iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 4 **Issue:** I **Month of publication:** January 2016

DOI:

International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Review on the Effect of Uniaxial Forces on Moony Rivline Hyperelastic Material Model

Prashant P Nimbalkar[1], Dr.Suhas P Deshmukh[2]
[1]M. E. (Design) Mechanical Engg. Department, SAOE, Kondhwa, Pune.
[2]Professor Mechanical Engg. Department, SAOE, Kondhwa, Pune.

Abstract—***Analytical material models do not entirely describe the stress-strain relationship in a material under all loading conditions. Therefore, experimental data needs to be created such that a reasonable material model may be selected to define material behaviour pertinent to the application of interest. Loading variables such as strain rate, strain direction, maximum strain, relaxation, cyclic behaviour and plasticity are outlined to provide a context for the design of experiments needed to define hyperelastic and plastic material models based on the needs of the application. The work presents a review of some problems associated with the simulation of hyperelastic fabrics using Finite Element Analysis. The precision and accuracy of experimentally measured material properties are critical for correct fabric modelling. In general, a combination of uniaxial tension (compression), biaxial tension, and simple shear is required for the characterization of an hyperelastic material. However, the use of these deformation tests to obtain the mechanical properties of a fabric may be complicated and also expensive. A methodology for characterising the fabric employing one single experimental test is presented and the error induced by this approximation is determined. The effects of experimental limitations on the characterization of material and the pros and cons of choosing a constitutive model are discussed.***
Keywords- Hyperelastic, FEA, Moony Rivline, Non linear FEA, Stress- strain etc.

I. INTRODUCTION

The principal problem in the Elasticity Theory is to find the relation between the stress and the strain in a body under certain forces. Hooke's Law is applied when the strains are small. However at large deformations, new expressions to characterize the behaviour of materials like rubber are required. Hyperelasticity is the capability of a material to experience large elastic strain due to small forces, without losing its original properties. A hyperelastic material has a nonlinear behaviour, which means that its answer to the load is not directly proportional to the deformation. Several woven fabrics like Lycra have hyperelastic behaviour and are widely employed by textile companies.

Performing a finite element analysis (FEA) on a hyperelastic material is difficult due to nonlinearity, large deformation, and material instability. This paper provides a brief review of the hyperelastic theory and discusses several important issues that should be addressed when using ANSYS.

\

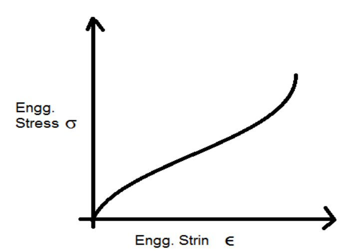

Fig 1-Nonlinear stress-strain relationship observed in rubber.

II. LITERATURE REVIEW

[1] Arun U Nair, Hubert Lobo and Anita M Bestelmeyer, perform review on the experimental procedures for characterizing the mechanical behaviour of hyperelastic materials with an emphasis on the equivalency between different tests and estimation of tear strength.

[2] Kurt Miller, Ann Arbo, they perform on the stress-strain relationship in a material under all loading conditions experimental data needs to be created such that a reasonable material model may be selected to define material behaviour pertinent to the application of interest. Loading variables such as strain rate, strain direction, maximum strain, relaxation, cyclic behaviour and plasticity are

Volume 4 Issue I, January 2016
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

outlined to provide a context for the design of experiments needed to define hyperelastic and plastic material models based on the needs of the application.

[3] Xiao-Yan Gong and Riyad Moe are found that the finite element analysis (FEA) on a hyperelastic material is difficult due to nonlinearity, large deformation, and material instability. This paper provides a brief review of the hyperelastic theory and discusses several important issues that should be addressed when using ANSYS.

[4] E. Boudaia and L. Bousshine basically they studied the use of elastomer properties (large deformations, damping) in industries such as aerospace, automotive, construction and civil engineering or even the entertainment industry.

[5] Mustafa Resit Usal, Melek Usal & Ahmet Kabul in his research they perform the constitutive equation has been obtained that characterizes nonlinear mechanical behavior of fiber reinforced hyperelastic material, whose matrix material is an isotropic.

[6] Manuel J. Garcia R, Oscar E. Ruiz S they focused among the elastic fabrics, there is a whole range of polymers (rubbers) that can be modelled with hyperelastic constitutive equations.

