be .25 and the densities ( $\rho$ ) are calculated from the P-wave velocities ( $V_p$ ) using the relation  $0.32V_p+0.77$  ([1]). Starting from initial estimates, the model parameters are iteratively improved until a good fit between the theoretical and observed dispersion curves is obtained. During the inversion, a number of criteria were adapted to calculate the goodness of fit. These criteria are the standard error of estimate (SE), mean residual (MR), average absolute residual (AR), weighted root mean square error (RMS) and the percent of signal power fit (SPF). These criteria are computed by [9]:

$$SE = \sqrt{\frac{\sum_{i=1}^N (obs_i - mean)^2}{N - 1}} \tag{8}$$

$$MR = \frac{\sum_{i=1}^N (obs_i - pred_i)}{N} \tag{9}$$

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$$AR = \frac{1}{N} \left| \sum_{i=1}^N \frac{(obs_i - pred_i)}{SE} \right| \quad (10)$$

$$RMS = \sqrt{\frac{\sum_{i=1}^N \frac{(obs_i - pred_i)^2}{mean}}{N(N-1)}} \quad (11)$$

$$SPF = \left( 1 - \frac{\sum_{i=1}^n (obs_i - pred_i)^2}{\sum_{i=1}^N (obs_i)^2} \right) \times 100 \quad (12)$$

Where ‘obs’ is the observed group velocity at each period, ‘mean’ is the mean of the observed group velocities, N is the number of observations at each period and ‘pred’ is the predicted group velocity of the current model. Estimated errors of the models are shown in table 3.

### VI. CRUSTAL THICKNESS MEASUREMENT

Group velocity dispersions are estimated in this section using graphical method (Eqn.1) and Haskell modified matrix method (Equation. 5) as discussed below:

#### A. Group Velocity Estimation from Earthquake data

Group velocity is computed for the earthquake data recorded at Dhaka University seismic station, Bangladesh (located at 23°44.10' N and 90°23.45' E) equipped with a three component digital broad-band sensor (Figure 1) and earthquake source parameters are shown in table 1.

Figure 3 shows order number (n) versus travel time (t) plot and Figure 4 shows group velocity variation with time period. This dispersion relation is computed by graphical method (Equation 1).

