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A Study on IFM/IFG/1 Vacation Queueing System with Breakdown, Repair and Server Timeout in (Triangular, Trapezoidal & Pentagon) Intuitionistic Fuzzy Set using α-Cuts

Y. Saritha[1], K. Chandan[2]
[1]Research Scholar, [2]Professor, Department of Statistics, Acharya Nagarjuna University, Guntur, Andhra Pradesh, India

Abstract: In this, we studied and investigated IFM/IFG/1 vacation queueing system/waiting line with server breakdowns, repair and server timeout, by using (Triangular Trapezoidal and Pentagonal) IF (Intuitionistic Fuzzy) numbers with the application of IFS (Intuitionistic fuzzy set). Here we operate single server vacation queueing system in the following manner; when the system finds empty, the server waits for fixed time 'c' known as server timeout. At the expiration of this time, if no one arrives into the system, then server takes the vacation. If anyone arrived in the system during the timeout period as well as in vacation the server commences the service otherwise, he will go for another vacation. If the system had occurred with a breakdown, just after a break down the server undergoes for repair. After the repaired process is completed the server restarts the service to the arrived customer. By the approach of IFS properties, we develop the membership function of the system performance are of fuzzy nature. Based on IFS α-cut approach the Intuitionistic fuzzy queues are reduced to a family of ICS (Intuitionistic Crisp Set). The numerical results are illustrated to the model.

Keywords: Server timeout, Breakdown, Repair, Membership function of (Triangular, Trapezoidal & Pentagonal) IF numbers, IFS α-cuts and IFS Properties.

I. INTRODUCTION

At the present queue with vacation had wider applications or simply called vacation models attracted with great attention of queueing researchers and became an active research area due to their wide applications in many areas like computer systems, Banking and communication networks, etc., Queueing system with server vacations idea was first discussed by Levy and Yechiel [5] they introduced the system idle time in the utilization of idle time in an M/G/1 queueing system.

When the system is empty, the server will wait for a fixed time known as server timeout. E. Ramesh Kumar and Y. Praby Loit [4] derived an expression for the mean waiting time of a vacation queueing system in which server does not take immediately another vacation upon returning from a vacation and finding system empty, as in multiple vacation scheme or wait indefinitely for a customer to arrive. Y. Saritha, K. Satish Kumar and K. Chandan [8] derived the expected system length using different bulk size distributions for MX/G/1 vacation Queueing systems with server timeout. Y. Saritha, K. Satish Kumar, V. N. RamaDevi and K. Chandan derived the expected system length for M/G/1 Vacation Queueing System with breakdown, repair and Server Timeout [10]. K. Satish Kumar, K. Chandan and Y. Saritha derived the explicit expressions for the system length in steady and transient states for N-Policy $M/E_k/1$ Vacation Queueing System with Server Start-Up and Time-Out. Also designed cost structure of this queuing system in both steady and transient states and find the total expected cost [7]. But in fuzzy queuing theory, it is best-described arrival rate and service rates through linguistic terms of very low, low, moderate, high and very high. A. Kaufmann, [3] had introduced the Theory of Fuzzy Subsets and these models are formulated to calculate the lower and upper limits and can be split into eleven distinct points through the α-cuts. Among the various generalizations of fuzzy sets such as Intuitionistic fuzzy sets and interval-valued fuzzy sets have gained much attention from the researchers. In fuzzy set theory, the membership of an element of a fuzzy set is a single value and it lies in between zero and one. However, in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus of the membership degree because there may be some hesitation degree. Atanassov [2] derived Intuitionistic fuzzy sets (IFS) which incorporate the degree of hesitation called hesitation margin (and is defined as 1 minus of the sum of membership and non-membership degrees respectively). A. Nagoor Gani and Mohamed [1] proposed and derived the method for ranking and Generalized Trapezoidal Intuitionistic Fuzzy numbers. k. Ponnivalavan and Pathinathan [6] introduced Intuitionistic Pentagon fuzzy numbers with basic arithmetic operations and used the Accuracy function as a Ranking parameter. Y. Saritha, K. Satish Kumar, K. Chandan and G. Sridhar [9] study and derived on N-policy FM/FG/1 vacation queuing system with server timeout in triangular, trapezoidal and pentagonal fuzzy numbers using α-cuts.

