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Inverse Sum Status Energy of a Graph

K. N. Prakasha[1], Padmanava Samanta[2], Niyati Misra[3]
[1]*Department of Mathematics, Vidyavardhaka College Engineering, Mysuru-570002, INDIA*
[2]*Department of Mathematics, Berhampur University, Odisha, INDIA*

Abstract: Motivated by the inverse sum status index, we introduce the inverse sum status matrix $ISS(G)$ *as* $ISS(G) = (ISS)_{n\times n}$

$$ISS = \begin{cases} \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} & if\ u_i \sim v_j, \\ 0 & otherwise \end{cases}$$

Thus we also obtained the results for well known graphs.

Keywords: Inverse sum status energy, Inverse Sum Indeg matrix. 2010 AMS Subject Classification: 05C50.

I. INTRODUCTION

Status of the vertex *'u'* is defined as the sum of distance of all other vertices from *'u'*… Inverse Sum Status index is introduced by V. R. Kulli [35]. He defined it as

$$ISS(G) = \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)}$$

Where σ_u represents the status of the vertex *u.*

Motivated by the Inverse Sum Status index, We introduce the Inverse Sum Status matrix $ISS(G)$ *as* $ISS(G) = (ISS)_{n\times n}$

$$ISS = \begin{cases} \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} & if\ v_i \sim v_j \\ 0 & otherwise \end{cases}$$

The Inverse Sum Status energy is given by

$$ISSE(G) = \sum_{i=1}^{n} |\beta_i| \qquad (1)$$

The inverse sum status energy is given by

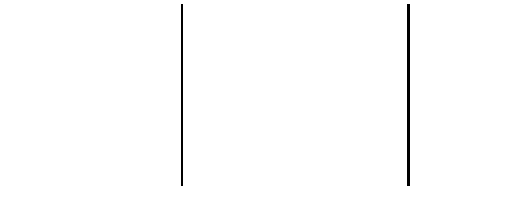

Figure 1. Graph with 6 vertices

$$ISSE(G) = \sum_{i=1}^{n} |\beta_i|$$

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II. INVERSE SUM STATUS ENERGY OF SPECIFIC GRAPH STRUCTURES

1) Theorem-2.1. Inverse sum status energy of complete graph K_n is $ISSE(K_n) = ISIE(K_n) = 2\sqrt{2n-4}$. [?]

Proof. For each and every vertex *u* in K_n , $\sigma(u) = (n-1)$. Then the every ij^{th} – entry of the Inverse Sum Status matrix will be $n-1$.

$$ISS\ (K_n) = \begin{bmatrix} 0 & n-1 & n-1 & \cdots & n-1 & n-1 \\ n-1 & 0 & n-1 & \cdots & n-1 & n-1 \\ n-1 & n-1 & 0 & \cdots & n-1 & n-1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n-1 & n-1 & \cdots & n-1 & 0 & n-1 \\ n-1 & n-1 & \cdots & n-1 & n-1 & 0 \end{bmatrix}$$

This matrix will be same as Inverse Sum Indeg Matrix. Then the energy will be also as same as Inverse Sum indeg energy. Therefore, $ISSE(K_n) = (n-1)^2$.

2) Theorem-2.2. The inverse sum status energy of the star $K_{1,n-1}$

$$ISSE(K_{1,n-1}) = \frac{\sqrt{n-1}}{2^{n-1}},$$

Proof. The status of the central vertex in $n-1$ and the pendant vertices is $2n-3$. Thus the inverse sum status matrix is

$$(K_{1,n-1}) = \begin{bmatrix} 0 & \frac{2n^2-5n+3}{3n-4} & \frac{2n^2-5n+3}{3n-4} & \cdots & \frac{2n^2-5n+3}{3n-4} & \frac{2n^2-5n+3}{3n-4} \\ \frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & 0 & 0 \\ \frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & 0 & 0 \\ \frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

Clearly, the characteristic equation is $\left|\beta I - ISS(K_{1,n-1})\right| = 0$

$$\begin{vmatrix} \beta & -\frac{2n^2-5n+3}{3n-4} & -\frac{2n^2-5n+3}{3n-4} & \cdots & -\frac{2n^2-5n+3}{3n-4} & -\frac{2n^2-5n+3}{3n-4} \\ -\frac{2n^2-5n+3}{3n-4} & \beta & 0 & \cdots & 0 & 0 \\ -\frac{2n^2-5n+3}{3n-4} & 0 & \beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & \beta & 0 \\ -\frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & 0 & \beta \end{vmatrix} = 0$$