III. HYPERELASTIC MATERIALS

The relationship between stress and deformation is explained by a constitutive equation. The most general way by which represent a linear relation between the stress tensor σ_{ij} and the strain tensor ϵ_{kl} is given by Hooke's law.

$$\sigma_{ij} = C_{ikl}\,\epsilon_{kl}$$

Where, σ_{ij} are components of the Cauchy stress tensor, ϵ_{kl} are components of the strain tensor and C_{ijkl} is called the elastic constants tensor of fourth order.

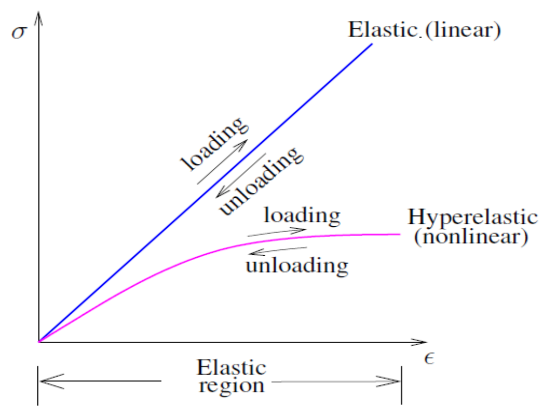

Fig 2: Stress-Strain Response

The principal problem in the elasticity theory is to find the relation between the stress and the strain in a body under specific forces. Hooke's Law is applied to the system when the strains are small. Hyperelasticity is the capability of a material to experience large elastic strain due to small forces applied, without missing its original properties. A hyperelastic material has a nonlinear behaviour, which means that its answer to the load is not directly proportional to the deformation.

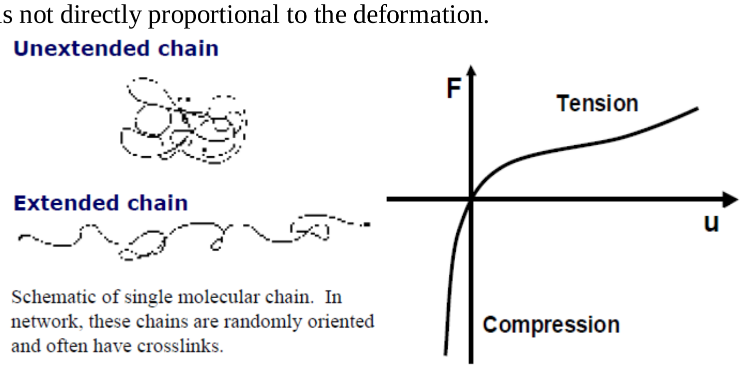

Fig 3: Rubber behaviour

The material, which show large deformation such rubbers are called elastomers. Elastomers involve natural and synthetic rubbers, which are amorphous and are comprised of long molecular chains. The molecular chains are highly twisted, coiled, and randomly oriented in an unreformed state. These chains become partially straightened and untwisted under a tensile load. Upon removal of the load, the chains revert back to its original configuration. Strengthening of the rubber is achieved by forming crosslink's between molecular chains through a vulcanization process.

IV. HYPERELASTIC MATERIAL MODELS

This section introduces the concept of hyperelasticity and mentions some aspects important about hyperelastic models. Hyperelasticity is the capability of a material to experience large elastic strain due to small forces, without losing its original properties. An hyperelastic material has a nonlinear behaviour, which means that its deformation is not directly proportional to the load applied. An elastic material is hyperelastic if there is a scalar function, denoted by W = W(E) : $R^{n\times n}\rightarrow R$ called strain energy function (or stored energy function), such that

$$S_{ij} = \frac{\partial W(E)}{\partial E_{ij}} = 2\frac{\partial W(E)}{\partial C_{ij}}$$

Where S_{ij} are the components of the second Piola-Kirchhoff stress tensor, W is the strain energy function per unit volume unreformed, E_{ij} are the components of the Green–Lagrange strain tensor and C_{ij} are the components of the right Cauchy–Green strain tensor. By algebraic manipulation of the components of Cauchy (true) stress tensor [σ] can be determinate.

$$\sigma_{ij} = -p\delta_{ij} + 2\frac{\partial W(E)}{\partial I_1}C_{ij} - 2\frac{\partial W(E)}{\partial I_2}\frac{1}{c_{ij}}$$