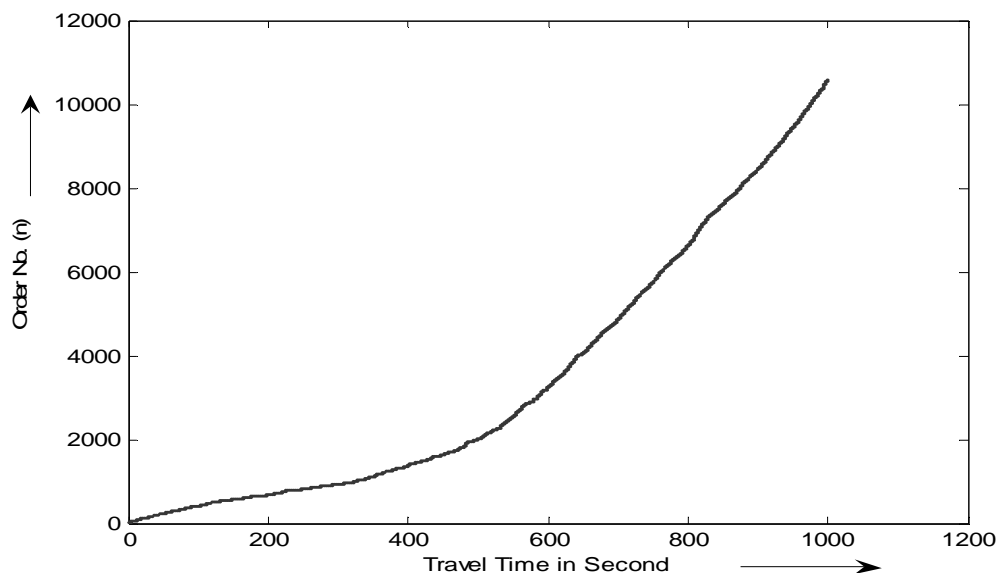


Fig. 2 Order number versus travel time curve for Sikim earthquake data

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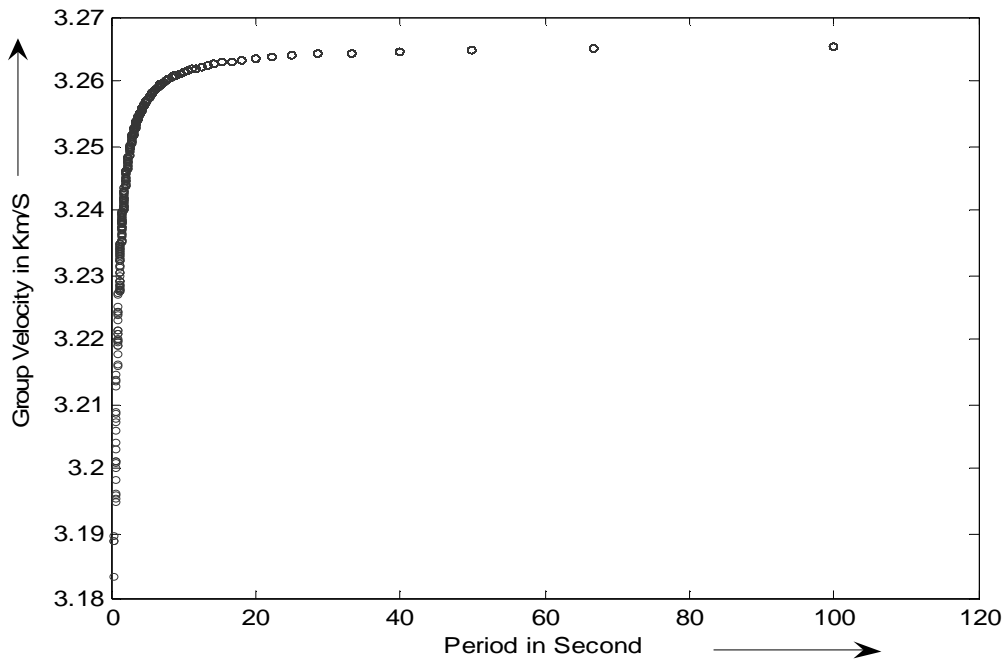


Fig. 3 Group velocity dispersion curve for Sikim earthquake data

### VII. GROUP VELOCITY ESTIMATION FROM MODEL

There are eight models (A-H) are considered in this work. Model based group velocity is computed using modified Haskell matrix method (Equations 2-7). The computed group velocity according to model parameters are shown in Figures. 5-12 also show the group velocity computed by graphical method