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$\beta^{n-2}\left(\beta+\frac{\left(\sqrt{n-1}\right)\left(2n^2-5n+3\right)}{3n-4}\right)\left(\beta-\frac{\left(\sqrt{n-1}\right)\left(2n^2-5n+3\right)}{3n-4}\right)=0$. Therefore, the spectrum would be

$$Spec_{ISS}\left(K_{1,n-1}\right)=\begin{pmatrix}\frac{\left(\sqrt{n-1}\right)\left(2n^2-5n+3\right)}{3n-4} & -\frac{\left(\sqrt{n-1}\right)\left(2n^2-5n+3\right)}{3n-4} & 0\\ 1 & 1 & n-1\end{pmatrix}$$

Therefore,

$$ISSE\left(K_{1,n-1}\right)=\frac{\left(2\sqrt{n-1}\right)\left(2n^2-5n+3\right)}{3n-4}.$$

3) Theorem-2.3. The inverse sum status energy of crown graph S_n^0 is

$$ISSE\left(S_n^0\right)=6n(n-1).$$

Proof. The inverse sum status matrix is

$$\begin{bmatrix}
0 & 0 & 0 & \cdots & 0 & 0 & \frac{3n}{2} & \cdots & \frac{3n}{2} & \frac{3n}{2}\\
0 & 0 & 0 & \cdots & 0 & \frac{3n}{2} & 0 & \cdots & \frac{3n}{2} & \frac{3n}{2}\\
0 & 0 & 0 & \cdots & 0 & \frac{3n}{2} & \frac{3n}{2} & \cdots & 0 & \frac{3n}{2}\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\
0 & 0 & 0 & \cdots & 0 & \frac{3n}{2} & \frac{3n}{2} & \cdots & \frac{3n}{2} & 0\\
0 & \frac{3n}{2} & \frac{3n}{2} & \cdots & \frac{3n}{2} & 0 & 0 & \cdots & 0 & 0\\
\frac{3n}{2} & \frac{3n}{2} & 0 & \cdots & \frac{3n}{2} & 0 & 0 & \cdots & 0 & 0\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\
\frac{3n}{2} & \frac{3n}{2} & 0 & \cdots & \frac{3n}{2} & 0 & 0 & \cdots & 0 & 0\\
\frac{3n}{2} & \frac{3n}{2} & \frac{3n}{2} & \cdots & 0 & 0 & 0 & \cdots & 0 & 0
\end{bmatrix}$$

$\left|\beta I-RTZ_3\left(K_n\right)\right|=$

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$$\begin{bmatrix} \beta & 0 & 0 & \cdots & 0 & 0 & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & -\frac{3n}{2} \\ 0 & \beta & 0 & \cdots & 0 & -\frac{3n}{2} & 0 & \cdots & -\frac{3n}{2} & -\frac{3n}{2} \\ 0 & 0 & \beta & \cdots & 0 & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & 0 & -\frac{3n}{2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & 0 \\ 0 & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & \beta & 0 & \cdots & 0 & 0 \\ -\frac{3n}{2} & 0 & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & 0 & \beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{3n}{2} & -\frac{3n}{2} & 0 & \cdots & -\frac{3n}{2} & 0 & 0 & \cdots & \beta & 0 \\ -\frac{3n}{2} & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & 0 & 0 & 0 & \cdots & 0 & \beta \end{bmatrix} = 0$$

Characteristic equation is

$$\left(\beta - \frac{3n}{2}\right)^{n+1} \left(\beta + \frac{3n}{3}\right)^{n-1} \left(\beta(n-1) - \frac{3n}{2}\right)\left(\beta - (n-1) - \frac{3n}{2}\right) = 0$$

spectrum is $Spec_{ISS}\left(S_n^0\right)$

$$= \begin{pmatrix} \frac{3n}{2}(n-1) & -\frac{3n}{2}(n-1) & -\frac{3n}{2} & \frac{3n}{2} \\ 1 & 1 & n-1 & n-1 \end{pmatrix}$$

Therefore, $ISSE\left(S_n^0\right) = 6n(n-1)$.

