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Inverse Sum Status Energy of a Graph

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Abstract: Motivated by the inverse sum status index, we introduce the inverse sum status matrix $\text{ISS}(G)$ as $\text{ISS}(G) = (\text{ISS})_{n \times n}$

$$\text{ISS} = \begin{cases} \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} & \text{if } u_i \sim v_j \\ 0 & \text{otherwise} \end{cases}$$

Thus we also obtained the results for well known graphs.

Keywords: Inverse sum status energy, Inverse Sum Indeg matrix. 2010 AMS Subject Classification: 05C50.

I. INTRODUCTION

Status of the vertex ‘ u ’ is defined as the sum of distance of all other vertices from ‘ u ’... Inverse Sum Status index is introduced by V. R. Kulli [35]. He defined it as

$$\text{ISS}(G) = \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)}$$

Where σ_u represents the status of the vertex u .

Motivated by the Inverse Sum Status index, We introduce the Inverse Sum Status matrix $\text{ISS}(G)$ as $\text{ISS}(G) = (\text{ISS})_{n \times n}$

$$\text{ISS} = \begin{cases} \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} & \text{if } v_i \sim v_j \\ 0 & \text{otherwise} \end{cases}$$

The Inverse Sum Status energy is given by

$$\text{ISSE}(G) = \sum_{i=1}^n |\beta_i| \quad (1)$$

The inverse sum status energy is given by



Figure 1. Graph with 6 vertices

$$\text{ISSE}(G) = \sum_{i=1}^n |\beta_i|$$

II. INVERSE SUM STATUS ENERGY OF SPECIFIC GRAPH STRUCTURES

1) *Theorem-2.1.* Inverse sum status energy of complete graph K_n is $ISSE(K_n) = ISIE(K_n) = 2\sqrt{2n-4}$. [?]

Proof. For each and every vertex u in K_n , $\sigma(u) = (n-1)$. Then the every ij^{th} – entry of the Inverse Sum Status matrix will be $n-1$.

$$ISS(K_n) = \begin{bmatrix} 0 & n-1 & n-1 & \cdots & n-1 & n-1 \\ n-1 & 0 & n-1 & \cdots & n-1 & n-1 \\ n-1 & n-1 & 0 & \cdots & n-1 & n-1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n-1 & n-1 & \cdots & n-1 & 0 & n-1 \\ n-1 & n-1 & \cdots & n-1 & n-1 & 0 \end{bmatrix}$$

This matrix will be same as Inverse Sum Indeg Matrix. Then the energy will be also as same as Inverse Sum indeg energy. Therefore, $ISSE(K_n) = (n-1)^2$.

2) *Theorem-2.2.* The inverse sum status energy of the star $K_{1,n-1}$

$$ISSE(K_{1,n-1}) = \frac{\sqrt{n-1}}{2^{n-1}},$$

Proof. The status of the central vertex is $n-1$ and the pendant vertices is $2n-3$. Thus the inverse sum status matrix is

$$(K_{1,n-1}) = \begin{bmatrix} 0 & \frac{2n^2 - 5n + 3}{3n-4} & \& \frac{2n^2 - 5n + 3}{3n-4} & \& \dots & \frac{2n^2 - 5n + 3}{3n-4} & \& \frac{2n^2 - 5n + 3}{3n-4} \\ \frac{2n^2 - 5n + 3}{3n-4} & 0 & & & & & 0 & & \dots & 0 & 0 \\ \frac{2n^2 - 5n + 3}{3n-4} & 0 & & & & & 0 & & \dots & 0 & 0 \\ \vdots & \vdots & & & & & \vdots & & \ddots & \vdots & \vdots \\ \frac{2n^2 - 5n + 3}{3n-4} & 0 & & & & & 0 & & \dots & 0 & 0 \\ \frac{2n^2 - 5n + 3}{3n-4} & 0 & & & & & 0 & & \dots & 0 & 0 \end{bmatrix}$$

Clearly, the characteristic equation is $|\beta I - ISS(K_{1,n-1})| = 0$

$$\begin{vmatrix} \beta & -\frac{2n^2 - 5n + 3}{3n-4} & -\frac{2n^2 - 5n + 3}{3n-4} & \& \dots & -\frac{2n^2 - 5n + 3}{3n-4} & -\frac{2n^2 - 5n + 3}{3n-4} \\ -\frac{2n^2 - 5n + 3}{3n-4} & \beta & & & & & 0 & & \dots & 0 & 0 \\ -\frac{2n^2 - 5n + 3}{3n-4} & 0 & & & & & \beta & & \dots & 0 & 0 \\ \vdots & \vdots & & & & & \vdots & & \ddots & \vdots & \vdots \\ -\frac{2n^2 - 5n + 3}{3n-4} & 0 & & & & & 0 & & \dots & \beta & 0 \\ -\frac{2n^2 - 5n + 3}{3n-4} & 0 & & & & & 0 & & \dots & 0 & \beta \end{vmatrix} = 0$$

