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Simulation of Asteroid (99942 APOPHIS) Formation using N-Body Problem and Mission Design for Sample Return from the Asteroid

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Abstract: *In this paper gravitational forces that are acting on N body which leads to the formation of stable aggregate is computed. N body gravitational dynamics and collision dynamics of the particles are studied. Interaction between two particles due to gravity without any perturbation is called as a two-body problem. During the formation of stable asteroid (99942 Apophis) interactions between two particles and the parameters involved during collision is numerically evaluated. Using MATLAB numerical modelling of two body interaction due to gravity and collision between each other which forms a single mass aggregate is evaluated. It also involves simulating the aggregation of a range of particles and checking how their interactions mainly based on gravitation acts. A study on the change in the energy, force and momentum during the course of aggregation is done. Several parameters are set for the simulation of the asteroid aggregation process. In addition to N Body aggregation the mission design (round trip) is developed for the sample return from the asteroid (99942 Apophis). Various launch opportunities were considered and compared and effective launch window was chosen. Calculations for the optimum trajectory is performed.*

I. INTRODUCTION

N body problem plays a major role in understanding the formation of objects in space. Studying about formation of an object is important because, for an effective mission design to any celestial body, proximity corrections are required which includes their shape, size, irregularity etc. This can be predicted precisely in N body aggregation. N body problem also gives us a vast knowledge about how celestial objects would have been formed in past and the possibility of aggregation of new object in space. With N body problem, we can also understand the concept of breaking of particles due to collision and again forming an aggregate. These N particle systems can be confined to a two-body system for numerical evaluation of parameters between them. In a two-body problem the particles don't experience any perturbations. They can be numerically evaluated for gravitational and contact dynamics. N-body simulation helps in approximating the motion of particles, especially the ones that interact with one another through some form of physical forces. By this definition, the type of particles which can be simulated by n-body methods range from celestial bodies to even atoms of gas cloud. Motion of the particles on the bodies is neglected, as it adds an unnecessary number of particles to the simulation. To perform this simulation is by direct integration of the Newtonian gravitational force equation. With the help of the force calculation of each particle a new velocity and position can be calculated by using discretized time step.

Asteroids are small rocky bodies that are orbiting around the sun. Millions of asteroids exist and are shattered all over the solar system. One among those Asteroid is 99942 Apophis. It is near earth asteroid that got a closest approach to earth March 6th 2021. It is also expected to have one closest approach to earth on April 13 2029. Since asteroids have a contrasting orbit, mission design to 99942 Apophis has been very challenging. As there are no any past missions to 99942 Apophis the sample return mission design to the asteroid is exacting. Possible launch window and launch opportunities were taken by considering the Ephemeris data from year 2021-2030.

II. TWO BODY PROBLEM OF ASTEROID AGGREGATION

At first two particles are taken into study during the process of aggregation of asteroid. Since only two particles are taken into consideration, perturbations caused due to other particles can be neglected in a two-body problem. Approach of two particles depends both on gravitational and contact dynamics as they are the external force acting on them^[1]. This chapter presents the dynamics of two particles during aggregation and numerical modelling of a two-body problem in MATLAB.

A. Gravitation Dynamics

Newton's law of gravitation states that every object in the universe attracts the other with a gravitational force which is directly proportional to their masses and inversely proportional to the square of the distance between them and acts along the line joining the particles. Newton's law is used to solve gravitational interactions between N-Bodies which means each body moves with the gravitational attraction of the remaining N-1 Bodies.

$$F_i = -G \sum_{j=1, j \neq i}^N m_i m_j \frac{r_i - r_j}{|r_i - r_j|^3}$$

F_i = force on the particle i due to j, G (gravitational constant) = $6.674 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

m_j = mass of particle j , r_i = position vector of particle i, r_j = position vector of particle j

