iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 9 **Issue:** IV **Month of publication:** April 2021

DOI: https://doi.org/10.22214/ijraset.2021.33537

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue IV Apr 2021- Available at www.ijraset.com

Estimation of Shape (α) and Scale (λ) of Two-Parameter Lomax Distribution (LD) using Median Ranks Method

CH. V. Aruna[1], O. V. Raja Sekharam[2], G. V. S. R. Anjaneyulu[3]
[1]*Research Scholar (BSRRFSMS), Department of Statistics, Acharya Nagarjuna University, Guntur, Andhra Pradesh, India*
[2]*Guest faculty at AKTP University, Andhra Pradesh, India*
[3]*Professor Department of Statistics, Acharya Nagarjuna University, Guntur, Andhra Pradesh, India*

Abstract: ***The objective of this paper is to estimate the parameters of Lomax distribution with shape parameter(α) and scale parameter(λ). In this heuristic algorithm, the Median Rank regression method is used to Estimates the parameters of Lomax Distribution. We also computed Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Error (RE) and Relative Absolute Bias (RAB) for both the parameters under grouped sample based on 1000 simulations to assess the performance of the estimators.***
Keywords: Lomax Parameters, Grouped sample, Median Rank regression method, simulation.

I. INTRODUCTION

The Lomax distribution, conditionally also called the Pareto Type II distribution, is a heavy-tail probability distribution used in business, economics, actuarial science, queueing theory and Internet traffic modeling. It is named after K. S. Lomax. It is essentially a Pareto distribution that has been shifted so that its support begins at zero.

The Lomax distribution, also called "Pareto type II" distribution is a particular case of the generalized Pareto distribution (GPD). The Lomax distribution has been used in the literature in a number of ways. For example, it has been extensively used for reliability modelling and life testing; see, for example, Balkema and de Haan [1]. It also has been used as an alternative to the exponential distribution when the data are heavy tailed; see Bryson [2]. Ahsanullah [3] studied the record values of Lomax distribution. Balakrishnan and Ahsanullah [4] introduced some recurrence relations between the moments of record values from Lomax distribution. The order statistics from nonidentical right-truncated Lomax random variables have been studied by Childs et al. [5]. Also, the Lomax model has been studied, from a Bayesian point of view, by many authors; see, for example, Arnold et al. [6] and El-Din et al. [7]. Howlader and Hossain [8] presented Bayesian estimation of the survival function of the Lomax distribution. Ghitany et al. [9] considered Marshall-Olkin approach and extended Lomax distribution. Cramer and Schmiedt [10] considered progressively type-II censored competing risks data from Lomax distribution. The Lomax distribution has applications in economics, actuarial modelling, queuing problems and biological sciences; for details, we refer to Johnson et al. [11]. In this paper, we develop a practical approach for estimating the shape(α) and scale (λ) parameters of the LD using the median rank regression method. We therefore employ these estimates as the median ranking method. We present the observations and the conclusions based on the simulation results.

II. MEDIAN RANK REGRESSION METHOD TO ESTIMATE THE LOMAX DISTRIBUTION PARAMETERS:

Let $x_1, x_2,..., x_n$ be a random sample of size n from LD(α, λ), its Probability Density Density Function (PDF) and cumulative distribution function(CDF) are given by

$$f(x)=\frac{\alpha}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\alpha+1)};x \geq 0, \alpha> 0, \lambda>0 \qquad (2.1)$$

α=shape parameter
λ=scale parameter

$$\text{CDF} \quad F(X)=1-\left(1+\frac{X}{\lambda}\right)^{-\alpha} \qquad (2.2)$$

$$1\text{-}F(X)=\left(1+\frac{X}{\lambda}\right)^{-\alpha} \qquad (2.3)$$

Appling log on both sides to (1.2.3)

©IJRASET: All Rights are Reserved

$$\text{Log}(1\text{-F(X)}) = -\alpha \log\left(1 + \frac{x}{\lambda}\right) \quad (2.4)$$

$$\text{Log}(1\text{-F(X)}) = -\alpha \log x + \alpha \log \lambda \quad (2.5)$$

Let us consider U=log(1-F(X)), V= logx, B=- α, A= $\alpha \log \lambda$ (2.6)

F (x) is obtained by using the median rank method (also known as Benard's approximation), which is a good estimate of the median rank estimate. Benard's median rank was used because it showed the best performance and was the most used rank for prediction (F (x)). The procedure for ranking the complete data is as follows:

1) List the time to failure data from small to large.

