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Divisor Cordial Labeling Of Cycle Related Graphs

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Abstract- A divisor cordial labeling of a graph G is a bijection f from V (G) to {1, 2, . . . , |V (G)|} such that an edge uv is assigned the label 1 if f (u)| f (v) or f (v)| f (u) and the label 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a divisor cordial labeling is called a divisor cordial graph. In this paper we prove that cycle with one chord, cycle with twin chords, barycentric subdivision of cycle with one chord, barycentric subdivision of cycle with twin chords, vertex switching of cycle with one chord and vertex switching of cycle with twin chords are divisor cordial graphs.
Keywords— Divisor cordial labeling, Barycentric subdivision, Vertex switching. AMS Subject classification number: 05C78.

I. INTRODUCTION

The graphs considered here are finite, connected, undirected and simple. The vertex set and edge set of a graph G are denoted by V (G) and E (G) respectively. For various graph theoretic notations and terminology we follow Gross and Yellen[2]. The concept of divisor cordial labeling was introduced by R. Varatharajan, S. Navanaeethakrishnan, K. Nagarajan[4] and they proved that paths, cycles, wheels, stars are divisor cordial. They also proved that complete graphs Kn, $n \geq 4$ are not divisor cordial. In addition they also proved that dragons, coronas, wheels, and complete binary trees are divisor cordial. Vaidya and Shah[3] proved that the splitting graphs of stars and bistars are divisor cordial. They also proved that the shadow graphs and the squares of bistars are divisor cordial. A dynamic survey of graph labeling is published and updated every year by Gallian[1]. In this paper we prove that cycle with one chord, cycle with twin chords, vertex switching of cycle with one chord, vertex switching of cycle with twin chords, barycentric subdivision of cycle with one chord, barycentric subdivision of cycle with twin chords are divisor cordial graphs.

Note: Fundamental Theorem of Arithmetic

Any natural number (except for 1) can be expressed as the product of primes. For each natural number such an expression is unique.

II. MAIN RESULTS

Theorem 1. Cycle with one chord is divisor cordial, where chord forms a triangle with two edges of the cycle.

Proof. Let $v_1, v_2, \dots, v_n$ be the successive vertices of cycle C_n and $e = v_2 v_n$ be the chord of cycle C_n, where the edges $e_1 = v_1v_2$, $e_2 = v_1v_n$, $e = v_2v_n$ form a triangle. Our aim is to generate $\lceil (n+1)/2 \rceil$ edges having label 1 and edges having label 0. Label the vertex $v_1$by 1 which will generate 2 edges having label 1. Now it remains to generate $k = \lceil (n+1)/2 \rceil - 2$ edges with label 1. For the vertices $v_2, v_3, \dots, v_n$, assign the vertex labels as per the following ordered pattern up to it generate k edges with label 1.

$$2, 2^2, 2^3, \dots, 2^{k1},$$
$$3, 3 \times 2, 3 \times 2^2, \dots, 3 \times 2^{k2},$$
$$5, 5 \times 2, 5 \times 2^2, \dots, 5 \times 2^{k3},$$
$$\dots, \dots, \dots, \dots, \dots,$$
$$\dots, \dots, \dots, \dots, \dots,$$

where $(2m - 1)\, 2^{km} \leq n$, $m \in N$ and k_m is a non-negative integer.

Observe that $(2m - 1)\, 2^{\alpha} \mid (2m - 1)\, 2^{\alpha+1}$ and $(2m - 1)\, 2^{ki}$ does not divide $2m + 1$.

In view of above labeling pattern we have number of edges with label 0 = $\lfloor (n+1)/2 \rfloor$ and number of edges with label 1 = $\lceil (n+1)/2 \rceil$. Hence, cycle with one chord is divisor cordial graph.

Example 1. Divisor cordial labeling of cycle C8 with one chord is shown in Fig. 1.

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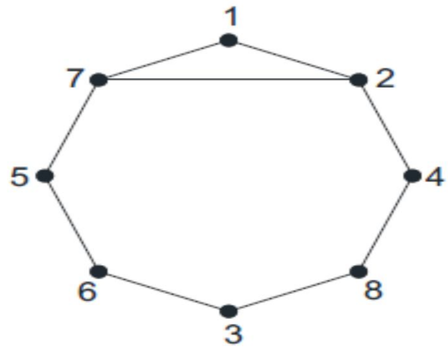

Fig. 1

Theorem 2. Cycle with twin chords Cn,3 is divisor cordial for all n, where chords form two triangles and one cycle Cn−2.