$\beta^{n-2} \left(\beta + \frac{(\sqrt{n-1})(2n^2 - 5n + 3)}{3n-4} \right) \left(\beta - \frac{(\sqrt{n-1})(2n^2 - 5n + 3)}{3n-4} \right) = 0$. Therefore, the spectrum would be

$$Spec_{SS}(K_{1,n-1}) = \begin{pmatrix} \frac{(\sqrt{n-1})(2n^2 - 5n + 3)}{3n-4} & -\frac{(\sqrt{n-1})(2n^2 - 5n + 3)}{3n-4} & 0 \\ 1 & 1 & n-1 \end{pmatrix}$$

Therefore,

$$ISS(K_{1,n-1}) = \frac{(2\sqrt{n-1})(2n^2 - 5n + 3)}{3n-4}.$$

3) *Theorem-2.3.* The inverse sum status energy of crown graph S_n^0 is

$$ISSE(S_n^0) = 6n(n-1).$$

Proof. The inverse sum status matrix is

$$\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & \frac{3n}{2} & \cdots & \frac{3n}{2} & \frac{3n}{2} \\ 0 & 0 & 0 & \cdots & 0 & \frac{3n}{2} & 0 & \cdots & \frac{3n}{2} & \frac{3n}{2} \\ 0 & 0 & 0 & \cdots & 0 & \frac{3n}{2} & \frac{3n}{2} & \cdots & 0 & \frac{3n}{2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \frac{3n}{2} & \frac{3n}{2} & \cdots & \frac{3n}{2} & 0 \\ 0 & \frac{3n}{2} & \frac{3n}{2} & \cdots & \frac{3n}{2} & 0 & 0 & \cdots & 0 & 0 \\ \frac{3n}{2} & \frac{3n}{2} & 0 & \cdots & \frac{3n}{2} & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{3n}{2} & \frac{3n}{2} & 0 & \cdots & \frac{3n}{2} & 0 & 0 & \cdots & 0 & 0 \\ \frac{3n}{2} & \frac{3n}{2} & \frac{3n}{2} & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$|\beta I - RTZ_3(K_n)| =$$

$$\begin{bmatrix} \beta & 0 & 0 & \cdots & 0 & 0 & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & -\frac{3n}{2} \\ 0 & \beta & 0 & \cdots & 0 & -\frac{3n}{2} & 0 & \cdots & -\frac{3n}{2} & -\frac{3n}{2} \\ 0 & 0 & \beta & \cdots & 0 & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & 0 & -\frac{3n}{2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & 0 \\ 0 & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & \beta & 0 & \cdots & 0 & 0 \\ -\frac{3n}{2} & 0 & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & 0 & \beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{3n}{2} & -\frac{3n}{2} & 0 & \cdots & -\frac{3n}{2} & 0 & 0 & \cdots & \beta & 0 \\ -\frac{3n}{2} & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & 0 & 0 & 0 & \cdots & 0 & \beta \end{bmatrix} = 0$$

Characteristic equation is

$$\left(\beta - \frac{3n}{2}\right)^{n+1} \left(\beta + \frac{3n}{2}\right)^{n-1} \left(\beta(n-1) - \frac{3n}{2}\right) \left(\beta - (n-1) - \frac{3n}{2}\right) = 0$$

spectrum is $\text{Spec}_{\text{ISS}}(S_n^0)$

$$= \begin{pmatrix} \frac{3n}{2}(n-1) & -\frac{3n}{2}(n-1) & -\frac{3n}{2} & \frac{3n}{2} \\ 1 & 1 & n-1 & n-1 \end{pmatrix}$$

Therefore, $\text{ISSE}(S_n^0) = 6n(n-1)$.

4) *Theorem-2.4.* For complete bipartite graph, the Inverse Sum Status energy is

$$\text{ISSE}(K_{m,n}) = 2(mn)^{\frac{5}{2}}$$

Proof. $\text{ISS}(K_{m,n}) = \text{ISS}(K_{m,n}) = (mn)^2 \begin{pmatrix} 0_{m \times m} & J_{m \times n} \\ J_{n \times m} & 0_{n \times n} \end{pmatrix}$

$$\text{Spec}_{\text{ISS}}(K_{m,n}) = \begin{pmatrix} 0 & (mn)^{\frac{5}{2}} & -(mn)^{\frac{5}{2}} \\ m+n-2 & 1 & 1 \end{pmatrix}$$

Therefore, $\text{ISSE}(K_{m,n}) = 2(mn)^{\frac{5}{2}}$.

