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# Special pairs of rectangles and sphenic number 

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#### Abstract

We present pairs of Rectangles, such that in each pair, the sum of their areas is two times the Sphenic number minus one. Also we present the number of pairs of primitive and non-primitive rectangles. Keywords-Pairs of Rectangles, Sphenic number, Primitive and non-primitive Rectangles.


## I. INTRODUCTION

Number theory is a broad and diverse part of Mathematics that developed from the study of the integers. Diophantine equations are numerously rich because of its variety. Diophantine problems have fewer equations that unknown variables and involve finding solutions in integers. There is no universal method available to know whether a Diophantine equation has a solution or finding all solutions if it exists. In this context one may refer [1-4].
A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Nasty number, Dhuruva numbers and Jarasandha numbers. These numbers have presented in [5-8]. In [9-14], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are presented.
In this communication, we search for pairs of Rectangles, such that in each pair, the sum of their areas is two times the sphenic numbers minus one, where the Sphenic number which are divisible by three satisfies this relation.

## II. BASIC DEFINITIONS

## A. Definition1: Sphenic Number

Sphenic number is a positive integer that is the product of three distinct prime numbers.

## B. Definition 2 : Primitive Rectangle

A primitive rectangle is a rectangle having integer sides $a$ and $b$ such that $\operatorname{gcd}(a, b)=1$.

## III.METHOD OF ANALYSIS

Let $R_{1}, R_{2}$ be two distinct Rectangles with generators $u, w(u>w>0)$ and $v, w(v>w>0)$ respectively.
Let $A_{1}, A_{2}$ be the areas of $R_{1}, R_{2}$ such that

$$
A_{1}+A_{2}=2 \text { times the Sphenic Number }-1 .
$$

where the Sphenic number which are divisible by 3 satisfies the above relation.
The above relation which leads to the equation

$$
u^{2}+v^{2}-2 w^{2}=2 \text { Times the Sphenic Number }-1 .
$$

After performing numerical computations we have presented below the values of $u, v, w$ and the corresponding sphenic numbers. Here we present 2-digit and 3-digit sphenic numbers which satisfies the above relation

TABLE I
RECTANGLE PAIRS AND SPHENIC NUMBERS

| $u$ | $v$ | $w$ | $A_{1}$ | $A_{2}$ | $\frac{A_{1}+A_{2}+1}{2}=$ Sphenic Number |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 10 | 9 | 40 | 19 | 30 |
| 15 | 14 | 13 | 56 | 27 | 42 |

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| 23 | 22 | 21 | 88 | 43 | 66 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 27 | 26 | 25 | 104 | 51 | 78 |
| 35 | 34 | 33 | 136 | 67 | 102 |
| 36 | 35 | 34 | 140 | 69 | 105 |
| 39 | 38 | 37 | 152 | 75 | 114 |
| 47 | 46 | 45 | 184 | 91 | 138 |
| 56 | 55 | 54 | 220 | 109 | 165 |
| 59 | 58 | 57 | 232 | 115 | 174 |
| 63 | 62 | 61 | 248 | 123 | 186 |
| 66 | 65 | 64 | 260 | 129 | 195 |
| 78 | 77 | 76 | 308 | 153 | 231 |
| 83 | 82 | 81 | 328 | 163 | 246 |
| 86 | 85 | 84 | 340 | 169 | 255 |
| 87 | 86 | 85 | 344 | 171 | 258 |
| 92 | 91 | 90 | 364 | 181 | 273 |
| 95 | 94 | 93 | 376 | 187 | 282 |
| 96 | 95 | 94 | 380 | 189 | 285 |
| 107 | 106 | 105 | 424 | 211 | 318 |
| 116 | 115 | 114 | 460 | 229 | 345 |
| 119 | 118 | 117 | 472 | 235 | 354 |
| 120 | 119 | 118 | 476 | 237 | 357 |
| 123 | 122 | 121 | 488 | 243 | 366 |
| 134 | 133 | 132 | 532 | 265 | 399 |
| 135 | 134 | 133 | 536 | 267 | 402 |
| 143 | 142 | 141 | 568 | 283 | 426 |
| 144 | 143 | 142 | 572 | 285 | 429 |
| 146 | 145 | 144 | 580 | 289 | 435 |
| 147 | 146 | 145 | 584 | 291 | 438 |
| 156 | 155 | 154 | 620 | 309 | 465 |
| 159 | 158 | 157 | 632 | 315 | 474 |
| 162 | 161 | 160 | 644 | 321 | 483 |
| 167 | 166 | 165 | 664 | 331 | 498 |
| 179 | 178 | 177 | 712 | 355 | 534 |
| 195 | 194 | 193 | 776 | 387 | 582 |
| 203 | 202 | 201 | 808 | 403 | 606 |
| 206 | 205 | 204 | 820 | 409 | 615 |
| 207 | 206 | 205 | 824 | 411 | 618 |
| 210 | 209 | 208 | 836 | 417 | 627 |
| 215 | 214 | 213 | 856 | 427 | 642 |
| 216 | 215 | 214 | 860 | 429 | 645 |
| 218 | 217 | 216 | 868 | 433 | 651 |
| 219 | 218 | 217 | 872 | 435 | 654 |
| 227 | 226 | 225 | 904 | 451 | 678 |
| 236 | 235 | 234 | 940 | 469 | 705 |
| 255 | 254 | 253 | 1016 | 507 | 762 |
| 260 | 259 | 258 | 1036 | 517 | 777 |
| 263 | 262 | 261 | 1048 | 523 | 786 |

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| :--- | :--- | :--- | :--- | :--- | :--- |
| 275 | 274 | 273 | 1096 | 547 | 822 |
| 279 | 278 | 277 | 1112 | 555 | 834 |
| 288 | 287 | 286 | 1148 | 573 | 861 |
| 296 | 295 | 294 | 1180 | 589 | 885 |
| 299 | 298 | 297 | 1192 | 595 | 895 |
| 300 | 299 | 298 | 1196 | 597 | 897 |
| 302 | 301 | 300 | 1204 | 601 | 903 |
| 303 | 302 | 301 | 1208 | 603 | 906 |
| 306 | 305 | 304 | 1220 | 609 | 915 |
| 315 | 314 | 313 | 1256 | 627 | 942 |
| 320 | 319 | 318 | 1276 | 637 | 957 |
| 324 | 323 | 322 | 1292 | 645 | 969 |
| 327 | 326 | 325 | 1304 | 651 | 978 |
| 330 | 329 | 328 | 1316 | 657 | 987 |

Thus it is seen that for each pair, the sum of the areas of a Rectangle is equal to twice the sphenic number minus one.
From the table 1, all the pairs of Rectangles are primitive. In each of the primitive pairs, one of the Rectangles is primitive and the other is non-primitive Rectangle.

## IV.CONCLUSION

One may search for the connections between the pairs of Rectangles and the other sphenic numbers of higher order.

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