iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 9 **Issue:** V **Month of publication:** May 2021

DOI: https://doi.org/10.22214/ijraset.2021.34801

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International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue V May 2021- Available at www.ijraset.com

Multi Integer Linear Programming and the Traveling Salesman Problem

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Abstract: ***The Travelling Salesman problem is considered as a binary integer problem. For this problem, several stop variables and subtours are discussed.***
The stops are generated and the distance between those stops are found, consequently the graphs are drawn. Further the variables are declared and the constraints are framed. Then the initial problem is visualised along with the subtour constraints in order to achieve the required output.
Keywords: Traveling Salesman, binary Integer, stop variables, subtour constraints.

I. INTRODUCTION

Mixed integer linear programming (MILP) demonstrates an effective mathematical modelling proceedings to solve tedious optimisation queries and seek the potential trade-offs between conflicting objectives, that can provide a better knowledge of bioenergy systems and hold up decision-makers briefing the sustainable roots towards bioenergy destinations.

II. MIXED-INTEGER PROGRAMMING (MIP) PROBLEMS

A mixed-integer programming (MIP) problem is the one in which constraining few of the decision variables to the values of integer occurs (i.e. non fractional or whole numbers such as -2, -1, 0, 1, 2, 3 etc.) at the optimal solution. The usage of integer variables majorly elaborates the view of helpful optimization sums that you can declare and solve.
A primary special type is a decision variable X1 which should be either 0 or 1 at the answer. That variables are known as binary integer or 0-1 variables and can be utilized for the modelling yes/no decisions, such as whether to build a plant or buy a piece of equipment. However, integer variables make an optimization problem non-convex, and therefore far more difficult to solve. Memory and solution time may rise exponentially as you add more integer variables.
Even with highly sophisticated algorithms and modern supercomputers, there are models with just a few hundred integer variables that have never been solved to optimality. This is because many combinations of specific integer values for the variables must be tested, and each combination requires the solution of a "normal" linear or nonlinear optimization problem. The number of combinations can rise exponentially with the size of the problem.

Optimizers solve mixed-integer and constraint programming problems using these methods:

A. Branch and Bound
B. Strong Branching
C. Pre-processing and Probing
D. Cut Generation
E. Integer Heuristics
F. Non-traditional Methods

III. TRAVELING SALESMAN PROBLEM: PROBLEM-BASED

This example shows how to use binary integer programming to solve the classic traveling salesman problem. This problem involves finding the shortest closed tour (path) through a set of stops (cities). In this case there are 200 stops, but you can easily change the Stops variable to get a different problem size.
You'll solve the initial problem and see that the solution has subtours. This means the optimal solution found doesn't give one continuous path through all the points, but instead has several disconnected loops. You'll then use an iterative process of determining the subtours, adding constraints, and rerunning the optimization until the subtours are eliminated.

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IV. PROBLEM FORMULATION

Formulate the traveling salesman problem for integer linear programming as follows:

1) Generate all possible trips, meaning all distinct pairs of stops.
2) Calculate the distance for each trip.
3) The cost function to minimize is the sum of the trip distances for each trip in the tour.
4) The decision variables are binary, and associated with each trip, where each 1 represents a trip that exists on the tour, and each 0 represents a trip that is not on the tour.
5) To ensure that the tour includes every stop, include the linear constraint that each stop is on exactly two trips. This means one arrival and one departure from the stop.

A. *Generate Stops*

Generate random stops inside a crude polygonal representation of the continental U.S.

```
load('usborder.mat','x','y','xx','yy');
rng(3,'twister') % Makes stops in Maine & Florida, and is reproducible
nStops = 200; % You can use any number, but the problem size scales as N^2
stopsLon = zeros(nStops,1); % Allocate x-coordinates of nStops
stopsLat = stopsLon; % Allocate y-coordinates
n = 1;
while (n <= nStops)
   xp = rand*1.5;
   yp = rand;
   if inpolygon(xp,yp,x,y) % tTest if inside the border
      stopsLon(n) = xp;
      stopsLat(n) = yp;
      n = n+1;
   end
end
```