5) *Theorem-2.5.* The Inverse Sum Status energy of $K_{n \times 2}$ is

$$ISSE(K_{n \times 2}) = 2n(n-1).$$

Proof. Here the status of each vertex is $2n$. hence Inverse Sum Status matrix is

$$\begin{bmatrix} 0 & 0 & n & n & \cdots & n & n & n & n \\ 0 & 0 & n & n & \cdots & n & n & n & n \\ n & n & 0 & 0 & \cdots & n & n & n & n \\ n & n & 0 & 0 & \cdots & n & n & n & n \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ n & n & n & n & \cdots & 0 & 0 & n & n \\ n & n & n & n & \cdots & 0 & 0 & n & n \\ n & n & n & n & \cdots & n & n & 0 & 0 \\ n & n & n & n & \cdots & n & n & 0 & 0 \end{bmatrix}$$

$$Spec_{ISS}(K_{n \times 2}) = \begin{pmatrix} n(n-1) & 0 & -n \\ 1 & n & n-1 \end{pmatrix}$$

$$ISSE(K_{n \times 2}) = 2n(n-1).$$

III. PROPERTIES OF INVERSE SUM STATUS ENERGY

The following results will give the initial coefficients of the Inverse Sum Status characteristic polynomial.

1) *Proposition-3.1.* In the Inverse Sum Status characteristic polynomial $\phi_{ISS}(G, \beta)$, the first three coefficients are 1,0 and $-\sum_{i=1}^n \left[\frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} \right]^2$ respectively.

2) *Proposition-3.2.*

$$\sum_{i=1}^n \beta_i^2 = 2 \sum_{i=1}^n \left[\frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} \right]^2$$

(Where If β_i represents Inverse sum status eigenvalues.)

Proof. We know that

$$\begin{aligned} \sum_{i=1}^n \beta_i^2 &= \sum_{i=1}^n \sum_{j=1}^n a_{ij} a_{ji} \\ &= 2 \sum_{i < j} a_{ij}^2 + \sum_{i=1}^n a_{ii}^2 \\ &= 2 \sum_{i < j} a_{ij}^2 \\ &= 2 \sum_{i=1}^n \left[\frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} \right]^2. \end{aligned}$$

3) *Theorem-1.3.* Let G be a graph with n vertices. Then

$$\text{ISSE}(G) \leq \sqrt{2n \sum_{i=1}^n \left[\frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} \right]^2}$$

Proof. We have Cauchy-Schwartz inequality as

$$\left(\sum_{i=1}^n x_i y_i \right)^2 \leq \left(\sum_{i=1}^n x_i^2 \right) \left(\sum_{i=1}^n y_i^2 \right)$$

Substitute $x_i = 1$ and $y_i = \beta_i$. Then

$$\left(\sum_{i=1}^n |\beta_i| \right)^2 \leq \left(\sum_{i=1}^n 1 \right) \left(\sum_{i=1}^n |\beta_i|^2 \right)$$

Which implies that

$$\text{ISSE}(G) \leq \sqrt{2n \left[\sum_{i=1}^n \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} \right]^2}$$

4) *Theorem-1.4.*

$$\text{ISSE}(G) \geq \sqrt{2 \sum_{i=1}^n \left[\frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} \right]^2 + n(n-1) [\text{Det}(\text{ISS}(G))]^{\frac{2}{n}}}$$

Proof. By definition,

$$\begin{aligned} (\text{ISSE}(G))^2 &= \left(\sum_{i=1}^n |\beta_i| \right)^2 \\ &= \sum_{i=1}^n |\beta_i| \sum_{j=1}^n |\beta_j| \\ &= \left(\sum_{i=1}^n |\beta_i|^2 \right) + \sum_{i \neq j} |\beta_i| |\beta_j|. \end{aligned}$$