Proof. Let $v_1, v_2, \ldots, v_n$ be the successive vertices of cycle Cn and let $e_1 = v_2v_n$ and $e_2 = v_3v_n$ be the chords of Cn.

Our aim is to generate $\lfloor (n+2)/2 \rfloor$ edges having label 1 and $\lceil (n+2)/2 \rceil$ edges having label 0. Label the vertex v_1 by 1 which generates 2 edges having label 1. Now it remains to generate $k = \lceil (n+2)/2 \rceil - 2$ edges with label 1.

For the vertices $v_2, v_3, \ldots, v_n$ assign the vertex labels as per the following ordered pattern up to it generate k edges with label 1.

$2, 2^2, 2^3, \ldots, 2^{k1}$,

$3, 3 \times 2, 3 \times 2^2, \ldots, 3 \times 2^{k2}$,

$5, 5 \times 2, 5 \times 2^2, \ldots, 5 \times 2^{k3}$,

. . . , . . . , . . . , . . . , . . .,

. . . , . . . , . . . , . . . , . . .,

where $(2m - 1)\, 2^{km} \leq n$, $m \in N$ and k_m is a non-negative integer.

Observe that $(2m - 1)\, 2^{\alpha} \mid (2m - 1)\, 2^{\alpha+1}$ and $(2m - 1)\, 2^{ki}$ does not divide 2m + 1.

In view of above labeling pattern we have, number of edges with label 0 = $\lceil (n+2) / 2 \rceil$ and number of edges with label 1 = $\lfloor (n+2)/2 \rfloor$.

Hence, cycle with twin chords is divisor cordial graph.

Example 2. Divisor cordial labeling of cycle C9 with twin chords is shown in Fig. 2.

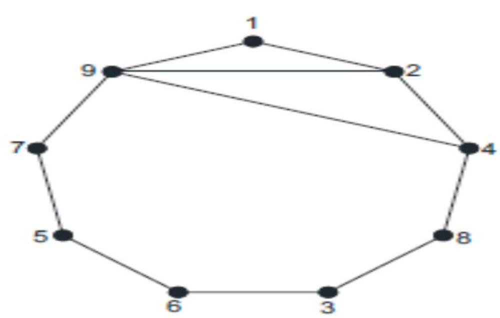

Fig. 2

Theorem 3. Vertex switching of cycle Cn ($n \geq 4$) with one chord admits divisor cordial labeling, where chord forms a triangle with two edges of cycle Cn.

Proof . Let G be the cycle Cn with one chord. Let $v_1, v_2, \ldots, v_n$ be the successive vertices of cycle Cn and $e = v_2v_n$ be a chord of cycle Cn. The edges $e = v_2v_n$, $e_1 = v_1v_2$, $e_2 = v_1v_n$ form a triangle.

Without loss of generality let the switched vertex be v_1 (of either degree 2 or degree 3) and let Gv_1 denote the vertex switching of G with respect to vertex v_1.

To define labeling function $f : V(Gv_1) \rightarrow \{1, 2, \ldots, n\}$ we consider the following cases.

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Case 1: The vertex v_1 is of degree 2. i.e. $\deg(v_1) = 2$.
{Here the number of vertices is n and number of edges is $2n - 4$.}
Subcase 1: $n \equiv 0(\mathrm{mod}\,4)$, $n \neq 4$.
$f(v_n) = 4$, $f(v_{n-1}) = n - 1$, $f(v_{n-2}) = n$,
$f(v_i) = i$, $1 \leq i \leq 3$.
$f(v_i) = i + 1$, $4 \leq i \leq n - 3$.
In view of the above labeling pattern we have $e_f(0) = e_f(1) = n - 2$.

Subcase 2: $n = 4$.
$f(v_i) = i$, $1 \leq i \leq n$.
In view of the above defined labeling pattern, $e_f(0) = e_f(1) = 2$.

Subcase 3: $n \equiv 1, 2, 3(\mathrm{mod}\,4)$.
$f(v_n) = 4$
$f(v_i) = i$, $1 \leq i \leq 3$.
$f(v_i) = i + 1$, $4 \leq i \leq n - 1$.
In view of the above defined labeling pattern, $e_f(0) = e_f(1) = n - 2$.