B. *Calculate Distances Between Points*

Because there are 200 stops, there are 19,900 trips, meaning 19,900 binary variables (# variables = 200 choose 2).
Generate all the trips, meaning all pairs of stops.

```
idxs = nchoosek(1:nStops,2);
```

Calculate all the trip distances, assuming that the earth is flat in order to use the Pythagorean rule

```
dist = hypot(stopsLat(idxs(:,1)) - stopsLat(idxs(:,2)), ...
          stopsLon(idxs(:,1)) - stopsLon(idxs(:,2)));
lendist = length(dist);
```

With this definition ifthe dist vector the length of a tour is

```
. dist'*trips
```

where trips is the binary vector representing the trips that the solution takes. This is the distance of a tour that you try to minimize.

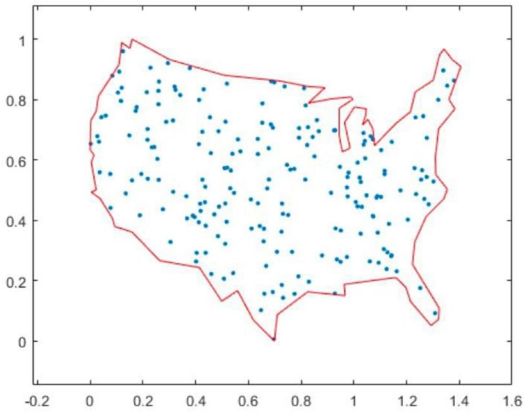
C. *Create Graph and Draw Map*

Represent the problem as a graph. Create a graph where the stops are nodes and the trips are edges.

```
G = graph(idxs(:,1),idxs(:,2));
```

Display the stops using a graph plot. Plot the nodes without the graph edges.

```
figure
hGraph = plot(G,'XData',stopsLon,'YData',stopsLat,'LineStyle','none','NodeLabel',{});
hold on
% Draw the outside border
plot(x,y,'r-')
hold off
```



D. Create Variables and Problems

Create an optimization problem with binary optimization variables representing the potential trips.

```
tsp = optimproblem;
trips = optimvar('trips',lendist,1,'Type','integer','LowerBound',0,'UpperBound',1);
```

Include the objective function in the problem.

```
tsp.Objective = dist'*trip;
```

E. Constraints

Create the linear constraints that each stop has two associated trips, because there must be a trip to each stop and a trip departing each stop.

Use the graph representation to identify all trips starting or ending at a stop by finding all edges connecting to that stop. For each stop, create the constraint that the sum of trips for that stop equals two.

```
constr2trips = optimconstr(nStops,1);
for stop = 1:nStops
    whichIdxs = outedges(G,stop); % Identify trips associated with the stop
    constr2trips(stop) = sum(trips(whichIdxs)) == 2;
end
tsp.Constraints.constr2trips = constr2trips;
```

F. Solve Initial Problem

The problem is ready to be solved. To suppress iterative output, turn off the default display.

```
opts = optimoptions('intlinprog','Display','off');
tspsol = solve(tsp,'options',opts)
tspsol = struct with fields:
    trips: [19900×1 double]
```

G. Visualize Solution

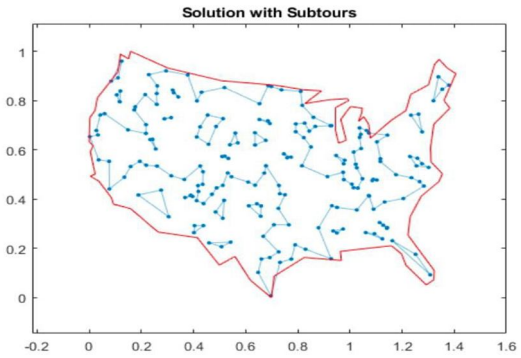
Create a new graph with the solution trips as edges. To do so, round the solution in case some values are not exactly integers, and convert the resulting values to logical.

```
tspsol.trips = logical(round(tspsol.trips));
Gsol = graph(idxs(tspsol.trips,1),idxs(tspsol.trips,2),[],numnodes(G));
% Gsol = graph(idxs(tspsol.trips,1),idxs(tspsol.trips,2)); % Also works in most cases
```

Overlay the new graph on the existing plot and highlight its edges.

```
hold on
highlight(hGraph,Gsol,'LineStyle','-')
title('Solution with Subtours')
```

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue V May 2021- Available at www.ijraset.com



As can be seen on the map, the solution has several subtours. The constraints specified so far do not prevent these subtours from happening. In order to prevent any possible subtour from happening, you would need an incredibly large number of inequality constraints.