Where I1 and I2 are the principal invariants of the [C] tensor. The three strain invariants of the strain tensor can be expressed as:

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$

$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_1^2\lambda_3^2$$

$$I_3 = \lambda_1^2\lambda_2^2\lambda_3^2$$

The strain energy functions of hyperelastic constitutive models are

Moony-Rivlin
Neo-hookean
Yeoh
Arruda-Bovce
Gent
ogden
Hyperfoam
Blatz-Ko

They are expressed as a function of strain invariants I1, I2, I3 or in terms of the principal stretches λ_1, λ_2 and λ_3 of strain tensor. In most of the models, in order to deduce the strain energy functions, it has been assumed, that the material is isotropic and with constant volume (isometric deformation $\lambda_1\lambda_2\lambda_3$ = 1). Unless indicated otherwise, hyperelastic materials are assumed to be nearly or purely incompressible

A. Polynomial model

The polynomial model offered here in the compressible form, based on the 1st and the 2nd invariant 1 I and 2 I of the deviatoric Cauchy-Green tensor, that is:

$$W = \sum_{i+j=1}^{N} C_{ij}(\bar{I_1} - 3)^i(\bar{I_2} - 3)^j + \sum_{i=1}^{N}\frac{1}{D_i}(J_{el} - 1)^{2i}$$

Where:

W = the strain energy potential (or strain energy density), that is the strain per unit of reference volume

J_{el} = The elastic volume ratio

$\bar{I_1}$ = The first invariants of the deviatoric strain,

$\bar{I_2}$ = The second invariants of the deviatoric strain

C_{ij} and D_i = Material constant

N = A positive determining the number of terms in the strain energy function (N = 1,2,3)

C_{ij} = Describes the shear behavior of the material

D_i= Introduces compressibility and is set equal to zero for fully incompressible materials

This model of strain energy function is usually used in modeling the stress-strain behavior of filled elastomers, with four to five terms.

B. Hyperelastic Models

1) Mooney–Rivlin Model: This material model is assumed to be incompressible and initially isotropic. Mooney and Rivlin proposed a strain energy function W as an infinite series in powers of (I1 − 3) and (I2 − 3) of the form,

$$W(I_1,I_2) = \sum_{ij=0}^{n\to\infty} C_{ij}\,(\overline{I_1} - 3)^i(\overline{I_2} - 3)^j + \sum_{k=1}^{m} D_k(J-1)^{2k}$$

Where,

W = strain energy potential

$\overline{I_1}$= First deviatoric strain invariant

$\overline{I_2}$= Second deviatoric strain invariant

C_{ij} And D_k = material constants.

Where C_{ij} is constants.

For example, the Mooney-Rivlin form with two parameters is

$$W = C_{10}(\overline{I_1} - 3) + C_{01}(\overline{I_2} - 3) + \frac{1}{d}(J-1)^2$$

A typical invariants based rubber material model is Moony-Rivilin model with the strain energy density function expressed as

$$W(I_1,I_2) = C_1(I_1 - 3) + C_2(I_2 - 3)$$

Where C_1 and C_1 are constant fitted from experimental data.

The Mooney-Rivlin model is simple and straight forward. However, experiments by Obata, Kawabata and Kawai showed that C_1 and especially C_2 vary with both I_1 and I_2 instead of staying constant. Further experiments demonstrated that the Mooney- Riviln model only works well with the rubber material for strains up to 200%. Hence, Mooney- Riviln enhanced the expression for strain energy function W as,

$$W(I_1,I_2) = \sum_{ij=0}^{n\to\infty} C_{ij}\,(\overline{I_1} - 3)^i(\overline{I_2} - 3)^j$$

2 Parameter Mooney-Rivlin

$$W = C_{10}(\overline{I_1} - 3) + C_{01}(\overline{I_2} - 3) + \frac{1}{d}(J-1)^2$$

Where

W = strain energy potential

$\overline{I_1}$= First deviatoric strain invariant

$\overline{I_2}$= Second deviatoric strain invariant

C_{10} and C_{01}= material constants characterizing the deviatoric deformation of the material

d = material incompressibility parameter

The initial shear modulus is defined as:

$$\mu = 2(C_{10} + C_{01})$$

and the initial bulk modulus is defined as:

$$K = \frac{2}{d}$$

where:

$$d = \frac{(1-2v)}{(C_{10}+C_{01})}$$

Volume 4 Issue I, January 2016
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