B. Contact Dynamics

Contact dynamics deals with the force, friction, stress, pressure experienced by that particle during contact. Although gravitational dynamics is sufficient to implement a simulation of N point like masses with long-range gravitational interactions, we also need to address the case where bodies come into contact during late stage of aggregation. Coulomb's frictional model is used to find the dry frictional forces during contact. Based on the coefficient of friction body can become elastic or inelastic after collision^[1]. Perhaps for a rigid body where penetration doesn't happen friction acts at the interface.

$$F_n = \hat{\mu} n$$

$$F_t = \hat{\mu} t$$

Where,

F_n = Normal friction force, F_t = tangential force, $\hat{\mu}$ = coefficient of friction,

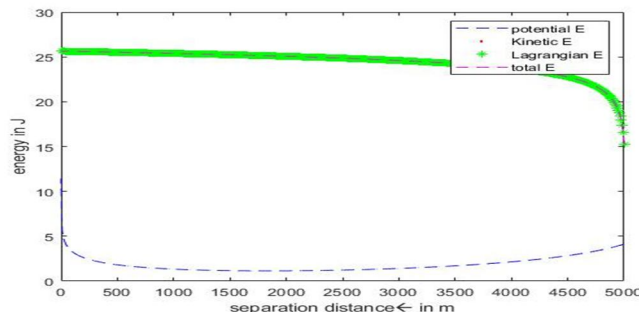
n = normal force, t = tangential force

Contact forces introduce a complication in the time integration of the system.^[1]

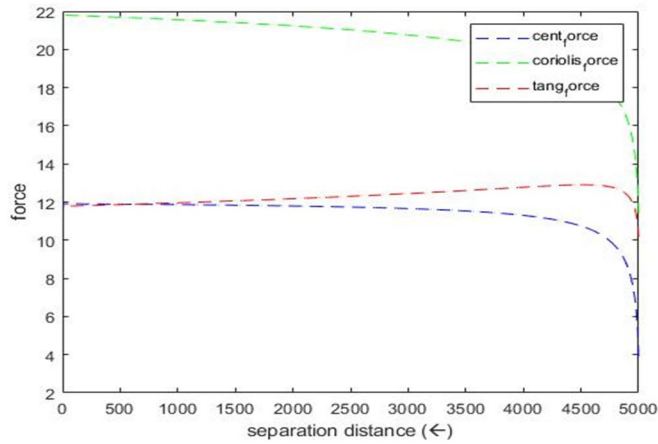
C. Numerical Modelling of two Body Problem

To start with the dynamics of two particles at first one particle is considered to be in origin while the other particle is taken at a distance of 5000 m^[1]. For precise calculation the distance between the particles is divided into 600 points. Where at each time integration step, at every point physical parameters for aggregation is calculated. Initial conditions were given to find the velocity at each point of approach in x, y, z direction within the given range of separation distance using Numerical methods (Runge-Kutta4). The number of particles are taken over a range of 900 to 9900. The particles are evaluated for a sample time aggregation of 5 hours. At every point of approach parameter like energy, angular momentum, virial radius, collision radius, pressure, virial ratio are calculated to understand the dynamics and estimate the point of aggregation^[2]. The formulas were coded in MATLAB and the values were refined by trial-and-error method.