The objective of this paper is to derive IFM/IFG/1 vacation queueing system with breakdown, repair and server timeout in (Triangular, Trapezoidal & Pentagonal) IFS. Numerical solution for various IFS to the single server vacation queueing system with the expected length is presented by using IFS α-cuts Properties for IF (Triangular, Trapezoidal & Pentagon).

II. FUZZY SET THEORY

1) *Set:* According to Cantor's definition, a set R is any collection of definite, distinguishable objects to our intuition or of our intellect to be conceived as a whole. The objects are called the elements or the members of R.

For a set in cantors view, some of the following properties are

a) $A \neq \{A\}$.
b) If $A \in A$ and $B \in B$ then $A \notin B$.
c) The set of all subsets of A is denoted as 2^A.
d) ϕ is the empty set and thus very important.

2) *Fuzzy Set:* A fuzzy set R in X is to be characterized by its membership function X: A $\rightarrow$ [1, 0]. Here X is represented with a non – empty set.

3) *Triangular Fuzzy Number:* A triangular fuzzy number $\tilde{E}$ is defined by (e1, e2, e3), where $e_i \in$ R and e1 $\leq$ e2$\leq$ e3 and its membership function is.

$$\mu_{\tilde{E}}(x) = \begin{cases} 0, \ for x < e_1 \\ \dfrac{x - e_1}{e_2 - a_1} \ fo e_1 \leq x \leq e_2 \\ \dfrac{e_3 - x}{e_3 - e_2} \ for e_2 \leq x \leq e_3 \\ 0, \ for e_3 > 0 \end{cases}$$

4) *Trapezoidal Fuzzy Number:* A trapezoidal fuzzy number $\tilde{E}$ is defined by (e1, e2, e3, e4) where $e_i \in$ R and e1≤e2≤ e3≤e4 and its membership function is.

$$\mu_{\tilde{E}}(x) = \begin{cases} 0, \ for x < e_1 \\ \dfrac{x - e_1}{e_2 - e_1} \ for e_1 \leq x \leq e_2 \\ 1, \ for e_2 \leq x \leq e_3 \\ \dfrac{e_4 - x}{e_4 - e_3} \ for e_3 \leq x \leq e_4 \\ 0, \ for e_4 > 0 \end{cases}$$

5) *Pentagonal Fuzzy Number:* A Pentagonal fuzzy number $\tilde{E}$ is defined by (e1, e2, e3, e4, e5) where $e_i \in$ R and e1≤e2≤ e3≤ e4≤ e5 and its membership function is.

$$\mu_{\tilde{E}}(x) = \begin{cases} 0, \ for x < e_1 \\ \dfrac{x - e_1}{e_2 - e_1} \ for e_1 \leq x \leq e_2 \\ \dfrac{x - e_2}{e_3 - e_2} \ for e_2 \leq x \leq e_3 \\ 1, \ if x = e_3 \\ \dfrac{e_4 - x}{e_4 - e_3} \ for e_3 \leq x \leq e_4 \\ \dfrac{e_5 - x}{e_5 - e_4} \ for e_4 \leq x \leq e_5 \\ 0, \ if e_5 > 0 \end{cases}$$

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6) *Intuitionistic fuzzy sets (IFS):* IFS is sets whose elements have degrees of membership and non-membership Functions. The term (IFS) have been introduced by Krassimir Atanassov and it was an extension of the Lotfi Zadeh's notion of fuzzy set, which has itself, extends to the classical notion of a set. In classical set theory, the elements of membership in a set are assessed in a binary term with according to a bivalent condition -if an element either belongs or does not belong to the set. As an extension, the fuzzy set theory permits the gradual assessment of the membership elements in a set; this is described with the aid of a membership function-valued in the real type unit interval [0, 1]. The theory of Intuitionistic fuzzy sets had further extended to both concepts by allowing the assessment of the elements into two functions: like for membership and also for non-membership, which belong to the real type unit interval [0, 1] and whose sum belongs to the same interval, as well. Here (IFS) generalize fuzzy sets, since the indicator functions of fuzzy sets are classified in the special cases of the membership and non-membership functions for the (IFS), in the case when the strict equality exists: i.e. the non-membership function fully complements to the membership function 1, not leaving room for any uncertainty.