3 Parameter Mooney-Rivlin

$$W = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) + C_{11}(\bar{I}_1 - 3)(\bar{I}_2 - 3) + \frac{1}{d}(J-1)^2$$

5 Parameter Mooney-Rivlin

$$W = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) + C_{20}(\bar{I}_1 - 3)^2 + C_{11}(\bar{I}_1 - 3)(\bar{I}_2 - 3) + C_{02}(\bar{I}_2 - 3)^2 + \frac{1}{d}(J-1)^2$$

9 Parameter Mooney-Rivlin

$$W = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) + C_{20}(\bar{I}_1 - 3)^2 + C_{11}(\bar{I}_1 - 3)(\bar{I}_2 - 3) + C_{02}(\bar{I}_2 - 3)^2 + C_{30}(\bar{I}_1 - 3)^2 + C_{21}(\bar{I}_1 - 3)^2(\bar{I}_2 - 3) + C_{12}(\bar{I}_1 - 3)(\bar{I}_2 - 3)^2 + C_{03}(\bar{I}_2 - 3)^3 + \frac{1}{d}(J-1)^2$$

2) Neo–Hookean Model

$$W(I_1, I_2) = \sum_{ij=0}^{n\to\infty} C_{ij}\,(\bar{I}_1 - 3)^i(\bar{I}_2 - 3)^j$$

By taking only first term,

$$W(I_1) = C_{10}(\bar{I}_1 - 3)$$

3) Ogden Model: Ogden deduced a hyperelastic constitutive model for large deformations of incompressible rubber-like solids. The strain energy is expressed as a function of principal stretches as:

$$W(I_1) = \sum_{r=0}^{n\to\infty} \frac{\mu_r}{\alpha_r}({\lambda_1}^{\alpha_r} + {\lambda_2}^{\alpha_r} + {\lambda_3}^{\alpha_r} - 3)$$

with μ_r and $\alpha_r$as material constants that can be determined by experimental tests.

4) Yeoh Model: The Yeoh model depends only on the first strain invariant I_1 . The Strain energy function W is obtained by,

$$W(I_1) = \sum_{n=1}^{i} C_{i0}(I_1 - 3)^i$$

It applies to the characterisation of elastic properties of carbon-black-filled vulcanized rubber.

5) Arruda–Boyce Model

The constitutive model for the large stretch behavior of elastic rubber materials is presented by Arruda and Boyce in. The strain-stress function is based on an eight-chain representation of the macromolecular structure of the rubber:

$$W(I_1) = G\left[\frac{1}{2}(I_1 - 3) + \frac{1}{20N}(I_1^2 - 9) + \frac{11}{1050N^2}(I_1^3 - 27)\right] + G\left[\frac{19}{7000N^3}(I_1^4 - 81) + \frac{519}{673750N^4}(I_1^5 - 243)\right] + \cdots$$

where the module $G = nK\theta$, where n is the chain density , k is Boltzmann's constant, N is the number of rigid links composing a single chain, and θ is the temperature . This model is also known as the eighth-chain model since it developed from a representative volume element where eight spring leave out from the center of cube to its corners.

6) Gent Model: The strain energy density in the Gent model is a simple logarithmic function of the first invariant I_1, involves two material constants, and the shear modulus μ and I_m which measures a limiting value for I_1-3. Gent proposed the strain energy density

$$W(I_1) = \frac{-\mu}{2} I_m \ln\left[1 - \frac{I_1 - 3}{I_m}\right]$$

7) Blatz–Ko Model: An application of finite elastic theory to the deformation of rubbery materials is given in. Incompressibility is not assumed. The strain energy function is cast in terms of the constants ν, μ, and f, which can be determined experimentally,

$$W(J_1, J_2, J_3) = \frac{\mu f}{2}\left[J_1 - 3 + \frac{1-2\nu}{\nu}\left\{J_3^{\frac{-2\nu}{(1-2\nu)-1}}\right\}\right] + \frac{\mu(1-f)}{2}\left[J_2 - 3 + \frac{1-2\nu}{\nu}\left\{J_3^{\frac{2\nu}{(1-2\nu)-1}}\right\}\right]$$

where ν is similar to the Poisson's ratio, μ is the shear modulus, and f is a material constant. Also, a new set of invariants is defined by,

$$J_1 = \sum \lambda_i^2\,,\ J_2 = \sum \lambda_i^{-2} \text{ and } J_3 = \prod \lambda_i$$

Accurate modeling of hyperelastic materials requires material properties data measured to large strains under different states of stress. Model coefficients are calculated from mechanical test data using least squares fit routines in the FE software.