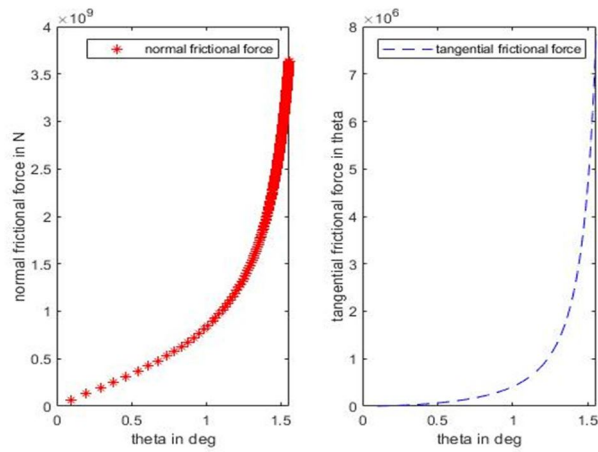
D. Graphs and Discussion



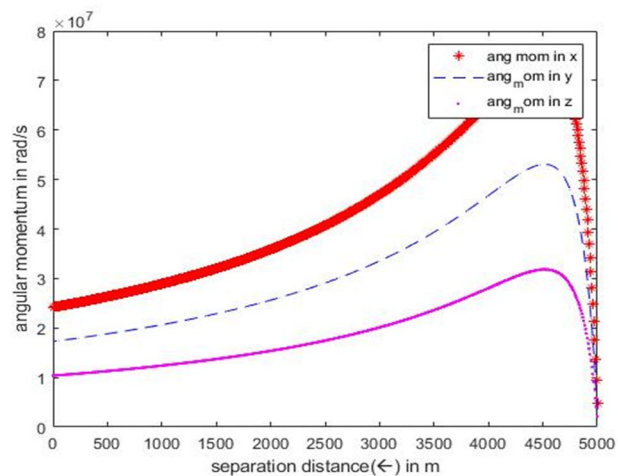
Graph (a)



Graph (b)



Graph (c)



Graph (d)

Graph a – separation distance (m) vs energy (N)

Graph b- separation distance (m) vs forces (N)

Graph c – theta (degree) vs frictional force (N)

Graph d – separation distance (m) vs angular momentum (rad/s)

In graph (a) Energy of the system increases. Since it's not an infinite time averaged system the energy is positive and linear. The potential energy is so small for the particle and it does not significantly affect the total energy. Total, kinetic and lagrangian energy are same.

In graph (b) the tangential force decreases as separation distance decreases which helps in aggregate formation. The Coriolis and centripetal force increases because of increase in spin of the particle due to gravitational attraction of other particle and the inward force required for aggregation respectively. As all the forces become constant a stable aggregate is formed.

In graph (c) Frictional force abruptly increases at theta 1.4 degrees. Since the particle is rotating on its own axis and due to irregularity in its shape there might be a possibility of breakdown of particles at high friction and contact force which is not favourable for aggregate formation.

In graph (d) the angular momentum rises to a peak in all three axes due to gravitational attraction, spin effects and reduction in separation distance. As the separation distance decreases angular momentum becomes stable and constant indicating the possibilities of stable aggregate formation.

III. SIMULATION OF ASTEROID FORMATION USING MATLAB

A MATLAB code is written based on gravitational N-body problem. A simulation of a dynamical system of particles is created that interacts with each other gravitationally. A system is assumed with mass m_i , position $\mathbf{r}_i = [x_i, y_i, z_i]$ and velocity $\mathbf{v}_i = [v_{x_i}, v_{y_i}, v_{z_i}]$. According to newtons law of gravitation the particles feel an acceleration

$$a_i = G \sum_{j \neq i} m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3}$$

where $G = 6.67 * 10^{-11} m^3 / kg / s^2$ is the Gravitational constant.

The MATLAB function is written to perform calculations and input a matrix for positions. Softening parameter is also included in the code to avoid interference when two particles are close to each other. The positions and velocities in the simulation are updated continuously using a leap-frog algorithm.

In this algorithm for every timestep increase there is a half kick to increase velocity followed by a full drift to terms of position and again a half kick (kick-drift-kick). This is highly accurate for second order and helps for preserving the energy of the whole system. The simulation specifies initial positions and velocities in a random Gaussian distribution for N=900, 1800, 3600, 5000, 8000 particles.

The total energy of the system is conserved during time evolution. In the addition to kinetic energy and potential energy, Lagrangian and absolute energies are found by numerical method.

With the help of mass and velocities, momentum and forces are found. The formulas for Momentum, Centripetal force, Coriolis force, Normal force and Frictional forces are used, and the data is stored, and graphs are plotted.