Case 2: The vertex v_1 is of degree 3. i.e. $\deg(v_1) = 3$.
{Here the number of vertices is n and number of edges is $2n - 5$.}
$f(v_3) = 4$, $f(v_4) = 3$
$f(v_i) = i$, $1 \leq i \leq n$, $i \neq 3$, $i \neq 4$.
In view of the above labeling pattern we have $e_f(0) = \lfloor (2n-5)/2 \rfloor$, $e_f(1) = \lceil (2n-5)/2 \rceil$.
Thus, $| e_f(0) - e_f(1) | \leq 1$.
Hence vertex switching of cycle Cn with one chord is a divisor cordial graph.

Example 3.
(a) Divisor cordial labeling of the graph obtained by switching of a vertex of degree 2 in cycle C_7 is shown in Fig. 3(a).
(b) Divisor cordial labeling of the graph obtained by switching of a vertex of degree 3 in cycle C_7 is shown in Fig. 3(b).

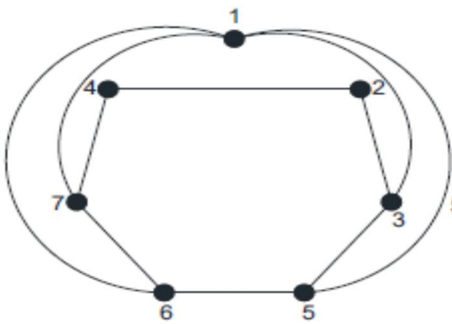

Fig. 3(a)

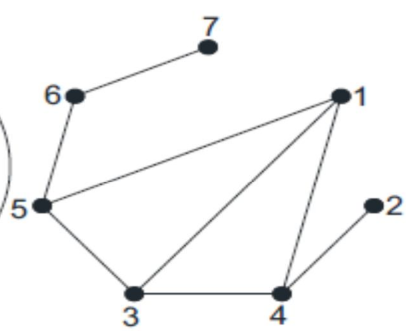

Fig. 3(b)

Theorem 4. Vertex switching of cycle Cn with twin chords Cn,3 admits divisor cordial labeling, where chords form two triangles and one cycle Cn-2.
Proof. Let G be the cycle Cn with twin chords. Let $v_1, v_2, \ldots, v_n$ be the successive vertices of G. Let $e_1 = v_nv_2$ and $e_2 = v_nv_3$ be the chords of cycle Cn which form two triangles and one cycle Cn-2.
Without loss of generality let v_1 (of either degree 2 or degree 3 or degree 4) be the switched vertex and let Gv_1 denote the vertex

switching of G with respect to vertex v_1.

To define labeling function $f : V(Gv_1) \rightarrow \{1, 2, \ldots, n\}$ we consider the following cases.

Case 1: The vertex v_1 is of degree 2. i.e. $\deg(v_1) = 2$

{Here the number of vertices is n and number of edges is $2n - 3$.}

$f(v_n) = 4$,

$f(v_i) = i, 1 \le i \le 3$.

$f(v_i) = i + 1, 4 \le i \le n - 1$.

In view of above labeling pattern:

When $n \equiv 0 (\mathrm{mod}\, 4)$, $e_f(0) = n - 2$, $e_f(1) = n - 1$.

Otherwise, $e_f(0) = n - 1$, $e_f(1) = n - 2$.

Case 2: The vertex v_1 is of degree 3. i.e. $\deg(v_1) = 3$

{Here the number of vertices is n and number of edges is $2n - 5$.}

$f(v_3) = 4$, $f(v_4) = 3$

$f(v_i) = i, 1 \le i \le n, i \neq 3, i \neq 4$.

In view of the above labeling pattern we have $e_f(0) = \lceil (2n-5) / 2 \rceil$, $e_f(1) = \lfloor (2n-5)/2 \rfloor$.

Case 3: The vertex v_1 is of degree 4. i.e. $\deg(v_1) = 4$

{Here the number of vertices is n and number of edges is $2n - 7$.}

$f(v_3) = 4$, $f(v_4) = 3$

$f(v_i) = i, 1 \le i \le n, i \neq 3, i \neq 4$.