7) *Triangular Intuitionistic Fuzzy Set (TIFS):* A Triangular Intuitionistic fuzzy number $\tilde{E}$ is defined by (e_1, e_2, e_3), where $e_i \in$ R, and $e_1 \le e_2 \le e_3$ and its membership and non membership functions are.

$$\mu_{\tilde{E}}(x) = \begin{cases} 0, \; for x < e_1 \\ \dfrac{x - e_1}{e_2 - a_1} \; foe_1 \le x \le e_2 \\ \dfrac{e_3 - x}{e_3 - e_2} \; fore_2 \le x \le e_3 \\ 0, \; fore_3 > 0 \end{cases} \qquad V_{\tilde{E}}(x) = \begin{cases} 0, \; for x < e_1 \\ \dfrac{e_2 - x}{e_2 - e_1} \; fore_1 \le x \le e_2 \\ \dfrac{x - e_2}{e_3 - e_2} \; fore_2 \le x \le e_3 \\ 1, \; for x > e_3 \end{cases}$$

8) *Trapezoidal Intuitionistic Fuzzy Set (TRIFS):* A Trapezoidal Intuitionistic fuzzy set $\tilde{E}$ is defined by (e1, e2, e3, e4), where $e_i \in$ R and e1≤e2≤e3≤e4 and its membership and non membership functions are.

$$\mu_{\tilde{E}}(x) = \begin{cases} 0, \; for x < e_1 \\ \dfrac{x - e_1}{e_2 - e_1} \; fore_1 \le x \le e_2 \\ 1, \; fore_2 \le x \le e_3 \\ \dfrac{e_4 - x}{e_4 - e_3} \; fore_3 \le x \le e_4 \\ 0, \; fore_4 > 0 \end{cases} \qquad V_{\tilde{E}}(x) = \begin{cases} 0, \; for x < e_1 \\ \dfrac{e_2 - x}{e_2 - e_1} \; fore_1 \le x \le e_2 \\ 0, \; fore_2 \le x \le e_3 \\ \dfrac{x - e_3}{e_4 - e_3} \; fore_3 \le x \le e_4 \\ 1, \; for x > e_4 \end{cases}$$

9) *Pentagonal Intuitionistic Fuzzy Set (PIFS):* A Pentagonal Intuitionistic fuzzy set $\tilde{E}$ is defined by (e_1, e_2, e_3, e_4, e_5) where $e_i \in$ R and $e_1 \le e_2 \le e_3 \le e_4 \le e_5$ and its membership and non membership functions are.

$$\mu_{\tilde{E}}(x) = \begin{cases} 0, \; for x < e_1 \\ \dfrac{x - e_1}{e_2 - e_1} \; fore_1 \le x \le e_2 \\ \dfrac{x - e_2}{e_3 - e_2} \; fore_2 \le x \le e_3 \\ 1, \; if x = e_3 \\ \dfrac{e_4 - x}{e_4 - e_3} \; fore_3 \le x \le e_4 \\ \dfrac{e_5 - x}{e_5 - e_4} \; fore_4 \le x \le e_5 \\ 0, \; if e_5 > 0 \end{cases} \qquad V_{\tilde{E}}(x) = \begin{cases} 0, \; for x < e_1 \\ \dfrac{e_2 - x}{e_2 - e_1} \; fore_1 \le x \le e_2 \\ \dfrac{e_3 - x}{e_3 - e_2} \; fore_2 \le x \le e_3 \\ 0, \; if x = e_3 \\ \dfrac{x - e_3}{e_4 - e_3} \; fore_3 \le x \le e_4 \\ \dfrac{x - e_4}{e_5 - e_4} \; fore_4 \le x \le e_5 \\ 1, \; for x > e_5 \end{cases}$$

10) *IFS α-cut:* The set of all the levels of α ∈ [0, 1] and, it represents the distinct α-cuts of a given IFS A are called a Level set of Λ (A). In other words Λ (A) = {α Λ γ / (x) = α γ, (x) ≥ 0 A, A α μ α γ for some x ∈ X}. Where Λ denotes the level set of IFS a defined on X