H. Subtour Constraints

Because you can't add all of the subtour constraints, take an iterative approach. Detect the subtours in the current solution, then add inequality constraints to prevent those particular subtours from happening. By doing this, you find a suitable tour in a few iterations.
Eliminate subtours with inequality constraints. An example of how this works is if you have five points in a subtour, then you have five lines connecting those points to create the subtour. Eliminate this subtour by implementing an inequality constraint to say there must be less than or equal to four lines between these five points.
Even more, find all lines between these five points, and constrain the solution not to have more than four of these lines present. This is a correct constraint because if five or more of the lines existed in a solution, then the solution would have a subtour (a graph with *n* nodes and *n* edges always contains a cycle).
Detect the subtours by identifying the connected components in Gsol, the graph built with the edges in the current solution. conncomp returns a vector with the number of the subtour to which each edge belongs.

```
tourIdxs = conncomp(Gsol);
numtours = max(tourIdxs); % Number of subtours
fprintf('# of subtours: %d\n',numtours);
```

# of subtours: 27

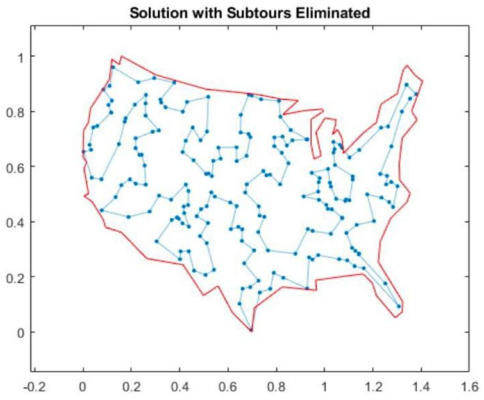
Include the linear inequality constraints to eliminate subtours, and repeatedly call the solver, until just one subtour remains.

```
% Index of added constraints for subtours
k = 1;
while numtours > 1 % Repeat until there is just one subtour
   % Add the subtour constraints
   for ii = 1:numtours
      inSubTour = (tourIdxs == ii); % Edges in current subtour
      a = all(inSubTour(idxs),2); % Complete graph indices with both ends in subtour
      constrname = "subtourconstr" + num2str(k);
      tsp.Constraints.(constrname) = sum(trips(a)) <= (nnz(inSubTour) - 1);
      k = k + 1;
   end

   % Try to optimize again
   [tspsol,fval,exitflag,output] = solve(tsp,'options',opts);
   tspsol.trips = logical(round(tspsol.trips));
   Gsol = graph(idxs(tspsol.trips,1),idxs(tspsol.trips,2),[],numnodes(G));
   % Gsol = graph(idxs(tspsol.trips,1),idxs(tspsol.trips,2)); % Also works in most cases
```

```
    % Plot new solution
    hGraph.LineStyle = 'none'; % Remove the previous highlighted path
    highlight(hGraph,Gsol,'LineStyle','-')
    drawnow

    % How many subtours this time?
    tourIdxs = conncomp(Gsol);
    numtours = max(tourIdxs); % Number of subtours
    fprintf('# of subtours: %d\n',numtours)
end
# of subtours: 20
# of subtours: 7
# of subtours: 9
# of subtours: 9
# of subtours: 3
# of subtours: 2
# of subtours: 7
# of subtours: 2
# of subtours: 1
title('Solution with Subtours Eliminated');
hold off
```