In view of the above labeling pattern we have $e_f(0) = \lfloor (2n-7)/2 \rfloor$, $e_f(1) = \lceil (2n-7) / 2 \rceil$.

Thus, $|e_f(0) - e_f(1)| \le 1$.

Hence vertex switching of cycle Cn with twin chords is a divisor cordial graph.

Example 4.

(a) Divisor cordial labeling of the graph obtained by switching of a vertex of degree 2 in cycle C_8 with twin chords is shown in Fig. 4(a).

(b) Divisor cordial labeling of the graph obtained by switching of a vertex of degree 3 in cycle C_8 with twin chords is shown in Fig. 4(b).

(c) Divisor cordial labeling of the graph obtained by switching of a vertex of degree 4 in cycle C_8 with twin chords is shown in Fig. 4(c).

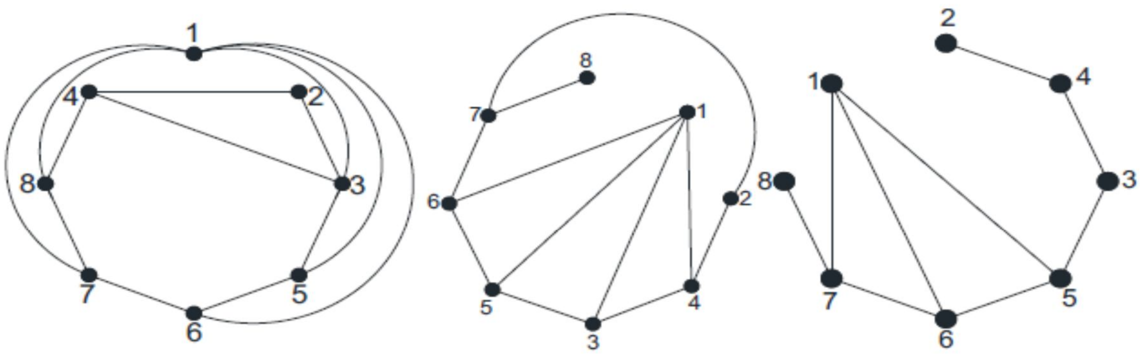

Fig. 4(a) Fig. 4(b) Fig. 4(c)

Theorem 5. The barycentric subdivision of cycle with one chord admits divisor cordial labeling, where chord forms triangle with two edges of the cycle.

Proof. Let G be the cycle Cn with one chord and let S(G) denote the barycentric subdivision of G. Here $|V(S(G))| = 2n + 1$ and $|E(S(G))| = 2n + 2$. Let $v_1, v_2, \ldots, v_{2n+1}$ be the successive vertices of S(G), where $v_1, v_3, \ldots, v_{2n-1}$ denote the vertices of cycle Cn and $e = v_3 v_{2n-1}$ be the chord of Cn. The vertices $v_2, v_4, \ldots, v_{2n}$ are newly inserted vertices due to barycentric subdivision of Cn and v_{2n+1} is the newly inserted vertex due to barycentric subdivision of the chord e of Cn.

We define labeling function $f : V(S(G)) \rightarrow \{1, 2, \ldots, 2n\}$ as follows.

Our aim is to generate n + 1 edges with label 1 and n + 1 edges with label 0.

f (v) = 1, which generates 2 edges having label 1. Now it remains to generate $k = n - 1$ edges with label 1.

For the vertices $v_1, v_2, \ldots, v_{2n}$ assign the vertex label as per following ordered pattern up to it generate k edges with label 1.

$2, 2^2, 2^3, \ldots, 2^{k1}$,

$3, 3 \times 2, 3 \times 2^2, \ldots, 3 \times 2^{k2}$,

$5, 5 \times 2, 5 \times 2^2, \ldots, 5 \times 2^{k3}$,

. . . , . . . , . . . , . . . , . . .,

. . . , . . . , . . . , . . . , . . .,

where $(2m - 1)\, 2^{km} \leq 2n+1$, $m \geq 1$ and $k_m \geq 0$.

Observe that $(2m - 1)\, 2^{\alpha} \mid (2m - 1)\, 2^{\alpha+1}$ and $(2m - 1)\, 2^{ki}$ does not divide 2m + 1.

In view of the above labeling pattern we have $e_f(0) = e_f(1) = n+1$.

Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the barycentric subdivision of cycle Cn with one chord admits divisor cordial labeling.

Example 5. Divisor cordial labeling of the graph obtained by the barycentric subdivision of cycle C_{10} with twin chords is shown in Fig. 5.

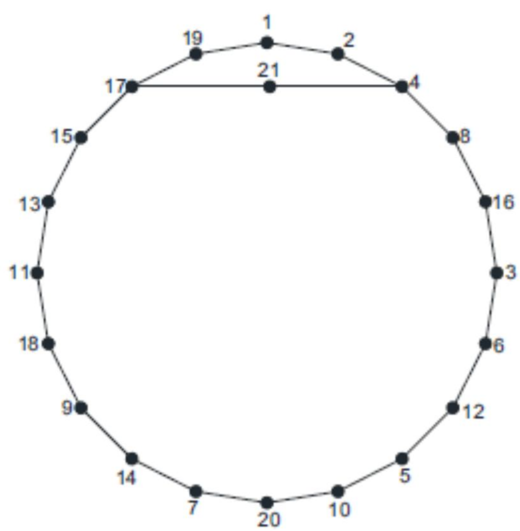

Fig. 5

Theorem 6. The barycentric subdivision of cycle Cn with twin chords Cn,3 admits divisor cordial labeling, where chords form two triangles and one cycle Cn−2.

Proof. Let G be the cycle Cn with twin chords and let S(G) denote the barycentric subdivision of G. Here $|V(S(G))| = 2n + 2$ and $|E(S(G))| = 2n + 4$. Let $v_1, v_2, \ldots, v_{2n+2}$ be the successive vertices of G.

Let $e_1 = v_n v_2$ and $e_2 = v_n v_3$ be the chords of cycle Cn. The vertices $v_2, v_4, \ldots, v_{2n}$ are newly inserted vertices due to barycentric subdivision of Cn, v_{2n+1} is the newly inserted vertex due to barycentric subdivision of the chord e_1 and v_{2n+2} is the newly inserted vertex due to barycentric subdivision of the chord e_2.

We define labeling function $f : V(G) \rightarrow \{1, 2, \ldots, 2n\}$ as follows. Our aim is to generate n + 2 edges with label 0 and n + 2 edges with label 1. f (v) = 1, which generates 2 edges with label 1. Now it remains to generate k = n edges with label 1. For the vertices $v_1, v_2, \ldots, v_{2n}$ assign the vertex label as per following ordered pattern up to it generate k edges with label 1.

$2, 2^2, 2^3, \ldots, 2^{k1}$,
$3, 3 \times 2, 3 \times 2^2, \ldots, 3 \times 2^{k2}$,
$5, 5 \times 2, 5 \times 2^2, \ldots, 5 \times 2^{k3}$,
$\ldots, \ldots, \ldots, \ldots, \ldots,$
$\ldots, \ldots, \ldots, \ldots, \ldots,$

where $(2m - 1)\, 2^{km} \leq n$, $m \in N$ and k_m is a non-negative integer.

Observe that $(2m - 1)\, 2^{\alpha} \mid (2m - 1)\, 2^{\alpha+1}$ and $(2m - 1)\, 2^{ki}$ does not divide 2m + 1.

In view of above labeling pattern, we have $e_f(0) = e_f(1) = n+2$.

Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the barycentric subdivision of cycle Cn with twin chords is divisor cordial.

Example 6. Divisor cordial labeling of the graph obtained by barycentric subdivision of cycle C_{11} with twin chords is shown in Fig. 6.

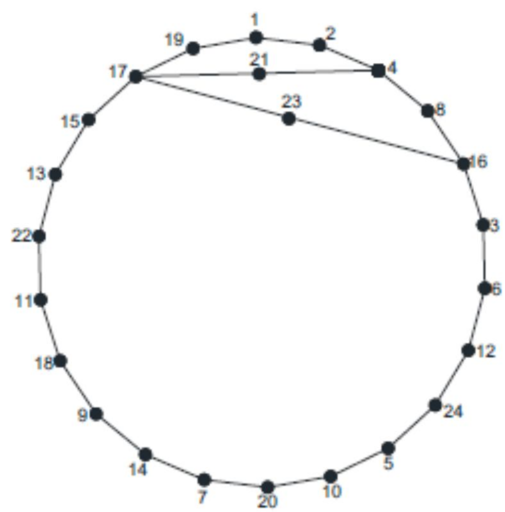

Fig. 6

III. CONCLUSION

We have contributed six new results in the theory of divisor cordial graphs. We have proved that cycle with one chord and cycle with twin chords are divisor cordial graphs. For these graphs divisor cordial labeling remains invariant under the operation vertex switching and barycentric subdivision.