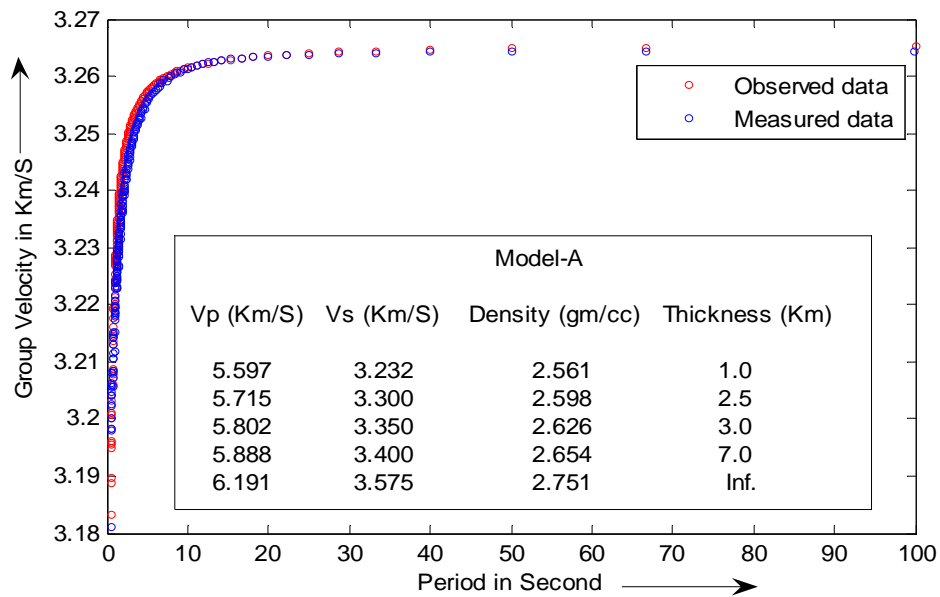


Fig. 4 Group velocity dispersion obtained from Sikim earthquake data and from modeling A. Rectangular box contained the model parameters.

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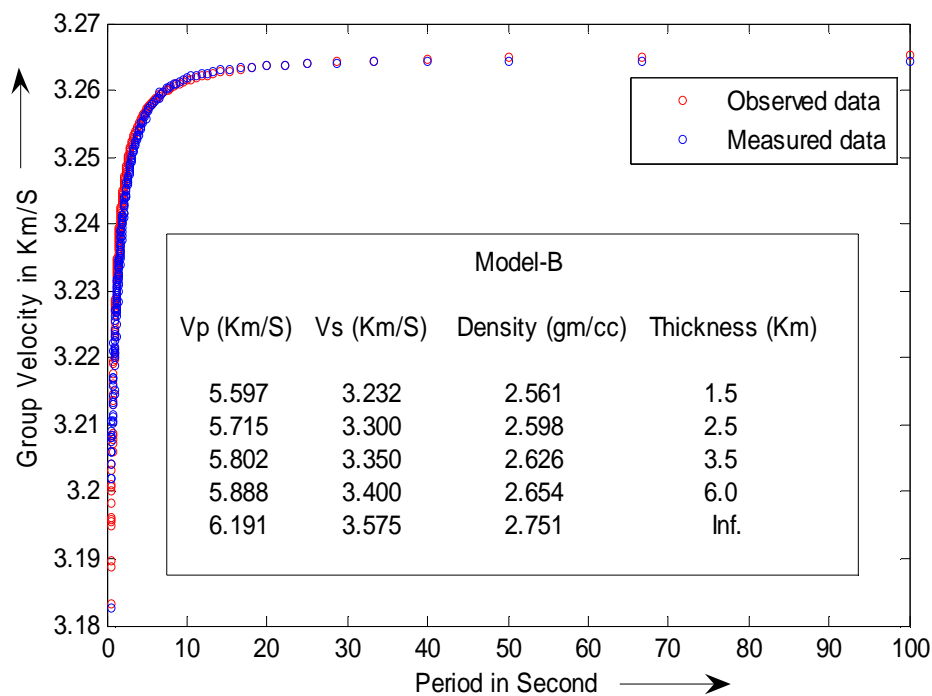


Fig. 5 Group velocity dispersion obtained from Sikim earthquake data and from modeling B. Rectangular box contained the model parameters.