At the start of the simulation and at the point of aggregation the particles are placed as shown in figure. Even though the simulation is done for a range of partcles from 900 to 8000, the results are shown for 3600 particles where a stable aggregate forms^[3].

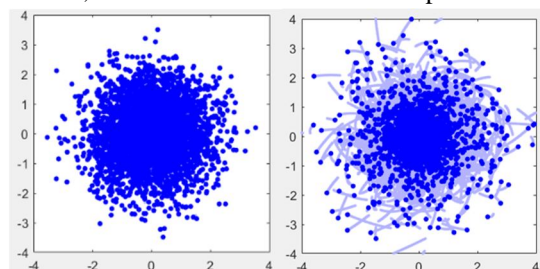
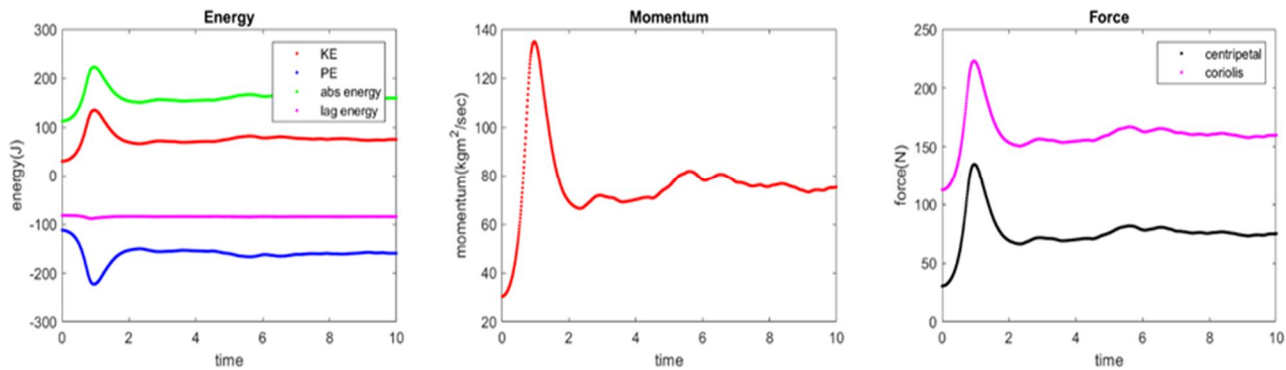
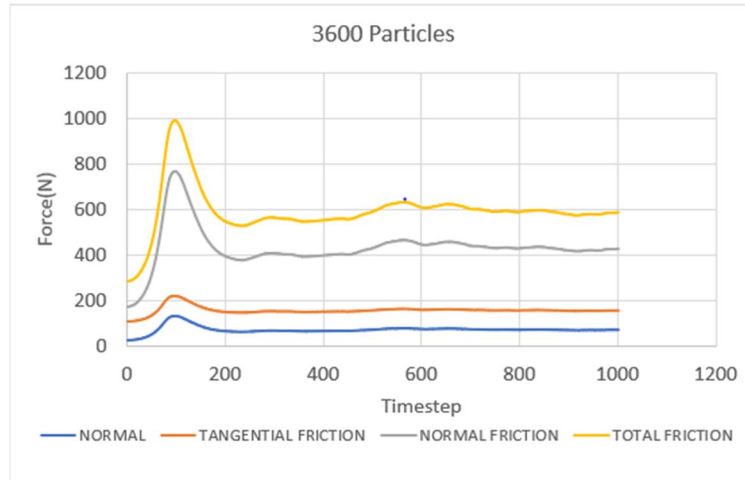


Figure 1 Simulation of 3600 particles

Once the simulation is over the graphs are plotted and it is found that at the point of aggregation there is huge increase in momentum, Kinetic energy, absolute energy, and the forces acting between particles. Once the collisions between particles become constant and transient the change in the parameters becomes a constant.