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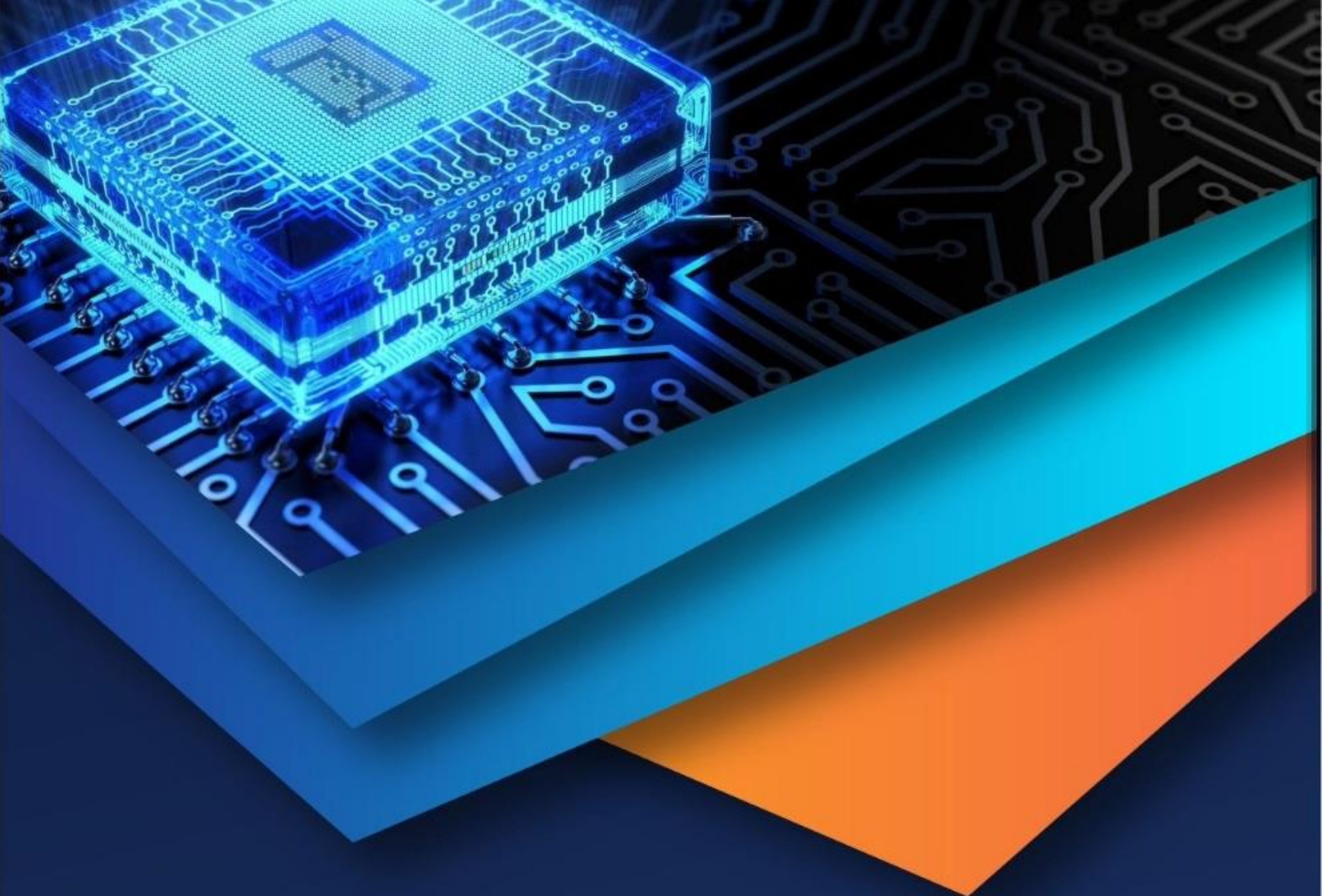

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