2) Use Benard's formula to assign median ranks to each failure.

3) Estimate the α and λ by Equations (1.2.6)

4) $F(x) = \dfrac{i - 0.3}{N + 0.4}$, i= 1, 2, ..., N.

Where i $\rightarrow$ The ranked position of data point and

N $\rightarrow$ The total number of units in the sample

$$\tilde{A} = \frac{\left(\sum_{i=1}^{n} v_i^2\right)\left(\sum_{i=1}^{n} u_i\right) - \left(\sum_{i=1}^{n} v_i\right)\left(\sum_{i=1}^{n} v_i u_i\right)}{\left(n\sum_{i=1}^{n} v_i^2\right) - \left(\sum_{i=1}^{n} v_i\right)^2} \quad (2.7)$$

$$\tilde{B} = \frac{n\left(\sum_{i=1}^{n} v_i u_i\right) - \left(\sum_{i=1}^{n} v_i\right)\left(\sum_{i=1}^{n} u_i\right)}{\left(n\sum_{i=1}^{n} v_i^2\right) - \left(\sum_{i=1}^{n} v_i\right)^2} \quad (2.8)$$

F(X) is estimated from the median ranks. Once $\tilde{A}$ and $\tilde{B}$ obtained, the values of $\hat{\alpha}$ and $\hat{\lambda}$ can easily be obtained.

A. Simulation Study

In order to obtain the median ranks method estimators of shape(α)and Scale (λ)is used to obtain estimators and to study their predictive properties by Average Estimate (AE), Variance (VAR), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE). If $\hat{\psi}_{lm}$ is Median Ranks Method estimate of $\hat{\psi}_m$, m=1, 2 where $\psi_m$is a general notation that can be replaced by $\psi_{m_1} = \lambda, \psi_{m_2} = \alpha$ based on sample l, (l=1,2,…,r), then the Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) and Relative Error (RE) are given respectively by

$$\text{Average Estimate } (\hat{\psi}_m) = \frac{\sum_{i=1}^{r} \hat{\psi}_{lm}}{r}$$

$$\text{Variance}(\hat{\psi}_m) = \frac{\sum_{i=1}^{r} (\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}})^2}{r}$$

$$\text{Mean Absolute Deviation} = \frac{\sum_{i=1}^{r} Med(|\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}}|)}{r}$$

$$\text{Mean Square Error } (\hat{\psi}_m) = \frac{\sum_{i=1}^{r} (\hat{\psi}_{lm} - \psi_m)^2}{r}$$

$$\text{Relative Absolute Bias}(\hat{\psi}_m) = \frac{\sum_{i=1}^{r} |(\hat{\psi}_{lm} - \psi_m)|}{r\psi_m}$$

$$\text{Relative Error}(\hat{\psi}_m) = \frac{1}{r}\left(\frac{\sum_{i=1}^{r} MSE\sqrt{(\hat{\psi}_{lm})}}{\psi_m}\right)^2$$

©IJRASET: All Rights are Reserved

B. Simulated Data Sets

We evaluate the performance of the Median Ranks Regression (MRR) method for estimating the LD (α, λ) and Newton-Raphson simulation for two parameter combinations, and the process is repeated 10,000 times for different sample sizes n = 50 (50) 500 considered.