The Absolute energy is positive as the Potential energy is negative. The Coriolis force is more than the centripetal force because it not only is found during the rotation of the particles and movement of them, but also on the formation of craters on their surfaces.



Graph 2 End result of Energies, Momentum, and forces for 3600 particles

IV. MISSION DESIGN FOR SAMPLE RETURN FROM THE ASTEROID

Any discussion of asteroid resources must start with an understanding of the asteroid’s composition, structure, and distribution throughout the solar system. The study of asteroids is an exceptionally specialized and consistently evolving field.

The concept of exploiting asteroids' natural resources predates the space program itself. However, the technology required to make this a possibility is now becoming available.

A. Mission Overview

Why sample return?

Asteroids are geologic remnants from the early Solar System and are considered as time capsules from the birth of the solar system. They are primitive, having undergone little geological change from the time of formation of solar system. It is expected that, after the successful sample collection from the asteroid, we can learn more about the formation and evolution of solar system, more knowledge on the origin and evolution of the planets, especially the origin of water and organic matters and its compounds that led to the formation of life on earth could be obtained.

B. Selection Of Target Asteroid

Selecting a suitable target for our mission began with the 500,000 or so asteroids known at the time of our proposal. Considerations for the delta-V of available launch vehicles suggested that Near-Earth asteroids (NEAs), whose orbits bring them close to the Earth, were a sensible place to begin the search. At the time, over 8000 NEAs were legendary. Further analysis revealed that of these, only 350 had orbits that were optimal for a sample return mission from the asteroid within the calendar constraints of the space organization, mass constraints, spacecraft design, and available launch vehicle delta-V. Of the 350 objects with optimal orbits, only 29 had diameters greater than 200 m. Objects smaller than 200 m often have fast rotation rates or exhibit non-principal axis rotation (tumbling) that could complicate proximity operations on the asteroid. Of the final 29 candidate objects, two asteroids were chosen for a further research.

After the formation of the solar system 4.5 billion years ago, these were most likely primitive asteroids with no physical change. It was these objects that were likely to yield the answers that we seek about the earliest stages of solar system formation. Two asteroids 153814 (2001 WN5) and 99942 Apophis (2004 MN4) were taken at the beginning considering the above factor and also its closest approach to Earth.

After a thorough research, Asteroid 99942 Apophis was chosen due to its orbit similarity with earth and its closest approach, a distance of 3.06×10^{-3} AU which is the size of a hair strand in space, figure 2 shows the closest approach of Earth and the asteroid. Considering these two factors, 99942 Apophis is fixed as the target and we proceeded further into the mission design.

C. Orbit

In order to successfully perform a sample return from the asteroid 99942 Apophis, the spacecraft must follow a designated mission design. 99942 Apophis is a good target because it has an orbit that is similar to Earth's, albeit at a 3.3 degrees inclination. Thus, the amount of propellant required reduces due to the similarity of orbits, which will save money and mass in order to meet the budget and spacecraft requirements.