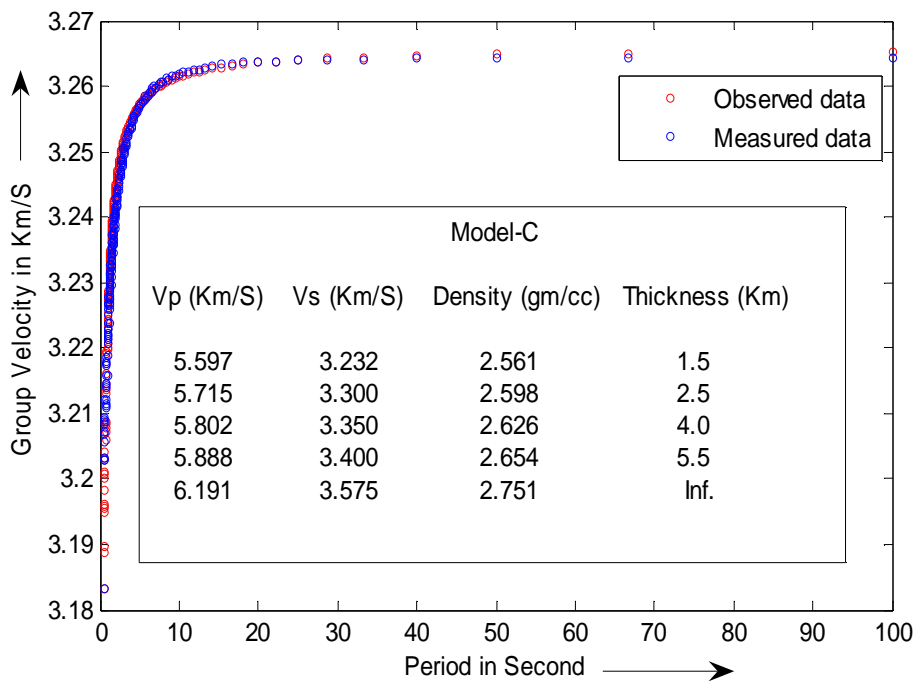


Fig.6 Group velocity dispersion obtained from Sikim earthquake data and from modeling C. Rectangular box contained the model parameters.

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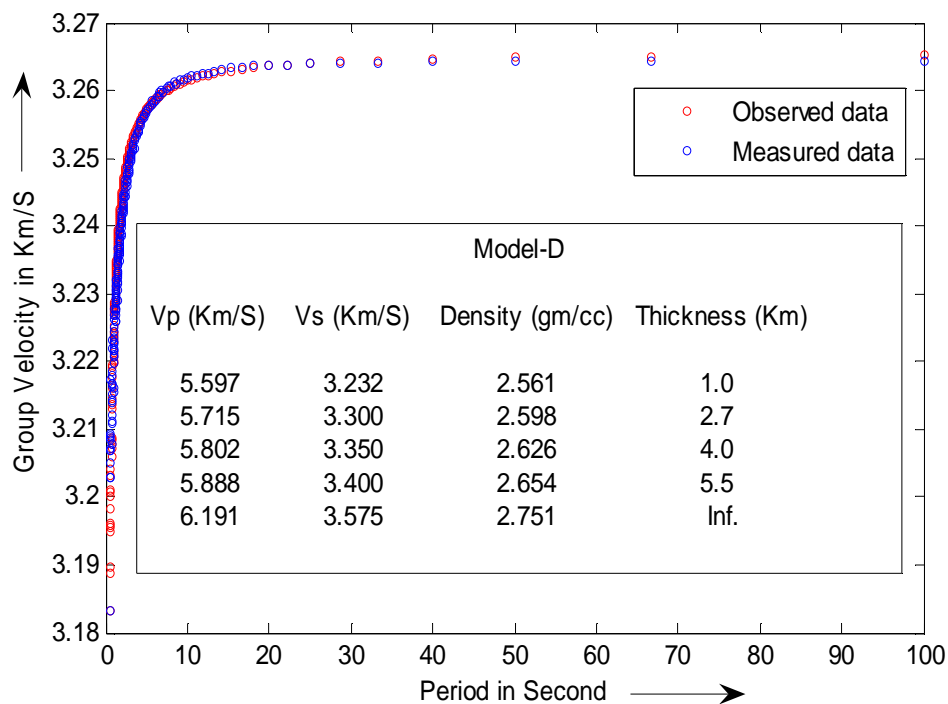


Fig. 7 Group velocity dispersion obtained from Sikim earthquake data and from modeling D. Rectangular box contained the model parameters.