4) Theorem-2.4. For complete bipartite graph, the Inverse Sum Status energy is

$$ISSE\left(K_{m,n}\right) = 2(mn)^{\frac{5}{2}}.$$

Proof. $ISS\left(K_{m,n}\right) = ISS\left(K_{m,n}\right) = (mn)^2 \begin{pmatrix} 0_{m\times m} & J_{m\times n} \\ J_{n\times m} & 0_{n\times n} \end{pmatrix}$

$$Spec_{ISS}\left(K_{m,n}\right) = \begin{pmatrix} 0 & (mn)^{\frac{5}{2}} & -(mn)^{\frac{5}{2}} \\ m+n-2 & 1 & 1 \end{pmatrix}$$

Therefore, $ISSE\left(K_{m,n}\right) = 2(mn)^{\frac{5}{2}}$.

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5) Theorem-2.5. The Inverse Sum Status energy of $K_{n\times 2}$ is

$$ISSE(K_{n\times 2}) = 2n(n-1).$$

Proof. Here the status of each vertex is 2*n*. hence Inverse Sum Status matrix is

$$\begin{bmatrix} 0 & 0 & n & n & \cdots & n & n & n & n \\ 0 & 0 & n & n & \cdots & n & n & n & n \\ n & n & 0 & 0 & \cdots & n & n & n & n \\ n & n & 0 & 0 & \cdots & n & n & n & n \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ n & n & n & n & \cdots & 0 & 0 & n & n \\ n & n & n & n & \cdots & 0 & 0 & n & n \\ n & n & n & n & \cdots & n & n & 0 & 0 \\ n & n & n & n & \cdots & n & n & 0 & 0 \end{bmatrix}$$

$$Spec_{ISS}(K_{n\times 2}) = \begin{pmatrix} n(n-1) & 0 & -n \\ 1 & n & n-1 \end{pmatrix}$$

$$ISSE(K_{n\times 2}) = 2n(n-1).$$

III. PROPERTIES OF INVERSE SUM STATUS ENERGY

The following results will give the initial coefficients of the Inverse Sum Status characteristic polynomial.

1) Proposition-3.1. In the Inverse Sum Status characteristic polynomial $\phi_{ISS}(G,\beta)$, the first three coefficients are 1,0 and $-\sum_{i=1}^{n}\left[\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2$ respectively.

2) Proposition-3.2.

$$\sum_{i=1}^{n}\beta_i^2 = 2\sum_{i=1}^{n}\left[\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2$$

(Where If β_i represents Inverse sum status eigenvalues.)

Proof. We know that

$$\sum_{i=1}^{n}\beta_i^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\,a_{ji}$$

$$= 2\sum_{i<j}a_{ij}^2 + \sum_{i=1}^{n}a_{ii}^2$$

$$= 2\sum_{i<j}a_{ij}^2$$

$$= 2\sum_{i=1}^{n}\left[\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2.$$

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3) Theorem-1.3. Let G be a graph with n vertices. Then

$$ISSE(G) \le \sqrt{2n\sum_{i=1}^{n}\left[\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2}$$

Proof. We have Cauchy-Schwartz inequality as

$$\left(\sum_{i=1}^{n} x_i y_i\right)^2 \le \left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i^2\right)$$

Substitute $x_i = 1 \; and \; y_i = \beta_i$. Then

$$\left(\sum_{i=1}^{n}|\beta_i|\right)^2 \le \left(\sum_{i=1}^{n}1\right)\left(\sum_{i=1}^{n}|\beta_i|^2\right)$$

Which implies that

$$ISSE(G) \le \sqrt{2n\left[\sum_{i=1(\sigma_u+\sigma_v)}^{n}\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2}$$

4) Theorem-1.4.

$$ISSE(G) \ge \sqrt{2\sum_{i=1}^{n}\left[\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2 + n(n-1)\left[Det\left(ISS(G)\right)\right]^{\frac{2}{n}}}$$

Proof. By definition,

$$(ISSE(G))^2 = \left(\sum_{i=1}^{n}|\beta_i|\right)^2$$

$$= \sum_{i=1}^{n}|\beta_i|\sum_{j=1}^{n}|\beta_j|$$

$$= \left(\sum_{i=1}^{n}|\beta_i|^2\right) + \sum_{i\ne j}|\beta_i||\beta_j|.$$

Using arithmetic mean and geometric mean inequality, we have

$$\frac{1}{n(n-1)}\sum_{i\ne j}|\beta_i||\beta_j| \ge \left(\prod_{i\ne j}|\beta_i||\beta_j|\right)^{\frac{1}{n(n-1)}}.$$