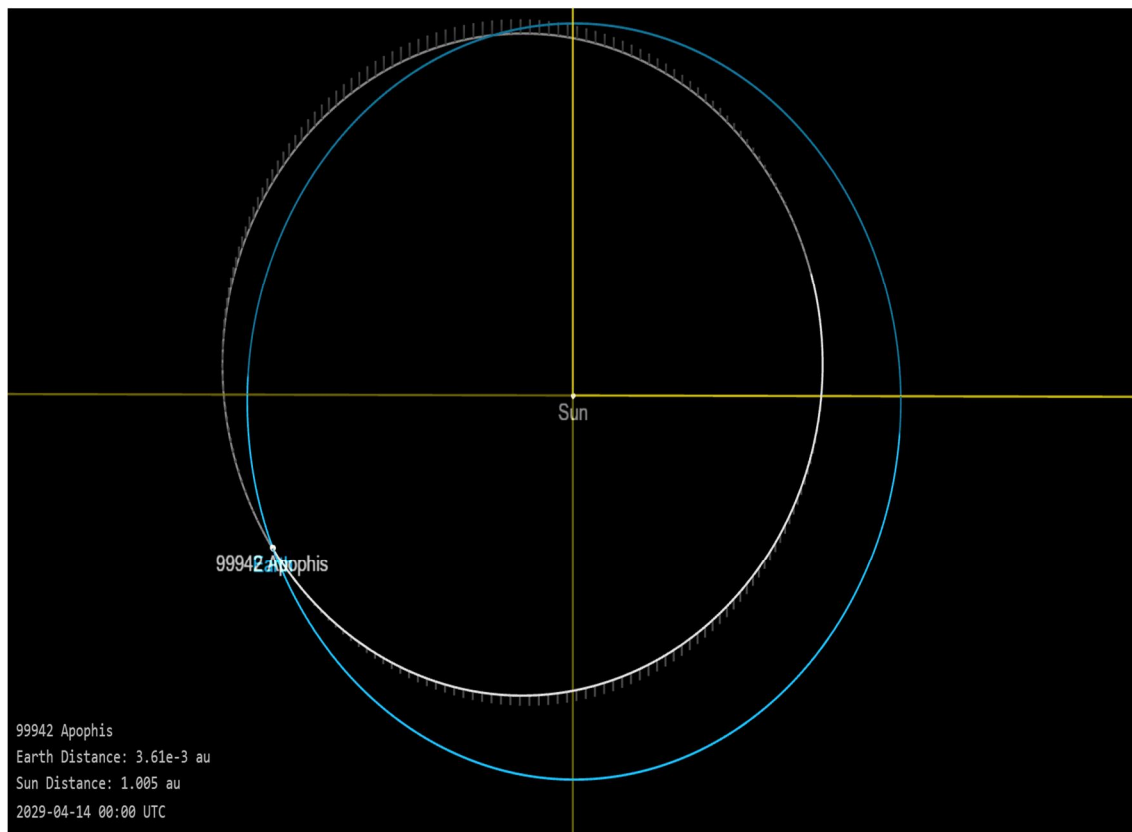


Figure 2 Orbits of Earth and 99942 Apophis (courtesy: JPL Small body database)

D. Mission Design Phases

A Hohmann transfer is chosen because it is the most efficient impulsive transfer orbit for orbits with relative sizes of that of the asteroid and Earth, will be used to reach 99942 Apophis after orbit raisings is utilized to account for the inclination change.

The number of phases for the mission design from earth to asteroid 99942 Apophis considered in our project is six and are follows,

- 1) Departure from Earth
- 2) Parking and coasting orbit
- 3) Orbit Raisings
- 4) DSM's and orbit correction maneuvers
- 5) Asteroid Approach
- 6) Final arrival on Asteroid 99942 Apophis.

These phases are considered for calculations to develop an optimum trajectory to reach the Asteroid.

E. Launch Window And Opportunities

Possible departure dates were considered with the help of the Ephemeris data from year 2021-2030 since the date of departure shall not exceed 2030 as the closest approach of 99942 Apophis with earth is in 2029. Using the formulae for interplanetary missions the ΔV was calculated for different dates and the results are compared to choose the efficient departure date. The date 3rd May 2027 was chosen as the optimum departure date as its ΔV is close to optimum. Accordingly, its arrival date is also obtained after interplanetary trajectory calculation as Jan 3rd, 2030.

The optimum return trajectory is possible after 5 years of stay in the Asteroid performing proximity operations and then departing the asteroid on 12th December 2035 and chase the earth during a close encounter and enter the sphere of influence of earth followed by the re-entry and landing of the sample return capsule tentatively on 12th April 2037. Thus, an estimated total mission duration of 10 years meeting the trajectory constraints is achieved.

- 1) *Trajectory Constraints:* The major mission constraints of our project are tabulated.