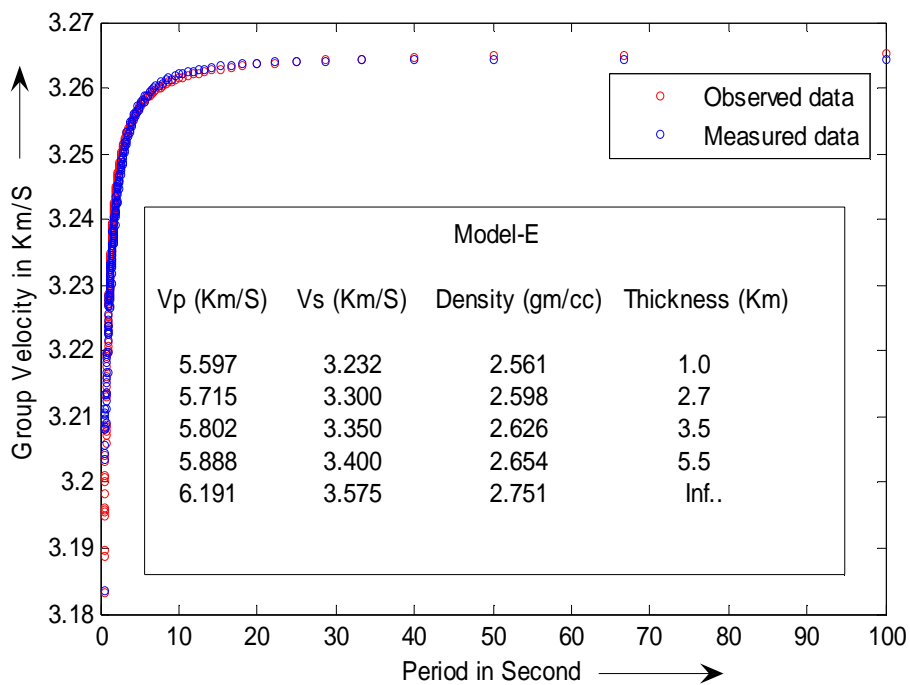


Fig. 8 Group velocity dispersion obtained from Sikim earthquake data and from modeling E. Rectangular box contained the model parameters.

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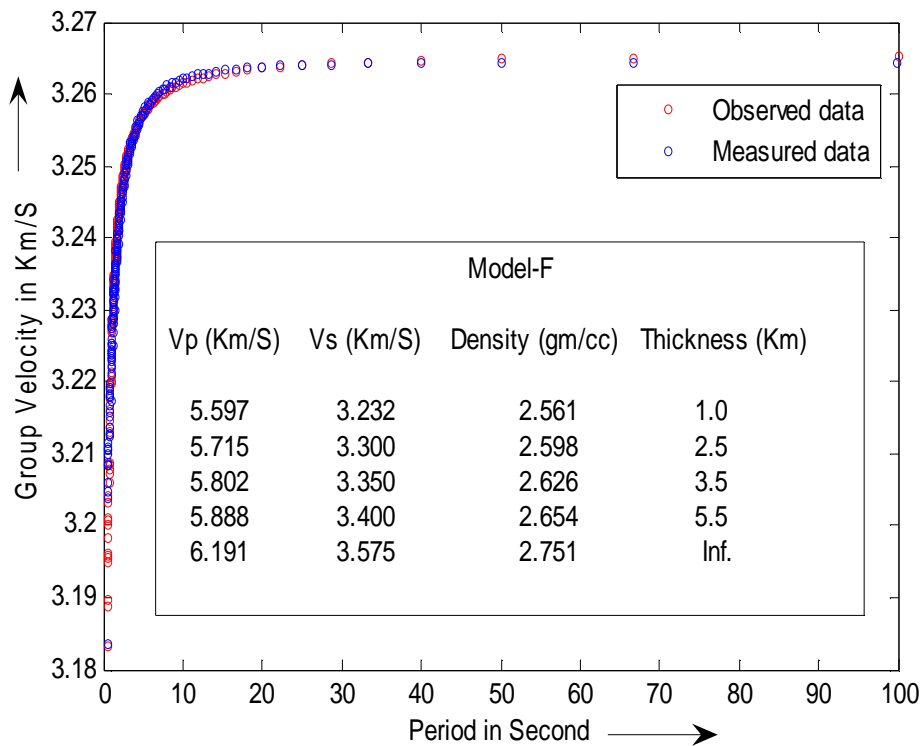


