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On (1, 2) - Domination of Certain Graphs

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Abstract: A vertex subset S of a graph G = (V,E) is called a (1,2)-dominating set if S is having the property that for every vertex v in V- S there is atleast one vertex in S of distance 1 from v and a vertex in S at a distance atmost 2 from v. The minimum cardinality of a (1, 2)-dominating set of G, denoted by $\Upsilon_{(1,2)}(G)$, is called the (1, 2)-domination number of G. In this paper we discuss about the (1, 2)-dominating set of Shell graph C(n,n-3,), Jewel graph J_n and Comb graph $P_n \Theta K_1$.
Keywords: (1, 2)-Domination, (1,2)-Domination number, Shell graph C(n,n-3), Jewel graph J_n and Comb graph $P_n \Theta K_1$.

I. INTRODUCTION

The concept of (1, 2)-domination was introduced by S.M. Hedetniemi, S.T. Hedetniemi and co-authors [6]. The author had explained the concept of (1, 2)-domination as an extension from secondary domination through an example. Initially he had proposed the idea in a general form as (1, k)-domination. And later he had concentrated on comparison between those dominations.
If S is a dominating set then every vertex, say $v \in V\text{-}S$ is adjacent to atleast one vertex u_1 of distance 1 and a vertex u_2 of distance atmost 2 from S, assuming the vertices of S are considered to be guard vertices of $V\text{-}S$. If there is a problem in any of the vertices of $V\text{-}S$ then it is easy to send its adjacent vertex, say guard vertex at one step from S. In this situation if it is required to send another guard vertex to assist the previous one which could also be achieved. But the question that how much distance these guards have to be placed for convenient usage is what plays the major role. The focus on the length of time it takes a second guard to arrive at any vertex $v \varepsilon V$ is called Secondary domination and in that context (1, 2) domination was introduced.
In this paper we determine the (1, 2) domination number of Shell graph and Jewel graph and comb graph.

II. PRELIMINARIES

1) *Definition: 2.1 [5]*

A (1, 2)-dominating set of a graph $G= (V, E)$ is a set S having the property that for every vertex v in $V\text{-}S$ there is atleast one vertex in S of distance 1 from v and a vertex in S at a distance atmost 2 from v.

2) *Definition: 2.2 [5]*

The order of the smallest (1, 2)-dominating set of G is called the (1, 2) - domination number of G and we denote it by $\Upsilon_{(1,2)}$.

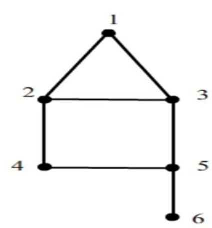

Figure 2.1 : (1,2)-dominating set of G ={1,3,5,6}, a minimum dominating set of G= {1,3,5} and $\Upsilon_{(1,2)}(G)$= 3

III. MAIN RESULTS

1) *Definition: 3.1 Shell Graph C (n, n-3): [1]*

A shell graph is defined as a cycle C_n with n-3 chords sharing a common end point called the apex. Shell graphs are denoted by C (n; n - 3), $n \geq 4$.

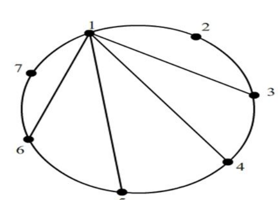

Figure 3.1: Shell graph C (7, 4)

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2) Definition: 3.2 Jewel Graph J_n: [4]

The Jewel graph J_n is a graph with vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and an edge set $E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$.

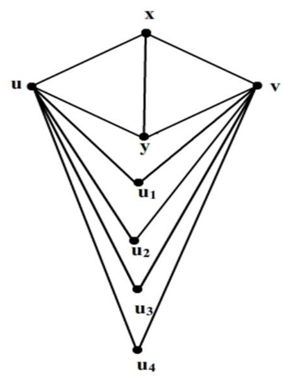

Figure 3.2: Jewel graph J_4

3) Definition: 3.3 Comb Graph $P_n \odot K_1$: [5]

Let P_n be a Path graph with n vertices. The Comb graph is defined as $P_n \odot K_1$. It is denoted by P_n^{+}. It has 2n vertices and 2n-1 edges.

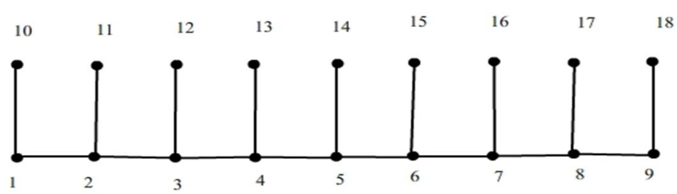

Figure 3. 3: Comb graph $P_9 \odot K_1$

a) Theorem :3.1 The (1,2)-domination number of Shell graph C(n; n - 3), $n \geq 4$, is $\Upsilon_{(1,2)}(C(n,n-3)) = 3$.