III. MODEL DESCRPTION

Let us consider an M/G/1 queueing system with breakdown, repair and server timeout. Here customers are assumed to arrive according to a Poisson process with rate λ. Here the service times assumed to follow the general distribution with rate μ and commence the service in FIFS discipline. Whenever the system becomes empty, the server waits for fixed time 'c', is called server timeout. At this time if a customer arrives, then the server return to the system and does service. At the expiration of the time if no customer arrives, then the server takes the vacation. During vacation, if the server finds the customer is waiting in the queue then the server returns to the system and commences service to the waiting or arrived customer exhaustively; otherwise, it takes another vacation. However, if the server works continuously, service is interrupted due to Server breakdown. Here the server is unable to work unless the machine gets repaired, therefore the server should undergo in the repair process. When the repair work is completed the server immediately returns to the service system and does the service for the waiting customer as well as the arrived customer in a queue.

An expression of expected system length for M/G/1 vacation queueing system to breakdown, repair and server timeout.

$$G_L^1(1) = E(L) = \frac{\lambda^2(\lambda+\gamma)}{\gamma^3(1-e^{-\lambda c}) + \lambda\gamma(\lambda+\gamma)} + \frac{\alpha}{1+\alpha} + \frac{\lambda(1+\alpha d)}{\mu-\lambda(1+\alpha d)} + \frac{\alpha(\lambda d)^2}{2(\mu-\lambda(1+\alpha d))} \quad (1)$$

IV. BASIC PROPERTIES OF IFS

(Operations, relations, operators)

For every two Intuitionistic fuzzy sets and various relations and operations have been defined, most important of which are:

1) *Addition*

$$C + Diff\,(\forall x \in E)\{\mu_C(x) + \mu_D(x) = Z = V_C(x) + V_D(x)\}$$

2) *Subtraction*

$$C + Diff\,(\forall x \in E)\{\mu_C(x) + \mu_D(x) = Z = V_C(x) + V_D(x)\}$$

3) *Inclusion*

$$C \subset Diff\,(\forall x \in E)\{\mu_C(x) \le \mu_D(x)\ \&\ V_C(x) \ge V_D(x)\}$$

$$C \supset DiffD \supset C$$

4) *Equality*

$$C = Diff\,(\forall x \in E)\{\mu_C(x) = \mu_D(x)\ \&\ V_C(x) = V_D(x)\}$$

5) *Classical Negation*

$$\overline{C} = \{\langle x\mu_C(x), V_C(x)\rangle / x \in E\}$$

6) *Conjunction*

$$C \cap D = \{\langle x, \min(\mu_C(x), \mu_D(x)), \max(V_C(x), V_D(x)\rangle / (x \in E)\}$$

7) *Disjunction*

$$C \cup D = \{\langle x, \max(\mu_C(x), \mu_D(x)), \min(V_C(x), V_D(x) / (x \in E)\}$$

These operations and relations are defined for the above system length. More interesting are the model operators that can be defined over Intuitionistic fuzzy sets. These have no analogue in fuzzy set theory.

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V. DSW ALGORITHM

The DSW algorithm consists of the following steps:

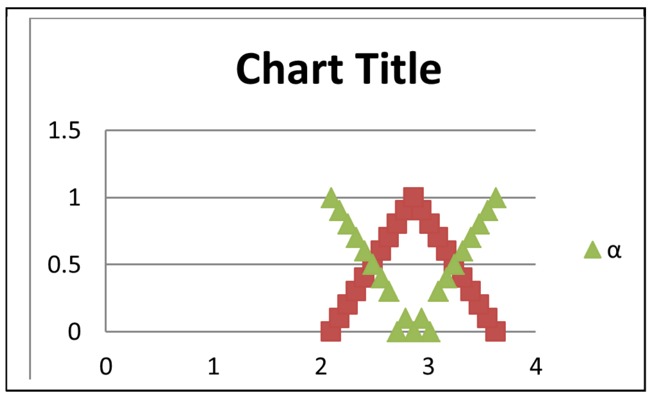
1) Select the α cut value where it lies in between $0 \le \alpha \le 1$.
2) Find the intervals in the input membership functions and also the input of non-membership function which correspond to this α.
3) Using standard binary interval operations and then compute the interval for the output of both the membership & non-membership function for the selectedα cut level.
4) Repeat the steps 1 – 3 for eleven different values of α to complete the α cut representation of both membership & non-membership function of the solution