Table 1 Trajectory constraints

Mission type	Round trip- Rendezvous
Launch year	2021-2030
Maximum Mission duration	10 years
Maximum Delta V	7 km/s
Semi major axis	0.922 AU
Solar range	0.89- 2.17 AU
Earth range	0-3.18 AU

With these trajectory constraints, preliminary mission designed was developed.

2) *Trajectory Itinerary*: As above mentioned, for each mission design phase, the launch window is taken into consideration and the interplanetary equations were used to solve for different velocities required for each of all the phases and its tentative dates are also listed below. The results are validated by comparing the data obtained from our calculations with those obtained from NASA Ames Research Centre Trajectory Browser and NASA Jet Propulsion Laboratory (JPL) asteroid search engine database. The trajectory itinerary for a round trip mission from earth to an asteroid 99942 Apophis with a sample collection is tabulated below,

Table 2 Trajectory itinerary

EVENTS	DATE	DELTA V
Earth departure	May 3 rd , 2027	4.23km/s
<i>1.11-year transfer</i>		
Transfer window	April 13 th , 2029	0.268 km/s
<i>21 days transfer</i>		
DSM and orbit correction	May 4 th , 2029	1.3088 km/s
<i>0.66-year transfer</i>		
Asteroid arrival	Jan 3 rd , 2030	-0.071 km/s
<i>5.11-year stay (Proximity operations)</i>		
Asteroid departure	December 12 th , 2035	
<i>1.4-year transfer</i>		
Earth re-entry	April 12 th , 2037	0.14011 km/s
Total mission plan – 9.94 years mission		

The calculations to support this mission design is provided further. The values from NASA Ames Research Centre Trajectory Browser and NASA Jet Propulsion Laboratory (JPL) asteroid search engine database used to validate the theoretical values as there may be errors in manual calculations. The results are validated and the trajectory that is feasible practically would be chosen.

V. SAMPLE RETURN TRAJECTORY TO THE ASTEROID

For a trajectory design to arrive at an optimum condition the most important parameters that we need to consider are the position of the asteroid and the ΔV required for the spacecraft for every orbit raising. By considering the launch dates after ΔV optimisation the calculations for the required parameters are performed. As we know the orbital velocity [V_o] of earth is given by

$$V_o = \sqrt{\frac{\mu_e}{r}}$$

which is 7.198 km/s. And the escape velocity [V_e] of the earth is given by $V_{esc} = \sqrt{\frac{2\mu_e}{r}}$

which is 11.2 km/s.

During departure and Arrival, the characteristic energy [C_3] plays a major role. Launch energy requirements should be considered while putting an optimum trajectory. The hyperbolic excess velocity

[V_{∞}] can also be calculated using the characteristic energy. The characteristic energy is given by $C_3 = V_{\infty}^2$ Also while putting an optimum trajectory the sphere of influences of the planet and the celestial body the mission design is made to is to be considered. The sphere of influence is given by the

$$R_{sot} = r \left[\frac{m_p}{m_s} \right]^{\frac{2}{5}}$$

where,

m_p = mass of the planet

m_s = mass of sun

r = radius of the planet

The parking orbit is considered at 350km from the surface of the earth. The spacecraft is sent to parking orbit initially and it is then transferred to the coasting orbit by using Hohman transfer at a ΔV of 3.95 km/s.

Orbit raisings are done by considering the distance of earth and the asteroid (99942 Apophis). As we know the asteroid 99942 Apophis is inclined 3.30 degrees. So, two orbit raisings are performed each with an inclination of 1.65 degrees. The distance between the sun and the asteroid on May 4th 2029 is 765941.098 km. For the calculation the first orbit is raised to half distance that is 365077.549 km. And the calculation for every orbit raising is performed. The ΔV required for the first orbit raising is 0.5608 km/s and for the second orbit raising is 0.268 km/s. After the second orbit raising different trajectory correction maneuverers [TCM] are performed between 4th September 2029 to 4th November 2029. The ΔV 's while performing the TCM's are listed in the Table 3.