Proof: Let G be a Shell graph C (n; n - 3) with n vertices. Label the vertices of the cycle C_n as 1, 2… n in clockwise direction. Fix vertex 1 to be the apex of the graph and construct (n - 3) chords sharing a common vertex with the apex. Now we claim that the set {1, 2, 3} is the minimum (1, 2)-dominating set.

Let S_1 = {1, 2}. All other vertices of V-S_1 is of distance one from vertex 1 and of distance two from vertex 2 except the vertex 3. Hence we need to include vertex 3 in S_1. Therefore the minimal (1, 2)-dominating set of a Shell graph C (n, n-3) is {1, 2, 3} and $\Upsilon_{(1,2)}(C(n,n-3)) = 3$.

Example: 3.1

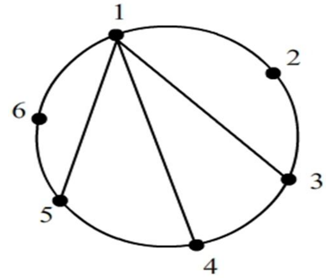

Figure 3.4: $\Upsilon_{(1, 2)}$ (C (6, 3)) =3

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b) Theorem :3.2 The (1,2)-domination number for Jewel graphs J_n, $n \geq 1$ is $\Upsilon_{(1,2)}(J_n) = 2$.

Proof: Let us consider G to be a Jewel graph J_n, $n \geq 1$, with vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and an edge set $E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : : 1 \leq i \leq n\}$.

Let $S = \{u, u_k\}$ for some k. Since all the vertices of V-S is at a distance 1 from the vertex u except the vertex v and at a distance 2 from the vertex u_k except v. Here v is of distance 1 from the vertex u_k and at a distance 2 from the vertex u. Therefore the minimum (1, 2)-dominating set of a Jewel graph is $\{u, u_k\}$ and $\Upsilon_{(1,2)}(J_n) = 2$.

Example: 3.2

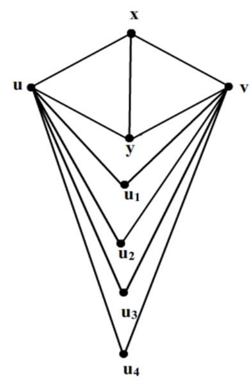

Figure 3.5: $\Upsilon_{(1, 2)}(J_4) = 2$

c) Theorem :3.3 The (1,2)-domination number for Comb graphs $P_n \odot K_1$, $n > 2$ is $\Upsilon_{(1,2)}(P_n \odot K_1) = n$.

Proof: Lest G be a Comb graph $Pn \odot K_1$, $n > 2$. Labelling the vertices of the path as {1, 2, 3… n} from left to right and the pendant vertices as {n+1, n+2... 2n} from left to right. Let us generate the minimum (1, 2)-dominating set S of a Comb graph by an algorithm explained below,

- *Step 1:* Since the vertex 1 is at distance 1 from vertex n+1 and at a distance 2 from vertex n+2 and also the vertex 2 is at a distance 1 from n+2 and at a distance 2 from the vertex n+1. We conclude the minimum (1, 2)-dominating set for vertices 1 and 2 is $S_1 = \{n+1, n+2\}$.
- *Step 2 :* Also if $S_2 = \{1,2\}$ dominates $V_2 = \{n+1,n+2,3\}$ but (n+3) has to be in S_2 as it does not have a vertex at a distance one from S_2 . Therefore either $S_2 = \{1,2,n+3\}$ dominate $V_2 = \{n+1,n+2,3\}$ or $S_2 = \{1,2,3\}$ dominates $V_2 = \{n+1,n+2,n+3\}$. And concluding the set S_2 is the minimum (1, 2)-dominating set of V_2.

On continuing the same procedure as above upto the n^{th} vertex and generating the final (1,2)-dominating set as $S_n = \{ n+1, n+2,\ldots, n+(n-1), 2n \}$ and $V_n = \{ 1, 2, \ldots, n-1, n \}$. And we see that the set S_n is the minimum (1, 2)-dominating set of the set V_n. At the n^{th} step we get V = S_n Ù V_n . The set S_n is considered as the minimum (1, 2)-dominating set S of Comb graph G.

Example: 3.3

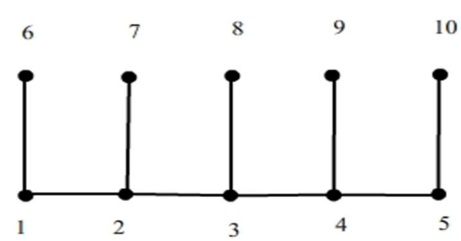

Figure 3.6: $\Upsilon_{(1, 2)}(P_5 \odot K_1) = 5$

IV. CONCLUSION

The problem of finding (1, 2) domination number is solved for Shell graph, Comb graph and Jewel graph and the (1, 2) domination numbers of the corresponding graphs are determined.

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