VI. NUMERICAL RESULTS

A. *Triangular Intuitionistic Fuzzy Numbers With Graphical Representation*

Take the arrival rate, service rate and vacation rate of Triangular Intuitionistic fuzzy number (TIFS) represented for eqn (1) by A = [1,2,3], B = [4,5,6], The length of the system of α is [1+α(2-1), 3− α(3-2)] and [4+ α(5-4), 6−α(6-5)],Where x = [1+α(2-1) , 3− α(3-2)] ,Y = [4+α(5-4), 6−α(6-5)]. Non-parameter value c=2. Z(= x+y)={(1+4)+ α[(2-1)+(5-4)],(3+6)- α[(3-2)+(6-5)]}

α-alpha	TIFS E(L) Lower limit	TIFS E(L) Upper limit
0	0.4134	0.3334
0.1	0.4078	0.3361
0.2	0.4024	0.3390
0.3	0.3973	0.3421
0.4	0.3923	0.3451
0.5	0.3875	0.3483
0.6	0.3829	0.3516
0.7	0.3785	0.3550
0.8	0.3742	0.3586
0.9	0.3701	0.3623
1	0.3661	0.3661

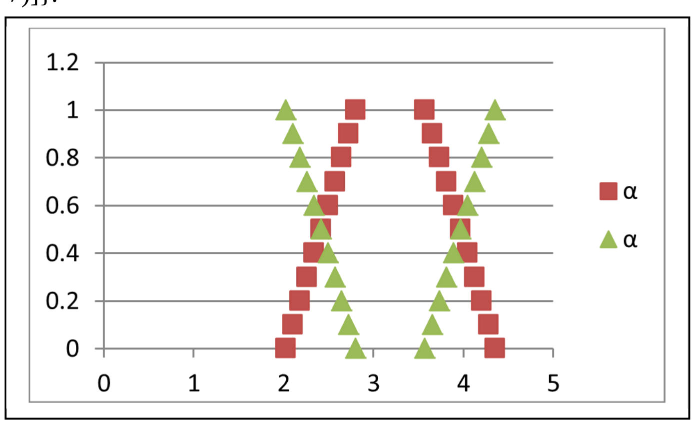


B. *Trapezoidal Intuitionistic Fuzzy Numbers With Graphical Representation*

Take the arrival rate, service rate and vacation rate of Trapezoidal Intuitionistic fuzzy number (TRIFS) represented for eqn (1) by A = [1,2,3,4], B = [5,6,7,8], The length of the system of α is [1+α(2-1), 4− α(4-3)] and [5+ α(6-5), 8−α(8-7)],Where x = [1+α(2-1),4−α(4-3)] ,Y = [5+α(6-5), 8−α(8-7)]. Non-parameter value c=2.

Z=(x+y)={(1+5)+ α[(2-1)+(6-5)],(4+8)- α[(4-3)+(8-7)]}.

α-alpha	TRIFS E(L) Lower limit	TRIFS E(L) Upper limit
0	0.4119	0.3042
0.1	0.4077	0.3116
0.2	0.4024	0.3137
0.3	0.3973	0.3159
0.4	0.3924	0.3182
0.5	0.3875	0.3205
0.6	0.3830	0.3228
0.7	0.3785	0.3253
0.8	0.3742	0.3279
0.9	0.3701	0.3305
1	0.3661	0.3332