Dates	Earth Distance (Au)	Delta V	Difference
4 Sep-29	0.938	5.268	0
11 Sep-29	0.976	5.3398	0.0718
18 Sep-29	1.009	7.5	2.1602
25 Sep-29	1.037	9	1.5
2 Oct-29	1.059	9.55	0.55
9 Oct-29	1.075	9.55	0
16 Oct-29	1.086	8.7	-0.85
23 Oct-29	1.092	8.45	-0.25
30 Oct-29	1.093	8.45	0
4 Nov-29	1.091	8.45	0
		Total delta V=	3.182

Table 3 TCM values before approach

After performing different TCM's the spacecraft orbit is now raising to enter to sphere of influence of the asteroid (99942 Apophis). The ΔV required by the spacecraft to enter into sphere of influence of the asteroid is -0.071 km/s. The hyperbolic excess velocity required by the spacecraft while arriving at the asteroid is 7.1 km/s and the characteristic energy is 50.41 km/s. The velocity obtained after calculations for the spacecraft to arrive at the asteroid is 7.10014 km/s.

After the successful landing on the asteroid surface as mentioned above the proximity operations are performed on the asteroid and then the return mission to the earth is made after collecting the sample. The return trajectory is a direct transfer. The spacecraft is first sent to the sphere of influence of the earth at ΔV of 0.14011 km/s.

The hyperbolic excess velocity required by the spacecraft while arriving on the earth is 5.9 km/s and the characteristic energy is 34.81 km/s. The velocity obtained after calculations for the spacecraft to arrive at the asteroid is 13.81 km/s.

A. Orbital Energy Calculations

The velocities that obtained is now converted into the energies. So, the orbital energy is given by

$$\Delta\xi = \frac{1}{2} (V_o^2 - V_{par_0}^2)$$

The values of orbital energies required by the spacecraft for every orbit raising is listed in the Table 4.

Parameters	PO to GSO	GSO to #1	#1 TO #2	#2 to SOI of Asteroid
Spacecraft velocity after impulsive shot (km/s)	3.08	0.98	0.718	0.052
Spacecraft velocity in parking orbit (km/s)	3.95	0.5608	0.268	-0.071
Orbital energy per unit mass (km ² /s ²)	3.058	-0.332	-0.221	0.0011

Table 4 orbital Energy values

B. Eccentricity Calculations

The eccentricity at which the spacecraft is transferred to the next orbit is given by

$$e = 1 - \left(\frac{r_i}{a_{tx_i}} \right)$$

Where, i= 1,2, 3.....,N

The eccentricities required for the mission to asteroid (99945 Apophis) is listed in the Table 5

Parameter	launch	PO to GSO	GSO to #1	#1 TO #2	#2 to SOI of Asteroid	Arrival at asteroid
eccentricity	1.4	0.73	0.81	0.309	0.28	0.108

Table 5 Eccentricity values

VI. CONCLUSION

Implementation of numerical evaluation of aggregate formation is presented. The capabilities of MATLAB have been exploited and dynamics of a 2-body aggregation and N particle aggregation is successfully carried out. To validate the values obtained in 2 body problem, significant special conditions were analysed during the formation of asteroid. It is found that separation distance between the particles plays a major role in determining the properties of the particle in motion. The results attained shows good agreement with N body problem theory and special conditions. Asteroid formation has been successfully done and graphs according to force, momentum and energies are plotted based on time step and separation distance for a range of particles.

A round-trip mission design for the sample return is explained. Selection and validation of optimum trajectory dates are also explained and required calculations are performed. In this delta V is the main criteria for the mission design. Both the formation and mission design for sample return have paved way for understanding deeply about asteroid (99942 Apophis) and gave a huge information to a lot of questions about various phenomenon.



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