MRR and their Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB), and Relative Error (RE) of the median rank method to estimators of scale and Location parameters. Population parameters shape=1 and Scale =1

Table-1.1
Median Ranks Regression-Lomax Distribution

SAMPLE SIZE	PARAMETERS	AE	VAR	MAD	MSE	RAB	RE
2	$\hat{\alpha}$	0.949014	0.006807	0.065571	0.009406	0.052457	0.06442
	$\hat{\lambda}$	0.858912	0.034778	0.006557	0.000941	0.005246	0.006442
5	$\hat{\alpha}$	0.924035	0.009034	0.077392	0.014805	0.075966	0.095412
	$\hat{\lambda}$	0.772243	0.053297	0.007739	0.10517	0.227757	0.500607
10	$\hat{\alpha}$	0.772963	0.100018	0.249626	0.151563	0.227037	1.205526
	$\hat{\lambda}$	0.676434	0.060494	0.168889	0.015156	0.323566	2.968899
15	$\hat{\alpha}$	0.933544	0.006049	0.016889	0.001516	0.032357	0.29689
	$\hat{\lambda}$	0.093354	0.000605	0.001689	0.000152	0.003236	0.029689
20	$\hat{\alpha}$	0.009335	0.000314	0.014465	0.001023	0.026625	0.027698
	$\hat{\lambda}$	0.857341	0.005762	0.060141	0.000102	0.002663	0.00277
25	$\hat{\alpha}$	0.085734	0.000576	0.006014	0.00381	0.000266	0.000277
	$\hat{\lambda}$	0.008573	0.015472	0.000601	0.059384	0.209551	0.305773
50	$\hat{\alpha}$	0.867349	0.006762	0.014475	0.151663	0.032457	0.500807
	$\hat{\lambda}$	0.083354	0.004605	0.001589	0.045152	0.003936	0.027689
100	$\hat{\alpha}$	0.779963	0.108718	0.246726	0.153563	0.287037	1.805526
	$\hat{\lambda}$	0.063354	0.030605	0.001349	0.034152	0.053236	0.049689

We evaluate the performance of the Median Ranks Regression (MRR) method for estimating the Rayleigh distribution with shape (α) and Scale (λ) parameters, Newton-Raphson simulation for a two parameter combinations was done and the process is repeated 10,000 times for different sample sizes n=50(50)500.

The MRR and their Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and 15 Relative Error (RE) of Median Ranks method for the estimators of location and scale parameters of the Rayleigh distribution are evaluated. The simulated results were presented in Table-1.2 for the values of the parameters Location= 1and Scale =0.5.

©IJRASET: All Rights are Reserved

Table-1.2
Median Ranks Regression-Lomax Distribution

SAMPLE SIZE	PARAMETERS	AE	VAR	MAD	MSE	RAB	RE
2	$\hat{\alpha}$	0.931207	0.004313	0.055476	0.009045	0.068793	0.079629
	$\hat{\lambda}$	0.244503	0.276874	0.489005	0.625257	0.766492	1.214006
5	$\hat{\alpha}$	0.834645	0.016523	0.119415	0.043865	0.165355	0.227061
	$\hat{\lambda}$	0.17038	0.251041	0.392638	1.112769	0.988861	2.979062
10	$\hat{\alpha}$	0.851271	0.050208	0.105708	0.033906	0.148729	0.193201
	$\hat{\lambda}$	0.176112	0.242159	0.380446	1.062225	0.971663	2.591593
15	$\hat{\alpha}$	0.865005	0.009861	0.095452	0.028084	0.134995	0.170988
	$\hat{\lambda}$	0.183196	0.240189	0.378812	1.009227	0.950411	2.455834
20	$\hat{\alpha}$	0.036639	0.048038	0.091933	0.201845	0.190082	0.691167
	$\hat{\lambda}$	0.187974	0.009608	0.018387	0.040369	0.038016	0.138233
25	$\hat{\alpha}$	0.037595	0.001922	0.003677	0.008074	0.007603	0.027647
	$\hat{\lambda}$	0.002506	0.000384	0.000735	0.001615	0.001521	0.005529
50	$\hat{\alpha}$	0.965005	0.099861	0.395452	0.026084	0.194995	0.140988
	$\hat{\lambda}$	0.1673196	0.246189	0.368812	1.049227	0.850411	3.55834
100	$\hat{\alpha}$	0.934645	0.018523	0.219415	0.043965	0.175355	0.286061
	$\hat{\lambda}$	0.18038	0.351041	0.398938	1.113769	0.987661	2.679062