Table 1 Comparison of models

Hyperelastic Model		Fit Test Data	Least Squared Error %
Mooney-Rivlin	2Parameters	-	60
	3 Parameters	Acceptable	15
	5 Parameters	Good	1
	9 Parameters	Best	0.01
Ogden	Order 1	-	50
	Order 2	-	54
	Order 3	-	54
Neo-Hookean		-	65
Arruda-Boyce		Acceptable	30
Yeoh	Order 1	-	60
	Order 2	-	40
	Order 3	Good	5
Blatz-Ko		-	200

From the above modes the Moony-Rivlin model is mostly used for the analysis for their acceptable results. The least squared error obtained from the Moony-Rivilin model is very less than the other models. Moony-Rivilin model of 9 parameter which is having best results to use for practical purpose but it is complicated to use. For that purpose 3 parameter and 5 parameter mostly preferred. Their disadvantage is that the material parameters must be obtained by experiments and they are not physically-based parameters. The fitting method can be complicated if the number of parameters is large.

V. CONCLUSIONS

This study concluded with following outcomes –

A. Hyperelastic materials are used to compute the large deformation response of nonlinear elastic materials (i.e. polymers, foams, and biological materials).

B. In this, we have analysed Mooney-Rivlin Materials constants. Mooney-Rivlin Material C_{10},C_{01} by using 6 deformation modes.

C. We determine principle stresses using Equibiaxial compression (Uniaxial Tension), Equibiaxial Tension (Uniaxial Compression), Pure shear.

D. Resultant values are taken as Cumulative values and the errors in the resultant values are minimised using Least-square fit Analysis.

According to this analysis, we can say that materials having high stress-strain values, mooney-rivlin model can be used to determine the material constants for hyperelastic materials.

REFERENCES

[1] Arun U Nair, Hubert Lobo and Anita M Bestelmeyer, "Characterization of Damage in Hyperelastic Materials Using Standard Test Methods and Abaqus", SIMULIA Customer Conference, 2009.
[2] Kurt Miller, Ann Arbo, "Experimental Loading Conditions Used to Implement Hyperelastic and Plastic Material Models." Axel Products, Inc.
[3] Xiao-Yan Gong and Riyad Moe, "On stress analysis for a hyperelastic material", Sulzer Carbomedics Inc.Austin, Te
[4] E. Boudaia, L. Bousshine, " Modeling of Large Deformations of Hyperelastic Materials", International Journal of Material Science Vol. 2 Iss. 4, December 2012.
[5] Mustafa Resit Usal, Melek Usal & Ahmet Kabul, "A Mathematical Model for fiber rainforced Hyperelastic Material & Result with FEM", Journal of Applied Science 5 (4); 1617-1631,2005 ISSN 1812-5654
[6] Matthew Wadham-Gagnon, Pascal Hubert, Christian Semler, Denys Lavoie, "Hyperelastic modeling of rubber in commercial finite element software", ANSYS™
[7] Cambridge Polymer Group, "Mooney Rivlin Experiments", Cambridge Polymer Group, Inc.(2013).
[8] I-Shih Liu, "A note on the Mooney-Rivlin material model", Instituto de Matematica, 21945-970
[9] Amir Khalilollahi,Brian P.Felker,Justin W. Wetzel, "Non-Linear Elastomeric Spring Design Using Mooney-Rivlin Constants", Pennsylvania State University, The Behrend College.
[10] N.Sombatsompop, "Practical Use of the Mooney-Rivlin equation for determination of degree of crosslinking of swollen NR vulcanisates." School of Energy and materials, king Mongkut's University of Technology Thonburi, Bangmod, Bangkok 10140, Thailand. J.Sci.Soc.Thailand,2 (1998) 199-204.