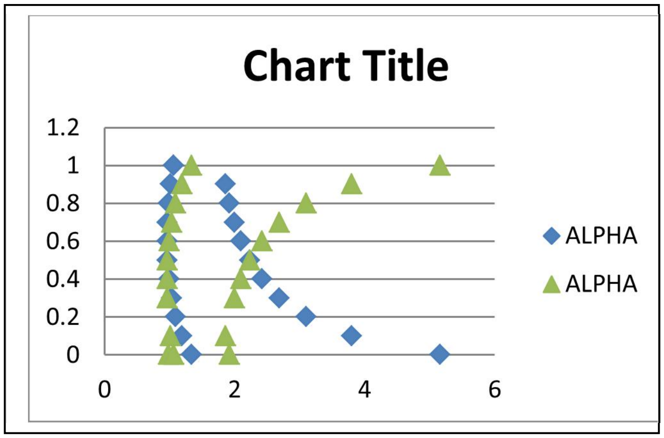


C. Pentagonal Intuitionistic Fuzzy Numbers With Graphical Representation

Take the arrival rate, service rate and vacation rate of Pentagonal Intuitionistic fuzzy number (PIFS) represented for eqn (1) by A= [1,2,3,4,5], B= [6,7,8,9,10], The length of the system of α is [1+α(2-1), 5− α(5-4)] and [6+ α(7-6), 10−α(10-9)],Where x = [1+α(2-1),5− α(5-4)] ,Y = [6+α(7-6), 10−α(10-9)]. Non-parameter value c=2.

Z(= x+y)={(1+6)+ α[(2-1)+(7-6)],(5+10)- α[(5-4)+(10-9)]}.

α-alpha	PIFS E(L) Lower limit	PIFS E(L) Upper limit
0	0.2402	0.6216
0.1	0.2492	0.5410
0.2	0.2572	0.4840
0.3	0.2644	0.4419
0.4	0.2716	0.4099
0.5	0.2779	0.3847
0.6	0.2843	0.3647
0.7	0.2908	0.3478
0.8	0.2983	0.3343
0.9	0.3053	0.3235
1	0.3133	0.3133



VII. CONCLUSION

By using the properties of Intuitionistic Fuzzy Set (IFS). We studied and performed IFM/IFG/1 vacation queueing system with Breakdown, Repair and Server Timeout in (Triangular, Trapezoidal & Pentagon) Intuitionistic Fuzzy set by using Intuitionistic α-cut method. Hence the system performance for the expected system length is of Intuitionistic Fuzzy natured for different IFS models.

REFERENCE

[1] Nagoor Gani and V.N. Mohamed, A method of Ranking Generalized Trapezoidal Intuitionistic Fuzzy Number, International Journal of Applied Engineering Research, ISSN 0973- 4562 Volume 10(2015) pp 25465-73.
[2] Atanasssov.K, (1986) Intuitionistic Fuzzy set, Fuzzy sets and Systems, Vol.20 pp. 87-96.
[3] A. Karfmann, (1975) Introduction to the Theory of Fuzzy Subsets, Vol. I, Academic Press, New York, 6.
[4] E.Ramesh Kumar and Y. Praby Loit (2016), A Study on Vacation Bulk Queueing Model with setup time and server timeout. IJCMS, 5(12), 81-89.
[5] Levy, Y. and Yechiale, U. (1975), Utilization of Idle Time in an M/G/1 Queueing System. Management Science, 22, 202-211.
[6] K.Ponnivalavan and T.Pathinathan, Intuitionistic Pentagonal fuzzy number, ARPN Journal of Engineering and Applied Sciences, Vol 10, No. 12(July 2015).
[7] K.Satish Kumar, K.Chandan and Y.Saritha (2018), Optimal Strategy Analysis of N-Policy $M/E_k/1$Vacation Queueing System with Server Start-Up and Time-Out. Journal of emerging Technologies and innovative Research, Volume 5, Issue 10, PP: 380-385.
[8] Y.Saritha, K.Satish Kumar and K.Chandan (2017), $M^x/G/1$ Vacation Queueing System with Server Timeout. International Journal of Statistics and Applied Mathematics, 2(5); 131-135.
[9] Y. Saritha, K. Satish Kumar, K. Chandan, G. Sridhar (2018). A study on N-policy FM/FG/1 vacation queuing system with server timeout in triangular, trapezoidal and pentagon fuzzy numbers using α-cuts. International Journal of Advance Research, Ideas and Innovations in Technology, 4(2).
[10] Y.Saritha, K.Satish Kumar and K.Chandan (2018), M/G/1 Vacation Queueing System with breakdown, repair and Server Timeout. Journal of emerging Technologies and innovative Research, vol 5, issue 2, 2349-5162.