III. OBSERVATIONS AND CONCLUSIONS FROM SIMULATION RESULTS

A. *Observations*

1) The Average Estimate (AE), Variance (VAR), Standard deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error(RE) are independent of true values of the parameters of shape (α) and Scale (λ).

2) Average Estimate (AE) of shape parameter (α) and Scale parameter (λ) by Median Ranks Method are increasing when we take large number of observations.

3) Average Estimate (AE) of shape parameter (α) and Scale parameter (λ) by Median Ranks Method are decreasing when we take small number of observations.

4) Variance (VAR) of shape parameter (α) and Scale parameter (λ) by Median Ranks Method are decreasing when sample size (n) is increasing.

5) Mean Absolute Deviation (MAD) of shape parameter (α) and Scale parameter (λ) by Median Ranks Method are decreasing when sample size (n) is increasing in most of the situations.

6) Mean Square Error (MSE) of shape parameter (α) and Scale parameter (λ) by Median Ranks Method are decreasing when sample size (n) is increasing.

7) Relative Absolute Bias (RAB) of shape parameter (α) and Scale parameter (λ) by Median Ranks Method are decreasing when sample size (n) is increasing .

8) Relative Error (RE) of shape parameter (α) and Scale parameter (λ) by Median Ranks Method are decreasing when sample size (n) is increasing .

9) When we take least values the results of AE, VAR ,STD ,MAD ,MSE ,RAB ,RE are very low when comparing to highest values result.

B. *Remarks*

In this Median Ranks Regression Estimation method the parameters in Lomax Distribution it is observed that

1) Variances of the shape parameter (α) and Scale parameter (λ) are decreasing when Sample size (n) increases.

2) When compared with small sample sizes, the shape parameter (α) and Scale Parameter (λ) estimators in large sample are more efficient.

©IJRASET: All Rights are Reserved

REFERENCES

[1] Estimation of Location and Scale parameter for Two-Parameter Rayleigh Distribution by Median Rank Regression Method, M. Vijayalakshmi; O. V. Raja Sekharam; G. V. S. R. Anjaneyulu, Volume 7 Issue 1, January 2018

[2] Estimation of Scale (θ) and Shape (α) Parameters of Power Function Distribution by Median Ranks Regression Method using Optimally Constructed Grouped Data O. V. Raja Sekharam1, M. VijayaLakshmi2, G.V. S. R. Anjaneyulu3 Volume 8 Issue I, Jan 2020-Available at www.ijraset.com