Fig. 9 Group velocity dispersion obtained from Sikim earthquake data and from modeling F. Rectangular box contained the model parameters.

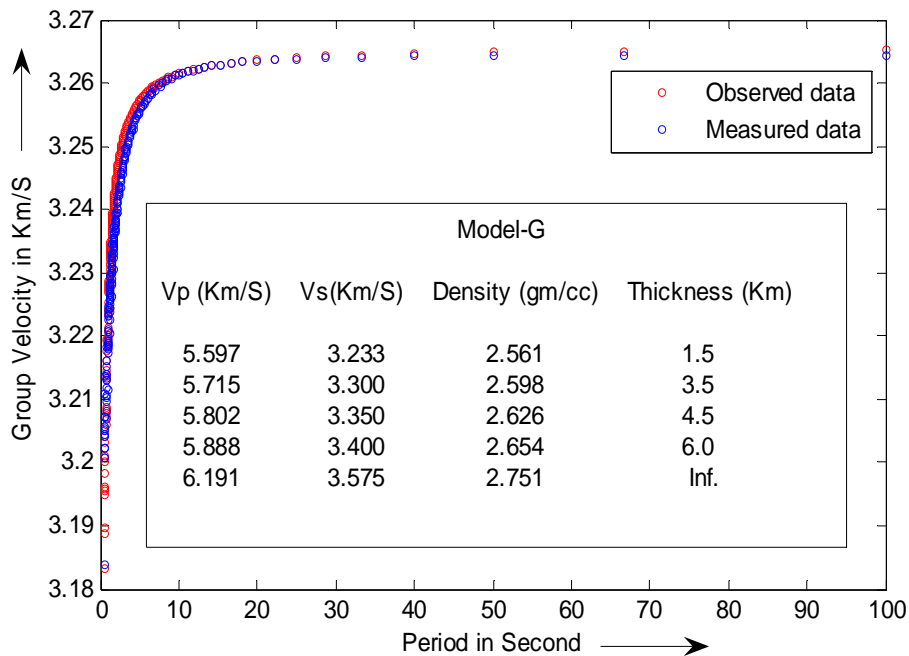


Fig. 10 Group velocity dispersion obtained from Sikim earthquake data and from modeling G. Rectangular box contained the model parameters.



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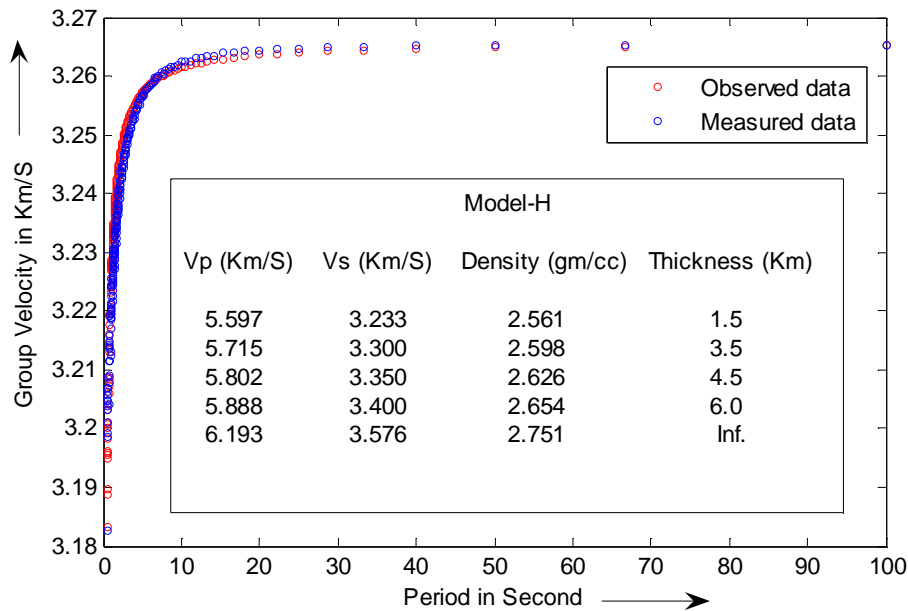