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Therefore,

$$(ISSE(G))^2 \geq \sum_{i=1}^{n} |\beta_i|^2 + n(n+1)\left(\prod_{i\neq j} |\beta_i||\beta_j|\right)^{\frac{1}{n(n-1)}}$$

$$\geq \sum_{i=1}^{n} |\beta_i|^2 + n(n+1)\left(\prod_{i=1}^{n} |\beta_i|^{2(n-1)}\right)^{\frac{1}{n(n-1)}}$$

$$= 2\left[\sum_{i=1}^{n} \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)}\right] + n(n-1)\,[[ISS]]^{\frac{2}{n}}.$$

$$ISSE(G) \geq \sqrt{2\left[\sum_{i=1}^{n} \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)}\right] + n(n-1)[[ISS]]^{\frac{2}{n}}} \ldots$$

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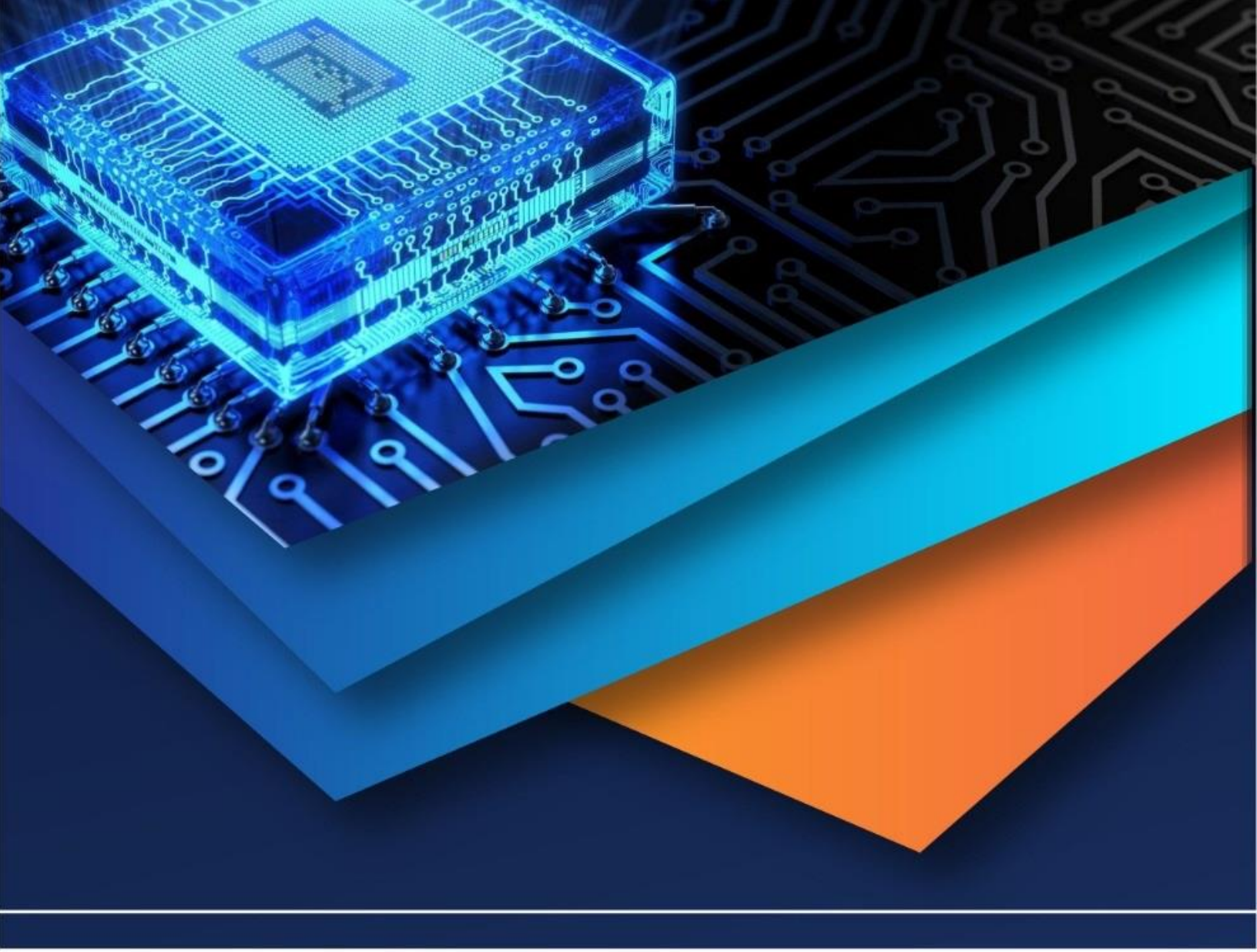

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