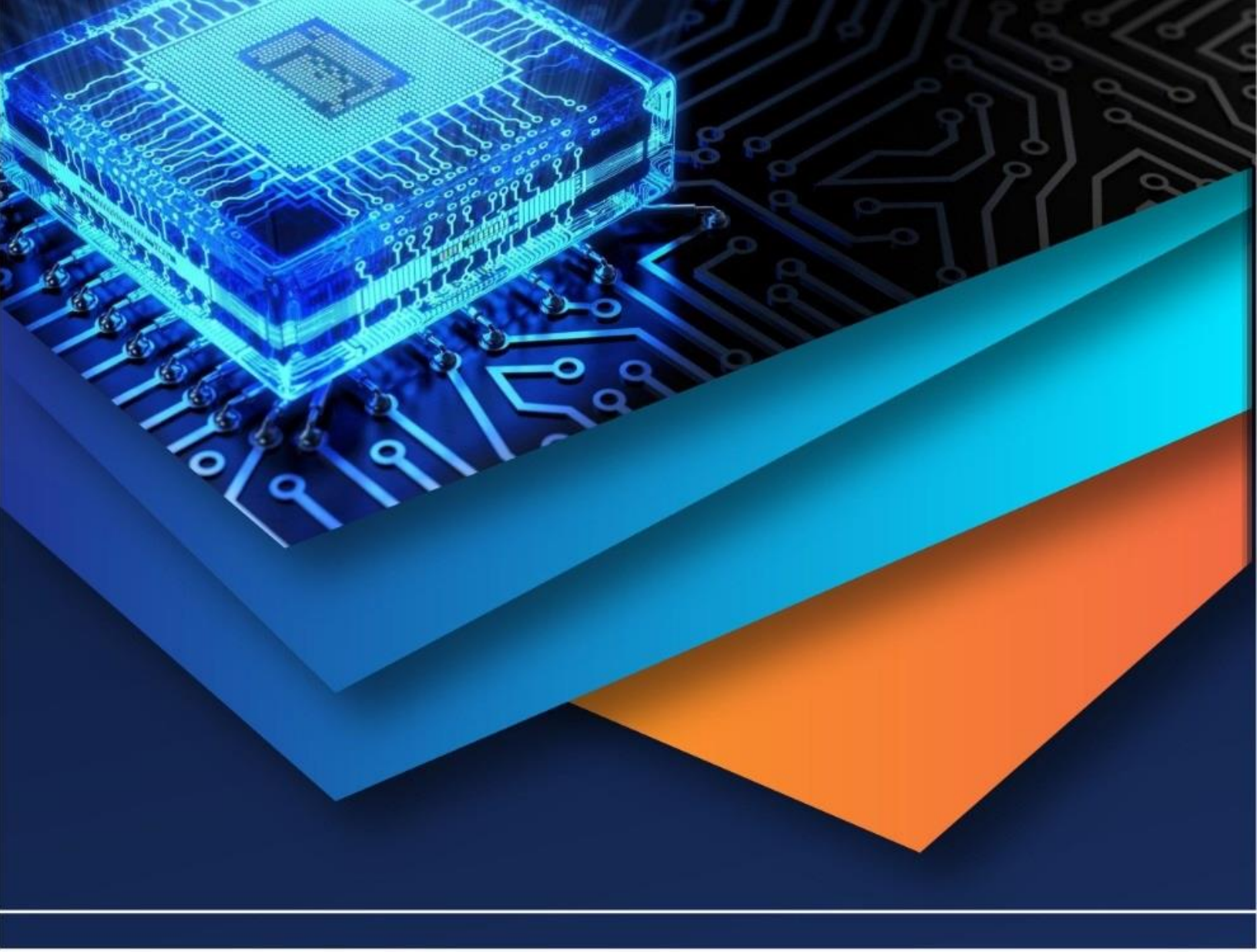
[3] Estimation of Location (μ) and Scale (λ) for Two-Parameter Half Logistic Pareto Distribution (HLPD) by Median Rank Regression Method. GVSRA M. Vijayalakshmi, International Journal of Research and Analytical Reviews 6 (6), 316-322

[4] Estimation of Location (θ) and Scale (λ) of Two-Parameter Half Logistic -Rayleigh Distribution (HLRD) Using Median Ranks and Least Square Regression Methods Authors :K. Sai Swathi and G. V. S. R. Anjaneyulu, INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH

[5] Swain, J., Venkataraman, S. & Wilson, J.R. (1988). Least squares estimation of distribution functions in Johnson's translation system. Journal of Statistical Computation and Simulation, 29, 271 – 297

[6] Abd-Elfattah, A. M., Hassan, A. S. &Ziedan, D. M. (2006). Efficiency of maximum likelihood estimators under different censored sampling schemes for Rayleigh distribution. Interstat, March issue, 1.

©IJRASET: All Rights are Reserved



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089 (24*7 Support on Whatsapp)