Fig. 11 Group velocity dispersion obtained from Sikim earthquake data and from modeling H. Rectangular box contained the model parameters.

### VIII. RESULT

From the modeling subsurface layers are estimated. It has seen that (Figures 5-12) group velocity obtained from earthquake data and from models have the similar characteristics as both are varying with period, and to a reasonable maximum velocity. Therefore, interpretations are made from model parameters as shown in the rectangular box in Figs. 4-11. None of the plots is found matched exactly and it should not match as the models consider only four variables, in fact there should be few more variables. Hence, statistical errors are analyzed as explained in section (4). Using Equations 8-12 the computed errors are shown in Table2.

TABLE 2  
 DATA FIT CRITERIA:

Model	SE	MR	AR	RMS	SPF
A	0.0055769	0.0034973	0.6271033	0.0000836	99.99975261
B	0.0055769	0.0015772	0.2828145	0.0001151	99.99984793
C	0.0055769	0.0010712	0.1920879	0.0001371	99.99987718
D	0.0055769	0.0010064	0.1804732	0.0001410	99.99985248
E	0.0055769	0.0003833	0.0687316	0.0002217	99.99987800
F	0.0055769	0.0001686	0.0302475	0.0003283	99.99987551
G	0.0055769	0.0031073	0.5571729	0.0000880	99.99975360
H	0.0055769	0.0027031	0.4847062	0.0000341	99.99980018

According to estimated statistical errors (Table 2) the model E is found more acceptable. Hence it can be said that the Sikim, India-Nepal border region earthquake wave is indicated that there are four major subsurface layers and layer thicknesses are shown in Figure 8.

### IX. CONCLUSIONS

In real cases the Poisson's ratio of 0.25 might be different for different subsurface layers and hence the interpreted crustal structure from model might not be appropriate. However, for the computational advantages  $V_p/V_s$  ratio or Poisson's ratio were kept fixed as it has seen in many contributions to use the value of 1.732 or 0.25 respectively ([1]).

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On other hand, it has shown that the thickness of the layers is a vital factor therefore, thickness setting in the model is also found to be difficult. However, from the investigations it is revealed that the setting of total depth rather than individual thicknesses of the subsurface layers can provide better interpretations that are more acceptable. Hence, total depth of 15.5 km is considered in our models. Instead of above limitations interpretation made from the four models are seemed good enough with the group velocities obtained by graphical methods as shown in Figs. 4-11. Group velocity dispersion from the eight models (A-H) (Figs. 4-11) and considering statistical error analysis (Table 2), it can be said that all the models are very nearer to an acceptable matching level though the statistical confidence level SPF should be 91.5% but our results are around 99.99987800%. Considering all errors studying in this research (Table. 2) model E is seemed more acceptable of Sikim, India-Nepal border region. Hence the interpreted subsurface layers of the studied Sikim earthquake data shows that there are four major subsurface layers having respectively the thickness and density of 1.0 km, 2.561 gm/cc; 2.7 km, 2.598 gm/cc; 3.5 km 2.626 gm/cc; 5.5 km, 2.654 gm/cc.

### X. ACKNOWLEDGMENT

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