I. Solution Quality

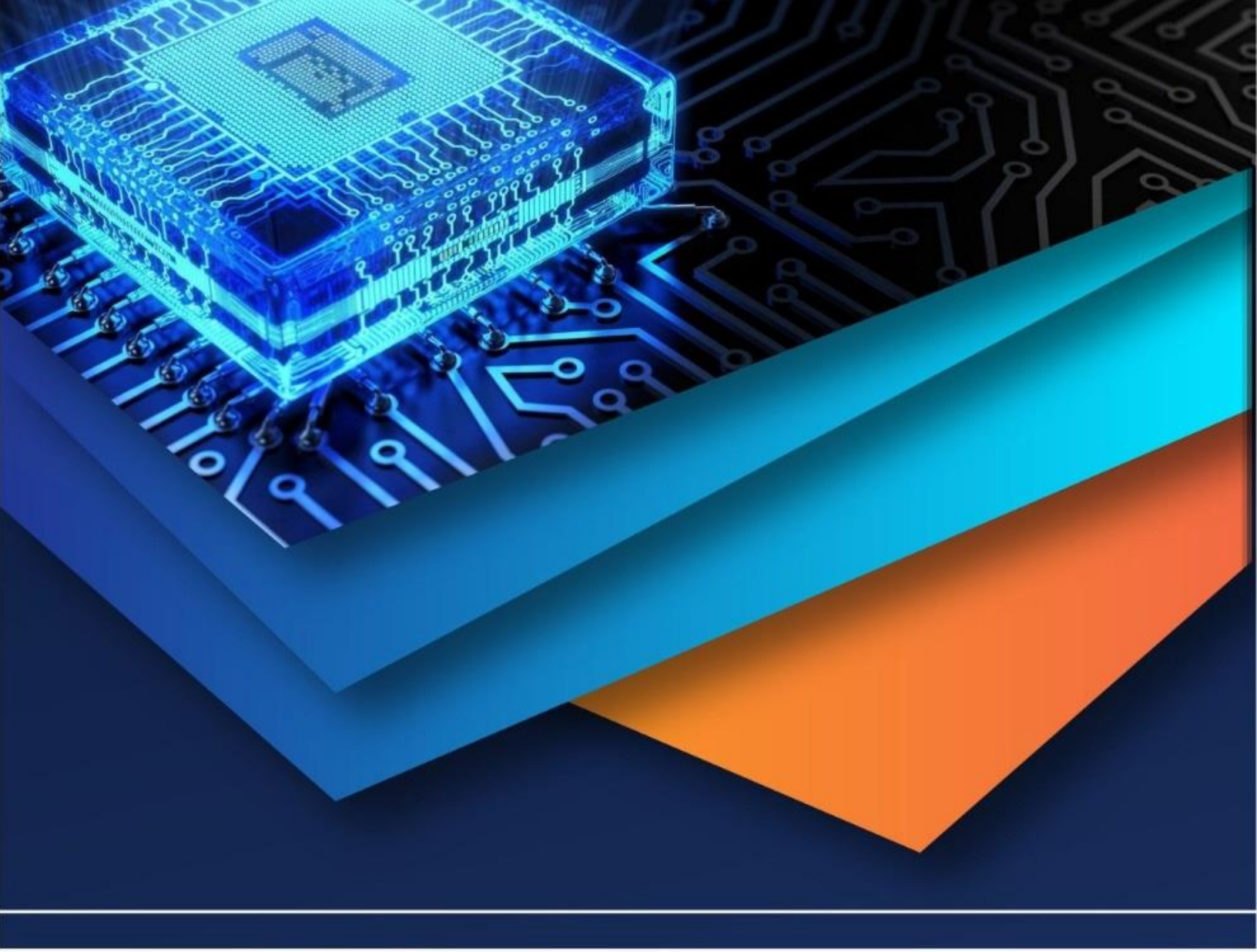
The solution represents a feasible tour, because it is a single closed loop. But is it a minimal-cost tour? One way to find out is to examine the output structure.

```
disp(output.absolutegap)
    0
```

The smallness of the absolute gap implies that the solution is either optimal or has a total length that is close to optimal.

V. CONCLUSION

In this Paper, we have made a detailed view about the Multi Integer linear programming and their classifications. And we made a elaborate study over the traveling salesman problem and its various methods for solving the problems. Which involves finding the shortest closed tour (path) through a set of stops (cities). We have formulated the problem and found the optimal solution successfully.



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