Using arithmetic mean and geometric mean inequality, we have

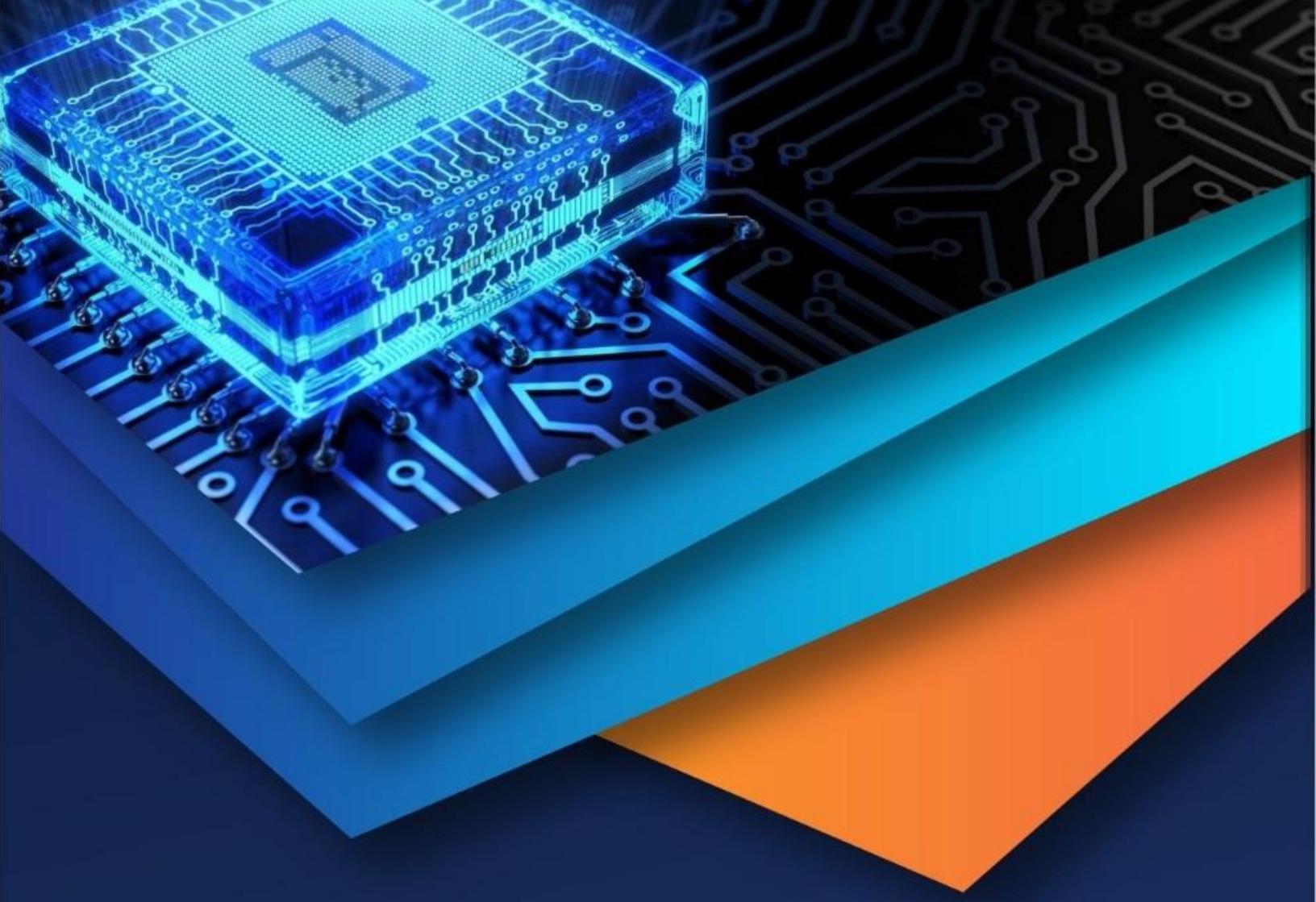
$$\frac{1}{n(n-1)} \sum_{i \neq j} |\beta_i| |\beta_j| \geq \left(\prod_{i \neq j} |\beta_i| |\beta_j| \right)^{\frac{1}{n(n-1)}}.$$

Therefore,

$$\begin{aligned}
 (\text{ISSE}(G))^2 &\geq \sum_{i=1}^n |\beta_i|^2 + n(n+1) \left(\prod_{i \neq j} |\beta_i| |\beta_j| \right)^{\frac{1}{n(n-1)}} \\
 &\geq \sum_{i=1}^n |\beta_i|^2 + n(n+1) \left(\prod_{i=1}^n |\beta_i|^{2(n-1)} \right)^{\frac{1}{n(n-1)}} \\
 &= 2 \left[\sum_{i=1}^n \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} \right] + n(n-1) [\text{ISS}]_n^2. \\
 \text{ISSE}(G) &\geq \sqrt{2 \left[\sum_{i=1}^n \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} \right] + n(n-1) [\text{ISS}]_n^2} ...
 \end{aligned}$$

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